VOL. 6, NO. 7, JULY 2011 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

INVESTIGATION OF AIRFOIL PROFILE DESIGN USING REVERSE ENGINEERING BEZIER CURVE

J. Fazil and V. Jayakumar
School of Mechanical and Building Sciences, VIT University, Vellore, Tamil Nadu, India
E-Mail: janifazil@gmail.com

ABSTRACT

Though it is easier to model and create an airfoil profile in CAD environment using camber cloud of points, after the creation of vane profile it is very difficult to change the shape of profile for analysis or optimization purpose by using cloud of points. In this paper, we investigate and describe the creation of airfoil profile in CAD (CATIA) environment using the control point of the camber profile. By means of changing the values of control points, the shape of the profile can be easily changed and also the design of the cambered airfoil is established without affecting the basic airfoil geometry. In this paper, the Quintic Reverse Engineering of Bezier curve formula is used to find the camber control points from the existing camber cloud of points.

Keyword: airfoil, quintic reverse engineering bezier curve, camber, NACA -4 digit series, CATIA.

1. INTRODUCTION

Each airfoil comprises an inner airfoil surface and outer airfoil surface. The inner airfoil and outer airfoil surface define a vane airfoil thickness [1]. Airfoil includes a leading edge positioned along a first inner and outer airfoil along a first airfoil surface junction, a trailing edge positioned along a second inner and outer surface junction. The airfoil inner and outer airfoil surface are specially configured to provide a airfoil camber line, positioned between the two airfoil profile and extending along the length of the airfoil that is substantially length of the camber line [2].

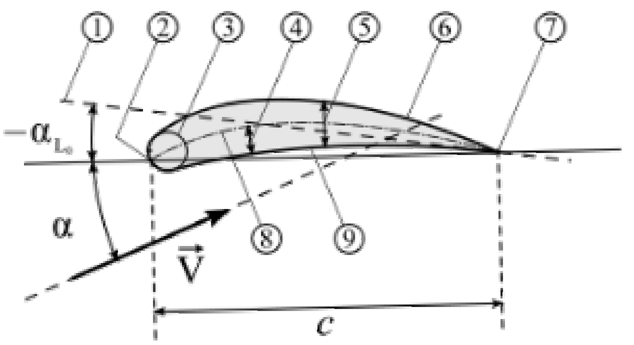


Figure-1. Profile geometry.

1: Zero lift line; **2:** Leading edge; **3:** Nose circle; **4:** Camber; **5:** Max thickness; **6:** Upper (outer) surface; **7:** Trailing edge; **8:** Camber mean-line; **9:** Lower (inner) surface; **10:** C-Chord length.

The "NACA airfoils" are airfoil shapes for aircraft wings developed by the "National Advisory Committee for Aeronautics (NACA)" [3]. The shape of the NACA airfoils is described using a series of digits following the word "NACA." The parameters in the numerical code can be entered into equations to precisely generate the cross-section of the airfoil and calculate its properties.

The NACA has different series of digit airfoil are [3]

(i) 4 Digit series airfoil; (ii) 5 Digit series airfoil;
(iii) 7 Digit series airfoil; (iv) 8 Digit series airfoil

We are taking the general example creation of standard airfoil like the NACA four-Digit airfoils sections are airfoil shapes for aircraft. The NACA four-digit wing sections define the profile by one digit describing maximum camber as percentage of the chord [4] NACA 2412.

(a) One digit describing the distance of maximum camber from the airfoil leading edge in tens of percents of chord.
(b) Two digits describing maximum thickness of the airfoil as percent of the chord

For example, the NACA 2412 airfoil [5] has a maximum camber of 2% located 40% (0.4 chords) from the leading edge with a maximum thickness of 12% of the chord. Four-digit series airfoils by default have maximum thickness at 30% of the chord (0.3 chords) from the leading edge.

Thickness equation

$$y=\frac{t}{0.2}c\left[\begin{array}{l}0.2969\sqrt{\frac{x}{c}}-0.1260\left(\frac{x}{c}\right)-0.3516\left(\frac{x}{c}\right)^{2}\\+0.2843\left(\frac{x}{c}\right)^{3}-0.1015\left(\frac{x}{c}\right)^{4}\end{array}\right] \quad (1)$$

The thickness equation (1), for example, is actually based on empirical studies conducted by NACA [5] back in the 1930s. Until that time, airfoil design was really little more than magic. Early aircraft designers had experimented with a number of different shapes and just happened to stumble across a few that worked very well. No one really understood why some shapes worked and others didn't, so there was no theory to guide designers in selecting the best airfoils for a given application. Picking the right shape was a matter of luck.

VOL. 6, NO. 7, JULY 2011 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

Researchers at NACA [5] were very curious about this subject, and one of the first major efforts undertaken by the organization after its founding was to bring some logic and reason to airfoil design. They started by trying to understand why some airfoils worked well and others did not. In the process, they realized that there were a lot of common features shared by the airfoils that were most successful. These features could be reproduced by a simple combination of a mean camber line and a thickness distribution. The mean camber line (or simply "mean line") is a line running along the center of the airfoil. It can be thought of as the average of the upper surface and lower surface of the airfoil shape. The thickness distribution defines how thick the airfoil is at any point along its length above and below that mean line.

2. DEFINITION OF NACA-0011 AIRFOIL

The NACA 0011 [5] airfoil is symmetrical, the 00 indicating that it has no camber. The 15 indicates that the airfoil has a 15% thickness to chord length ratio: it is 15% as thick as it is long. The thickness and camber formula (1 and 4) using the creation NACA-0011 airfoil.

The Thickness formula (1) for the shape of a NACA 0011 foil, with "11" being replaced by the percentage of thickness to chord is 0.11.

$$y=\frac{t}{0.2}c\left[\begin{array}{l}0.2969\sqrt{\frac{x}{c}}-0.1260\left(\frac{x}{c}\right)-0.3516\left(\frac{x}{c}\right)^2\\+0.2843\left(\frac{x}{c}\right)^3-0.1015\left(\frac{x}{c}\right)^4\end{array}\right]$$

Where,

- c is the chord length
- x is the position along the chord from 0 to c
- y is the half thickness at a given value of x (centerline to surface)
- t is the maximum thickness as a fraction of the chord (so 100 t gives the last two digits in the NACA 4-digit denomination) is 0.11.

The leading edge approximates a cylinder with a radius of:

$$r = 1.1019\ ct^2 \quad (2)$$

The below parabolic formula is used for finding the mean line of the aerofoil:

$$y_c=\frac{m}{p^2}\left(2px-x^2\right) \quad \text{From x=0 to x=p}$$

$$y_c=\frac{m}{(1-p)^2}\left[(1-2p)+2px-x^2\right] \quad \text{From x=p to x=c}$$

(3)

Where,

x = coordinates along the length of the airfoil, from 0 to c (which stands for chord, or length)

y = coordinates above and below the line extending along the length of the airfoil

These are either y_t for thickness coordinates or y_c for camber coordinates.

t = maximum airfoil thickness in tenths of chord (i.e. a 11% thick airfoil would be 0.11)

m = maximum camber in tenths of the chord

p = position of the maximum camber along the chord in tenths of chord.

Now the coordinates (x_U,y_U) of the upper airfoil surface, and (x_L,y_L) of the lower airfoil surface are:

$$x_U = x_L = x,\ y_U = +y \text{ and } y_L = -y \quad (4)$$

Deriving the above formula we get the cloud of points Coordinates of cambered airfoil, Upper Camber (x_U,y_U) and Lower Camber (x_L,y_L) in Equation (4). These are the formula behind the NACA 4-digit Series Profile Generator [6] (Figure-2).

Using the NACA 0011 aerofoil profile extract the cloud of points from NACA 4-Digit Series Profile Generator by giving the proper input [2]. The Input Variables are:

Maximum Camber: 0
Maximum Camber Position: 0
Thickness: 11
No of Streamwise Points: 40
PointsDistribution: 1
Points Size: 4

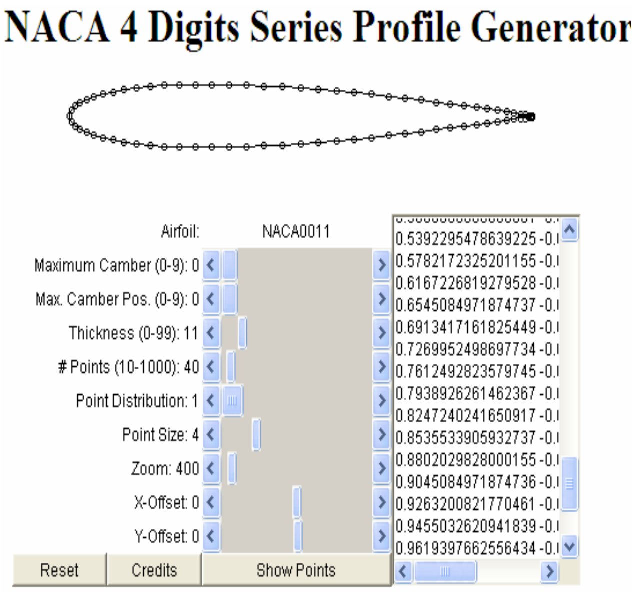


Figure-2. NACA Aerofoil profile Generator.

3. OUTPUT OF THE NACA 4-SERIES PROFILE GENERATOR

Output of the NACA 4-Series [6] Profile Generator-0011 (Figure-2) profile cloud points is calculated for Chord Length value is 1, After getting the Cloud of points Data, we multiply with our original Chord length Value, In this paper our Chord Length Value is 40mm and multiply with the parsed Cloud of points Data, We got the real scaled Cloud of points refer (Tables 1 and 2). In addition to that we need to divide the value 1 into 40 Division as shown in the Table-1- t division column.

Table-1. Upper camber cloud of points and 't' value.

X	Y	Z	t 1./40
0	**0**	**0**	**0**
0.246	0.495	0	0.025
0.552	0.728	0	0.051
0.978	0.949	0	0.076
1.522	1.157	0	0.102
2.179	1.351	0	0.128
2.947	1.529	0	0.153
3.819	**1.688**	**0**	**0.179**
4.791	1.828	0	0.205
5.857	1.946	0	0.230
7.011	2.042	0	0.256
8.244	2.116	0	0.282
9.55	2.166	0	0.307
10.92	2.194	0	0.333
12.34	2.2	0	0.35
13.81	**2.184**	**0**	**0.384**
15.33	2.149	0	0.410
16.87	2.095	0	0.435
18.43	2.025	0	0.461
20	1.941	0	0.487
21.56	1.844	0	0.512
23.12	1.737	0	0.538
24.66	1.622	0	0.564
26.18	**1.500**	**0**	**0.589**
27.65	1.374	0	0.615
29.07	1.245	0	0.641
30.45	1.115	0	0.666
31.75	0.986	0	0.692
32.98	0.859	0	0.717
34.14	0.737	0	0.743
35.20	0.620	0	0.769
36.18	**0.510**	**0**	**0.794**
37.05	0.408	0	0.820
37.82	0.317	0	0.846
38.47	0.237	0	0.871
39.02	0.170	0	0.897
39.44	0.116	0	0.923
39.75	0.077	0	0.948
39.93	0.054	0	0.974
40	**0**	**0**	**1**

Table-2. Lower camber cloud of point's value.

X	Y	Z
0	**0**	**0**
0.246	-0.49	0
0.552	-0.72	0
0.978	-0.94	0
1.522	-1.15	0
2.179	-1.35	0
2.947	-1.52	0
3.819	**-1.68**	**0**
4.791	-1.82	0
5.857	-1.94	0
7.011	-2.04	0
8.244	-2.11	0
9.55	-2.16	0
10.92	-2.19	0
12.34	-2.2	0
13.81	**-2.18**	**0**
15.33	-2.14	0
16.87	-2.09	0
18.43	-2.02	0
20	-1.94	0
21.56	-1.84	0
23.12	-1.73	0
24.66	-1.62	0
26.18	**-1.50**	**0**
27.65	-1.37	0
29.07	-1.24	0
30.45	-1.11	0
31.75	-0.98	0
32.98	-0.85	0
34.14	-0.73	0
35.20	-0.62	0
36.18	**-0.51**	**0**
37.05	-0.40	0
37.82	-0.31	0
38.47	-0.23	0
39.02	-0.17	0
39.44	-0.11	0
39.75	-0.07	0
39.93	-0.05	0
40	**0**	**0**

Total number of generating points in the output of the generator is only 80 points, Leading Edge created by using nose circle (2) and Trailing edge created by using angle. We need to find the Control points from calculated Cloud of points refer (Tables 1 and 2). For Quintic Reverse Engineering of Bezier Curve [7] formula, we need to pick only Six Cloud of point at Equidistance out of forty points of upper camber profile, similarly on lower camber profile please refer (Tables 1 and 2) picked points marked as BOLD letter and picked point plotted in the Tables 3 and 4.

ARPN Journal of Engineering and Applied Sciences

Table-3. Picked six cloud of points of upper camber and't' division.

Upper camber points				t
Des	X	Y	Z	
c0	0	0	0	0
c1 = f	3.8197	1.6885	0	0.1795 = u
c2 = g	13.819	2.1844	0	0.3846 = v
c3 = h	26.180	1.5003	0	0.5897= w
c4 = j	36.180	0.5102	0	0.7949 = r
c5	40	0.0462	0	1

Table-4. Picked six cloud of points of lower camber, 't' division.

Lower Camber points				t
Des	X	Y	Z	
c6	0	0	0	0
c7	3.819	-1.6885	0	0.1795 = u
c8	13.819	-2.1844	0	0.3846 = v
c9	26.180	-1.5003	0	0.5897 = w
c10	36.180	-0.5102	0	0.7949 = r
c11	40	-0.0462	0	1

4. QUINTIC BEZIER CURVE FORMULA

We created all input data for our Quintic Reverse Engineering Bezier Curve formula. In Bezier, the Control Points are the driving points to create a curve [8] .We need control points of the camber curve to create an airfoil in CATIA. We develop the Quintic Reverse Engineering of Bezier formula (6) from Quintic Bezier curve formula (5) to find out the control points from original camber cloud of points. We equally divide the 40 points Camber points' value into six Divisions refer (Tables 1, 2, 3 and 4). Using the Six Original camber point value, the Six Control point of the Camber Curve using formula (6) was find out.

Formula for Quintic Bezier Curve Formula for n=5 Points [8]:

$$B(t)=(1-t)^5*p0+5(1-t)^4*t*p1+10(1-t)^3*t^2*p2$$
$$+10(1-t)^2*t^3*p3+5(1-t)*t^4*p4+t^5*p5 \quad (5)$$

Where *p0,p1,p2,p3,p4* and *p5* are the Control points of the Bezier Curve and *c0,c1,c2,c3,c4* and *c5* are the original upper camber Point [9].

To define six original picked points and have a Bezier curve passing through all six origin points. Basically, given the upper profile six original points *c0,c1,c2,c3,c4* and *c5* , we will used find upper six control points of *p0, p1, p2, p3,p4,* and *p5* such that the Bezier curve calculated using points *p(i)*, will pass through the points *c(i)*.

Figure-3 and Table-3 shows that when't' is *zero*, the equation effectively collapses into just *p0*. When't' is *one*, there is no change in equation gives *p5*. When t is between zero and one, the resulting point lies on the curve itself, so iterating t from zero to one will give the Bezier curve. Since we know the curve will pass through *p0* and *p5*, we need to find *p1*, *p2*, *p3* and *p 4*.

Suppose we want the curve to pass through *p0* when t = 0, *f* when *t = u*, *g* when *t = v*, *h* when *t = w*, *j* when *t = r* and *p6* when *t =1*, where *f, g, h* and *j* are the points to be passed through. Next, we make sure that $0 < u, r < 1$ and u not equal to *r*. These conditions will ensure a solution can be found. Next, we substitute the desired points into the equation.

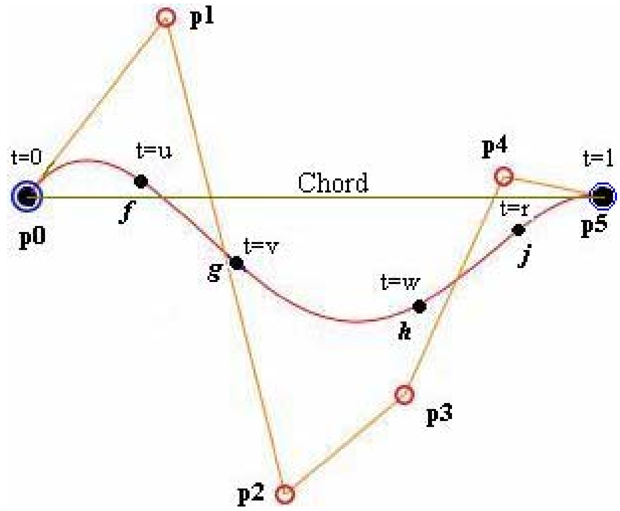


Figure-3. Example Quintic Bezier curve Figure.

5. QUINTIC REVERSE ENGINEERING OF BEZIER CURVE FORMULA

From Bezier curve formula (5), we developed a Quintic Reverse engineering Bezier curve formula.

$$B(t)-(1-t)^5*p0-t^5*p5=5(1-t)^4*t*p1$$
$$+10(1-t)^3*t^2*p2+10(1-t)^2*t^3*p3+5(1-t)*t^4*p4 \quad (6)$$

The above equations (6) simplifies into following steps we already know *t = u, v, w and r*

$$f=(1-u)^5*p0+5(1-u)^4*u*p1$$
$$+10(1-u)^3*u^2*p2+10(1-u)^2*u^3*p3$$
$$+5(1-u)*u^4*p4+u^5*p5$$

$$g=(1-v)^5*p0+5(1-v)^4*v*p1$$
$$+10(1-v)^3*v^2*p2+10(1-v)^2*v^3*p3$$
$$+5(1-v)*v^4*p4+v^5*p5$$

$$h=(1-w)^5*p0+5(1-w)^4*w*p1$$
$$+10(1-w)^3*w^2*p2+10(1-w)^2*w^3*p3$$
$$+5(1-w)*w^4*p4+w^5*p5$$

$$j=(1-r)^5*p0+5(1-r)^4*r*p1$$
$$+10(1-r)^3*r^2*p2+10(1-r)^2*r^3*p3 \quad (7)$$
$$+5(1-r)*r^4*p4+r^5*p5$$

The four equations often simplified into [9].

$$c=5(1-u)^4*u*p1+10(1-u)^3*u^2*p2$$
$$+10(1-u)^2*u^3*p3+5(1-u)*u^4*p4$$

$$d=5(1-v)^4*v*p1+10(1-v)^3*v^2*p2$$
$$+10(1-v)^2*v^3*p3+5(1-v)*v^4*p4$$

$$e=5(1-w)^4*w*p1+10(1-w)^3*w^2*p2$$
$$+10(1-w)^2*w^3*p3+5(1-w)*w^4*p4$$

$$k=5(1-r)^4*r*p1+10(1-r)^3*r^2*p2$$
$$+10(1-r)^2*r^3*p3+5(1-r)*r^4*p4 \quad (8)$$

Where c, d, e and k are:

$$c=f-\left((1-u)^5*p0+u^5*p5\right)$$
$$d=g-\left((1-v)^5*p0+v^5*p5\right)$$
$$e=h-\left((1-w)^5*p0+w^5*p5\right)$$
$$k=j-\left((1-r)^5*p0+r^5*p5\right) \quad (9)$$

This set of equations has a unique solution when $0 < u, r < 1$ and u not equal to r, and assuming *c*, *d*, *e* and *k* aren't both zero vectors. The equations have a unique solution because the *determinant* is not zero.

Let's transform the using the set of equations (8) and (9) into matrix form (10) before explaining what a determinant is AX=B.

$$\begin{pmatrix} 5(1-u)^4*u & 10(1-u)^3*u^2 & 10(1-u)^2*u^3 & 5(1-u)*u^4 \\ 5(1-v)^4*v & 10(1-v)^3*v^2 & 10(1-v)^2*v^3 & 5(1-v)*v^4 \\ 5(1-w)^4*w & 10(1-w)^3*w^2 & 10(1-w)^2*w^3 & 5(1-w)*w^4 \\ 5(1-r)^4*r & 10(1-r)^3*r^2 & 10(1-r)^2*r^3 & 5(1-r)*r^4 \end{pmatrix} * \begin{pmatrix} p1 \\ p2 \\ p3 \\ p4 \end{pmatrix} = \begin{pmatrix} c \\ d \\ e \\ k \end{pmatrix} \quad (10)$$

Next, we multiply the inverse of 4 by 4 matrix on the left sides of the equation and the determinant becomes- $X=A^{-1}B$ [9].

$AX = B$ then $X = A^{-1}B$

$X = A^{-1}*B$

$$\begin{pmatrix} p1 \\ p2 \\ p3 \\ p4 \end{pmatrix} = \text{inv} \begin{pmatrix} 5(1-u)^4*u & 10(1-u)^3*u^2 & 10(1-u)^2*u^3 & 5(1-u)*u^4 \\ 5(1-v)^4*v & 10(1-v)^3*v^2 & 10(1-v)^2*v^3 & 5(1-v)*v^4 \\ 5(1-w)^4*w & 10(1-w)^3*w^2 & 10(1-w)^2*w^3 & 5(1-w)*w^4 \\ 5(1-r)^4*r & 10(1-r)^3*r^2 & 10(1-r)^2*r^3* & 5(1-r)*r^4 \end{pmatrix} * \begin{pmatrix} c \\ d \\ e \\ k \end{pmatrix} \quad (11)$$

After calculating the inverse the 4x4 matrix [9] and multiplying with *c*, *d*, *e* and *k*, in equation (11) we got the Control points of the Upper Camber Control point are *p0 p1*, *p2*, *p3*, *p4* and *p5* (see Table-5) and similarly method is used to find the Lower Camber Control points are p6, *p7*, *p8*, *p9*, *p10* and *p11* (refer Table-6).

Table-5. Control point for upper camber profile.

Upper camber control point			
Des	**X**	**Y**	**Z**
p0	0.00000	0.000	0
p1	1.14034	2.44284	0
p2	11.1025	3.65375	0
p3	30.8809	1.13309	0
p4	39.9276	0.09712	0
p5	40.0000	0.04620	0

Table-6. Control point for lower camber profile.

Lower camber control point			
Des	**X**	**Y**	**Z**
p6	0.00000	0.00000	0
p7	1.14034	-2.44284	0
p8	11.1025	-3.65375	0
p9	30.8809	-1.13309	0
p10	39.9276	-0.09712	0

VOL. 6, NO. 7, JULY 2011 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

p11	40.0000	-0.04620	0

6. AIRFOIL DESIGN IN CATIA

After find out the control point of the both Camber profile using Quintic Reverse Engineering Bezier Curve formula, Using the Six Upper and Six Lower Camber Control Points create a airfoil profile in CATIA.

The Control Points are plotted in CATIA [10] using points coordinate option and after using the 3D curve option in free style shaper or Quick Surface Reconstruction Module in CATIA create a upper Camber and Lower Camber 3D Curve as shown in the Figure-4 and Figure-5.

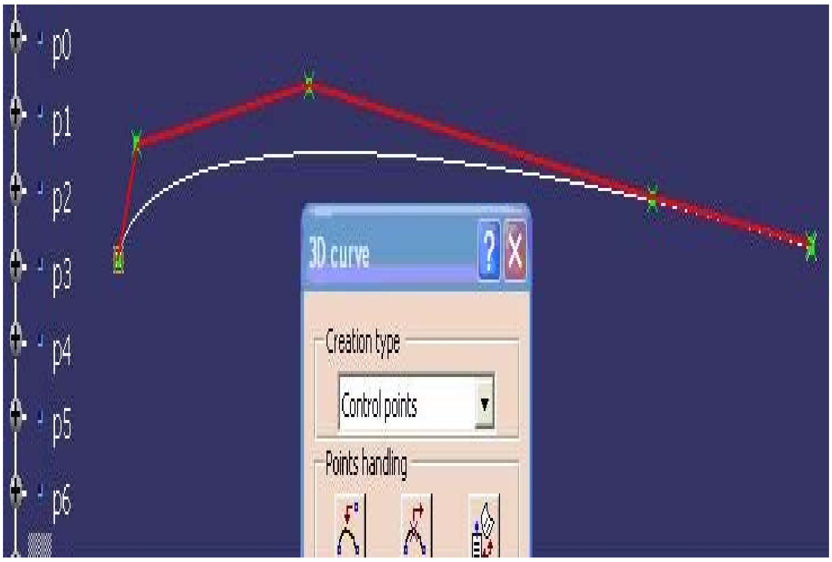


Figure-4. Upper camber profile 3D Curve creation.

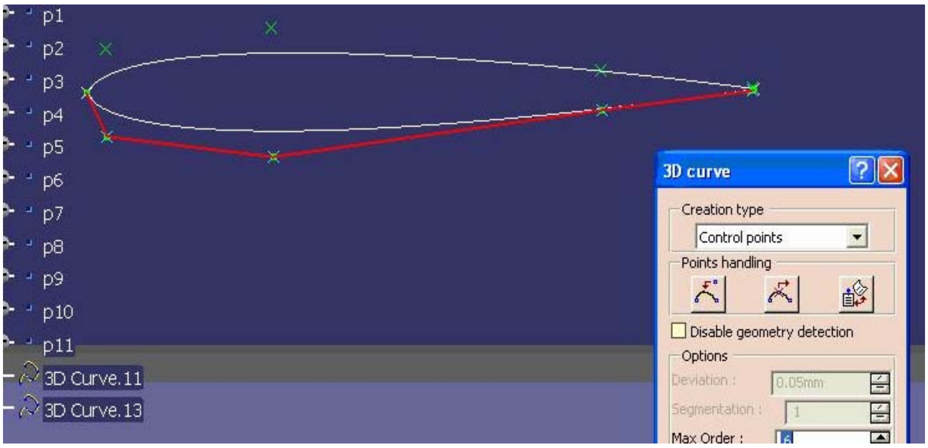
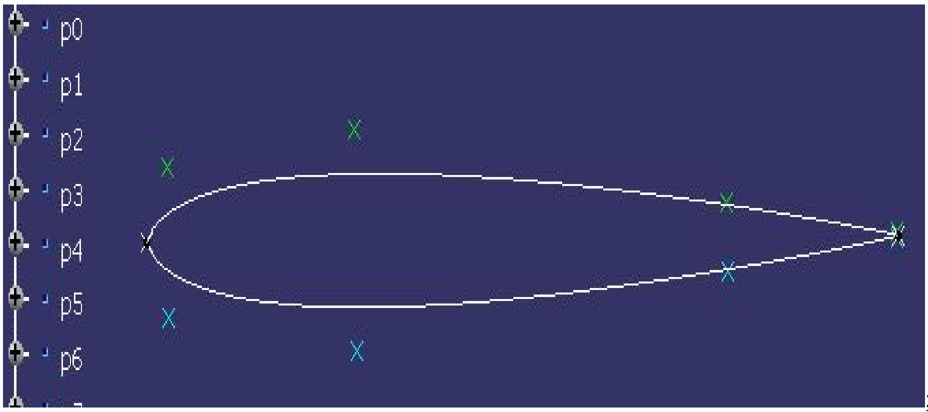


Figure-5. Lower camber profile 3D Curve creation.

Finally , created an airfoil profile design in CATIA using control points see Figure-6.



;

Figure-6. Final 2D airfoil profile design in CATIA.

7. VALIDATE THE AIRFOIL DESIGN

After created the airfoil profile in CATIA to validate the airfoil design with NACA 4-Digits profile Generator output data by means of read the cloud of points output data (Table-1 and Table-2) using digitized shape editor module and plot the cloud of points value in CATIA as shown in the Figure-7.

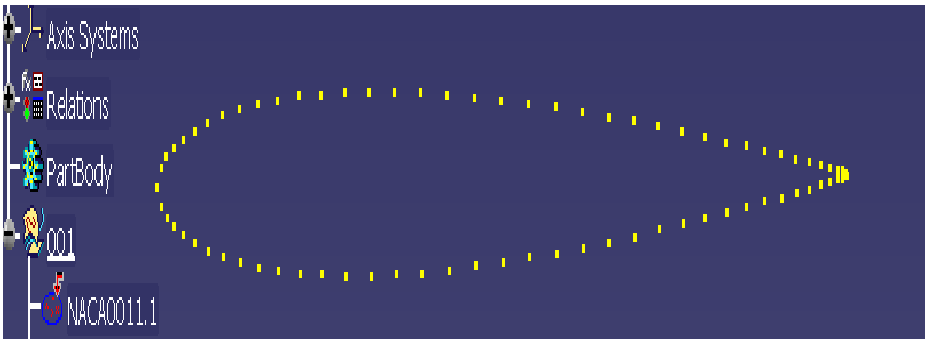


Figure-7. Airfoil created in CATIA using cloud of points.

Figure-7 is the base design created using the formula of the NACA profile Generator and coincide with our airfoil design which is created using the Control points (refer Figure-6) to validate the airfoil design. If it coincides perfectly, the design is validate. The Figure-8 shows that both the airfoil profile designs coincide perfectly.

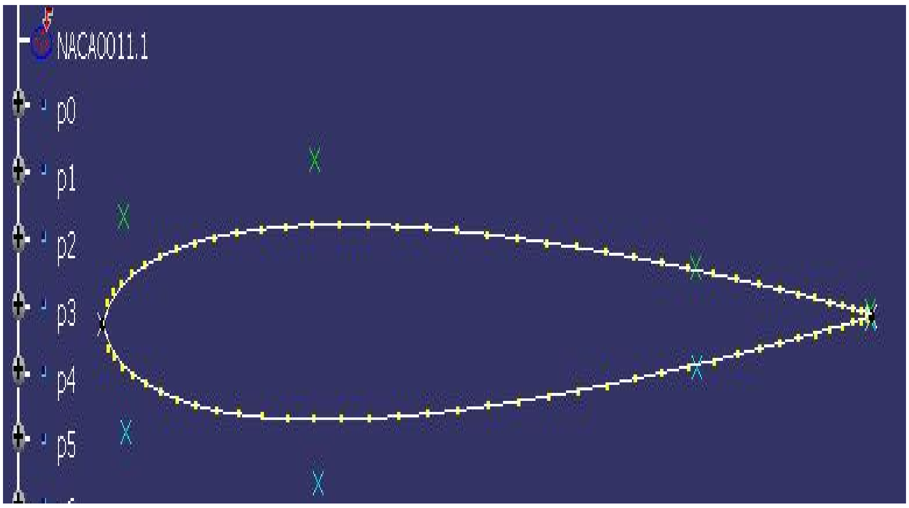


Figure-8. Coincide the 2D airfoil with cloud of points.

8. RELATION BETWEEN CONTROL POINTS

By assigning the Y axis length Difference formula relation [11] between the Upper and Lower

ARPN Journal of Engineering and Applied Sciences

Camber Control points to control the profile easily (refer Figures 9 and 10). By maintaining constant Y axis length difference between the Upper Control point *p2* and the Lower Camber Control point *p8* is shown in the Figures 9 and 10 and similarly maintains the Constant Y axis length difference between *p1* and *p7; p3* and *p9; p4 and p10*. For example Constant Y axis length Difference between the *p2* and *p8* is 7.308 as shown in the Table-7.

Table-7. Constant Y length difference between the p2 and p8.

Axis	P2-Upper camber control point (CP)	P8-Lower camber control point (CP)	Constant Y-axis length difference formula P8 = p2 + difference (X)
Y	3.654	-3.654	-3.654=3.654+X -3.645-3.654=-*7.308*=X
Constant Y value length difference *p8* is			-3.654 =3.654 -*7.308*

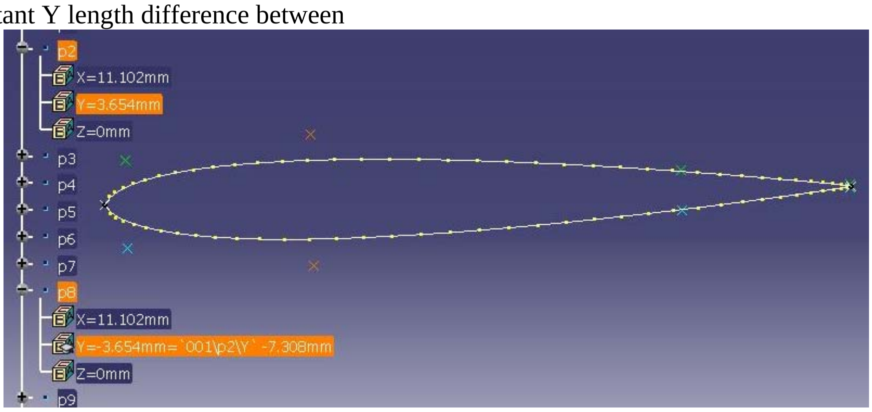


Figure-9. Constant Y length difference relation between upper *p2 (red)* and lower *p8 (red)* camber control point.

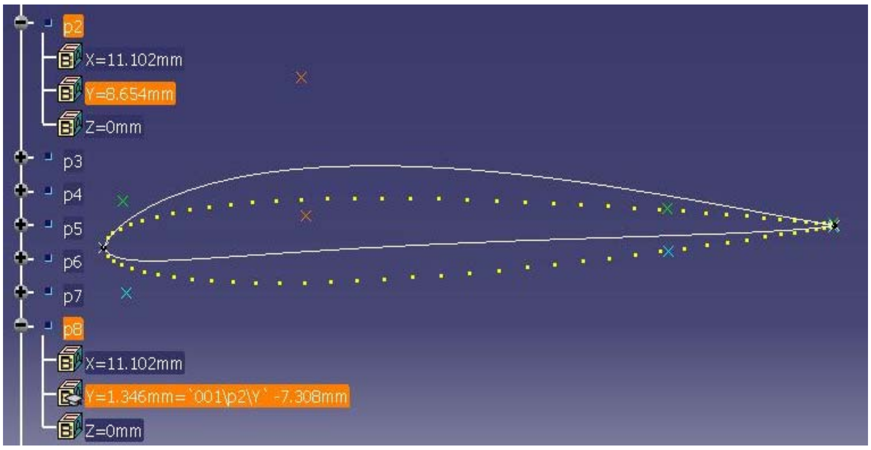


Figure-10. Constant Y Length difference relation between upper *p2 (red)* and lower *p8 (red)* camber control point.

By means of changing the control point *p*1**,** *p2,p3* and *p4* it automatically update the lower camber control point *p7***,** *p8, p9* and *p10* using formula relation as shown in Figures 9 and 10.

9. RESULTS

By changing the Y-value of the upper control Point *p1, p2, p3* and *p4* change the airfoil profile and without affecting the basic airfoil geometry. To create a camber airfoil, by changing upper camber control point Y-value which is situated 10%, 40% 80% and 95% of the chord length .It shows how the airfoil profile modified in CATIA as shown in Figures 11 to 14.

ARPN Journal of Engineering and Applied Sciences

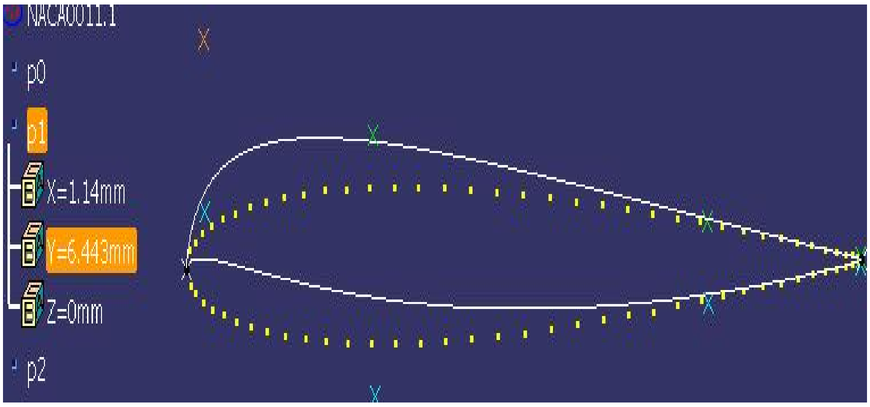


Figure-11. Control point *p1* (shown in red color) Y-Value changes which is located 10% of the length of the airfoil.

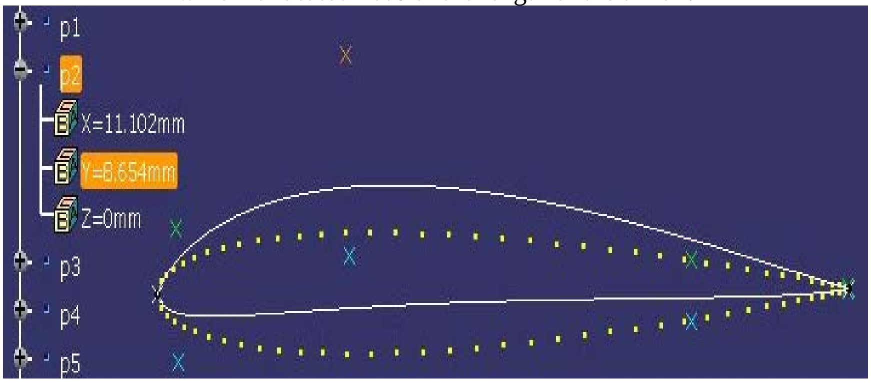


Figure-12. Control point *p2 (red)* Y-value changes which is located 40% of the length of the airfoil.

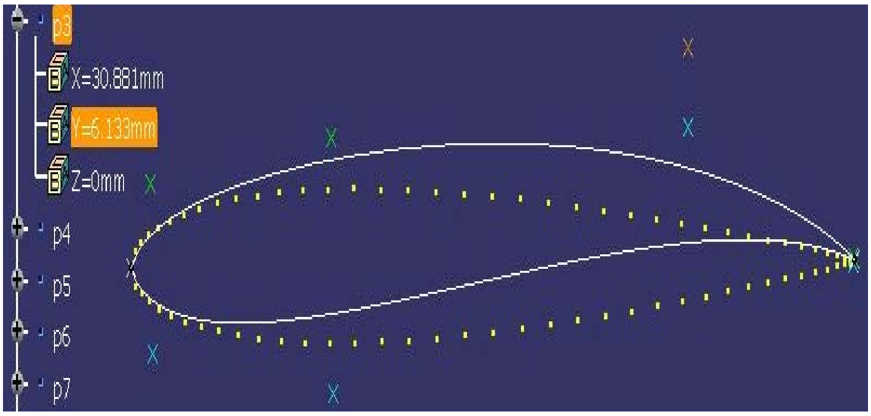


Figure-13. Control point *p3* (red) Y-value changes which is located 80% of the length of the airfoil.

VOL. 6, NO. 7, JULY 2011 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

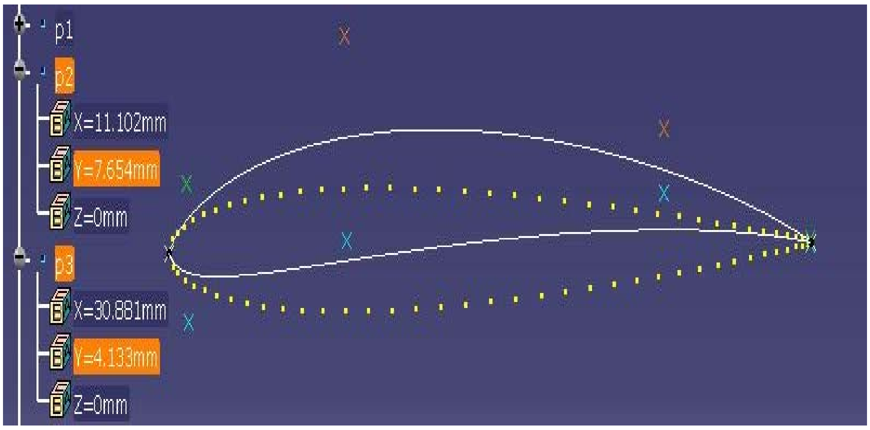


Figure-14. Control point *p2* and *p3* (red) Y- value changes which is located 40% and 80%of the length of the airfoil.

10. CONCLUSIONS

In this study, we have investigated and designed an airfoil Profile in CAD environment using control points. We have employed a Quintic Reverse Engineering Bezier curve formula to find out the control points of the camber profile which is used to create an upper and lower camber profile. By using the control points, we easily modify the shape of the profile so that to produce the cambered airfoil shape without affecting basic airfoil geometry. The objective of this work is to find a simple and accurate way to design the airfoil profile in CATIA using six camber control point position. However the proposed method is applied only for six camber control point position in the airfoil. The future work can be attempted to increase the number of camber point positions by means of increasing the degree '*n*' value of the Bezier curve formula and to employ a high order Reverse Engineering Bezier curve formula to find out the more camber control point locations on the airfoil design.

REFERENCES

[1] Joel P. Castan. 2003. Cambered airfoil use in Turbocharger. US patent. 7: 771, 162.

[2] Richard Eppler. 1990. Airfoil Design and Data.

[3] E. N. Jacobs, K. E. Ward and R. M. Pinkerton. 1933. The characteristics of 78 related airfoil sections from tests in the variable-density wind tunnel. NACA Report No. 460.

[4] Moran Jack. 2003. An introduction to theoretical and computational aerodynamics.

[5] Abbott and Von Doenhoff. 1959. Theory of Wing Sections -basics for all NACA airfoil surface.

[6] Jens Trapp and Robert Zores. www.ppart.de/aerodynamics/profiles/NACA4.html.

[7] H.Prautzsch W. Boehm. M. paluszny. 2002. Bezier and B-Spline Techniques.

[8] Farin Gerald. 1997. Curves and surfaces for computer-aided geometric design. 4th Ed., Elsevier Science and Technology Books.

[9] Strang Gilbert. 2003. Introduction to linear algebra. 3rd Ed. p. 71.

[10] Richard Cozzens. 2006. Advanced CATIA V5 Workbook: Knowledge ware and Work Benches Release 16.

[11] Sham Tickoo and Deepak Maini. 2006. CATIA V5R16 for Engineers and Designers.