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NONLINEAR DAMPED OSCILLATIONS - A CASE STUDY OF DYSFUNCTIONS IN SMOOTH PURSUIT EYE TRACKING-II

C.V. Pavankumar[1] and N. Ch. Pattabhi Ramacharyulu[2]
[1]Department of Humanities and Sciences, Faculty in Mathematics, Aurora's Scientific and Technological Institute, Hyderabad, India
[2]National Institute of Technology, Warangal, Andhra Pradesh, India
E-Mail: cheruvu.pavankumar76@gmail.com

ABSTRACT

This paper presents a simple deterministic model of dysfunctions of eye- tracking. The model is formulated as a second order nonlinear ordinary differential equation, incorporating non Hookesien cubic restoring force. The equation is solved by employing a perturbation technique with the nonlinear restoring force coefficient as the perturbation parameter. Numerical estimation of the angular velocity is computed for a wide spectrum of the eye dysfunction. The significance of the effects of time, the frequency and amplitude of the exiting force on displacement has been discussed by adopting ANOVA technique. The critical levels of time (τ) and frequency (Ω) have also been noted to be at τ =2.761sec, and Ω =0.181149375, corresponding to 0.05 level of significance.

Keyword: nonlinear oscillations, eye tracking, ANOVA.

INTRODUCTION

This paper is a sequel to the paper with a similar title by the present authors [1], which deals with the estimation of the angular displacement of the eye ball in eye dysfunction in schizophrenic patients.

The present communication concerns with the analysis of angular velocity of the eyeball oscillations as considered earlier [1]. The dysfunction dynamic parameter-Angular velocity has been computed numerically for a wide spectrum of eye dysfunction characteristics of schizophrenic patients. The influence of the amplitude and frequency of excitation on angular velocity has been highlighted by adopting the technique of ANOVA (analysis of variance), based upon the numerical computations.

It is observed that the variation of the amplitude (Γ) of the external excitation significantly influences the frequency of the eyeball oscillations; the same effect is noticed even with the time of observation while the effect of frequency (Ω) of the observation has no significant influence on the variation in the amplitude of the eyeball movement beyond $\Omega > 0.17$ at 5% level of tolerance these observations are in contrast with those noted while investigating the displacement of the eyeball dysfunction [1].

MATHEMATICAL MODEL EQUATION AND ANALYSIS

The assumption and the basic model equation formulated were presented in earlier communication [1]. Adopting the technique of regular perturbation explained in [1] an approximate solution of displacement is given by equation (15) of [1].

The angular velocity is given by:

$$\psi^{\bullet}(\tau) = \psi^{(0)^{\bullet}}(\tau) + \psi^{(1)^{\bullet}}(\tau) \tag{1}$$

$$\psi^{\bullet}(\tau) = -(c_1 + c_2\tau)e^{-\tau} + c_2 e^{-\tau} - \frac{\Gamma}{2}\sin 2\theta \sin(\Omega\tau - 2\theta)$$

$$-(c_3 + c_4\tau)e^{-\tau} + c_4 e^{-\tau} - e^{-3\tau}\left(\frac{3}{2}c_2(c_1+c_2\tau)^2 + \frac{15}{8}c_2{}^2(c_1+c_2\tau) + \frac{9}{8}c_2{}^3 + \frac{3}{4}(c_1+c_2\tau)^3\right)$$

$$+\Gamma\cos^4\theta e^{-2\tau}\begin{pmatrix} -3(c_1+c_2\tau)^2\{\tan\theta\sin\Omega\tau + 2\cos\Omega\tau\} \\ +6c_2(c_1+c_2\tau)\{\cos(\Omega\tau+2\theta) - 4\cos\theta\cos(\Omega\tau+\theta)\} \\ +c_2{}^2\{6\cos(\Omega\tau+2\theta) + \frac{9}{2}\cos(\Omega\tau+4\theta) + \frac{3}{2}\cos\Omega\tau - 36\cos^2\theta\cos(\Omega\tau+2\theta)\} \end{pmatrix}$$

$$+\frac{3\Gamma^2\cos^4\theta e^{-\tau}}{4c_2{}^2}\left(3c_2(c_1+c_2\tau)^2 - (c_1+c_2\tau)^3\right)$$

$$+\frac{3\Gamma^2\cos^3\theta e^{-\tau}}{8\sin^2\theta}\left(c_2\cot\theta\sin(5\theta-2\Omega\tau)+(c_1+c_2\tau)\cos(2\Omega\tau-5\theta)\right)$$

$$-\frac{3}{4}\Gamma^3\sin\theta\cos^7\theta\sin(\Omega\tau-4\theta)-\frac{3}{4}\Gamma^3\sin\theta\cos^5\theta\cos^2\alpha\sin(3\Omega\tau-6\theta-2\alpha) \quad (2)$$

Where

$$c_1=-\Gamma\cos^2\theta\cos(2\theta)\ ,\ c_2=-\Gamma\cos^2\theta \quad (3)$$

$$c_3=-\Gamma^3\{68\cos^{12}\theta-45\cos^{10}\theta-\frac{3}{4}\cos^8\theta+\cos^6\theta(\frac{1}{8}+\frac{15}{8}\cot^2\theta+\frac{1}{4}\cos^2\alpha\cos(6\theta+2\alpha))\}$$

$$c_4=-\Gamma^3\begin{bmatrix}4\cos^{12}\theta-36\cos^{10}\theta+\frac{69}{4}\cos^8\theta-\frac{199}{8}\cos^6\theta-\frac{99}{8}\cot^2\theta\cos^6\theta+\frac{1}{4}\cos^2\alpha\cos^6\theta\cos(6\theta+2\alpha)\\-18\cos^{10}\theta\cot^2\theta+27\cot^2\theta\cos^8\theta+24\cos^4\theta\cot^2\theta+\frac{3}{4}\sin\theta\cos^5\theta\cos^2\alpha\sin(6\theta+2\alpha)\end{bmatrix} \quad (4)$$

$$\theta=\tan^{-1}(\Omega)\ ,\ \alpha=\tan^{-1}(3\Omega) \quad (5)$$

Numerical estimation of the angular velocity have been carried out, for a wide spectrum of values of Ω = 0.1-(0.1)-0.5 and Γ = 0.1-(0.1)-0.5 within the time interval $0<\tau\leq 100$. Critical values of these dysfunction parameters at which their effects on the variations in angular displacement would be significant at 5% level of tolerance have been identified by employing ANOVA technique and linear interpolation. The computational details are not included in the present communication due to space limitations. However, these values are stated in the conclusions. Variations of the displacement versus time for different values of the amplitude (Γ) and angular frequencies (Ω) are illustrated (Figures 2 to 11) and phase portraits (Displacement-Velocity orbits) for the different values of Amplitude (Γ), frequency (Ω) are illustrated (Figures 12 to 17).

The following table gives the angular velocities for various values of amplitude and angular frequency.

Γ / Ω	**Γ = 0.1**	**Γ = 0.2**	**Γ = 0.3**	**Γ = 0.4**	**Γ = 0.5**
Ω=0.1	0.0164038316625493	0.032822854	0.049272258	0.065767235	0.082323
Ω=0.2	0.0164038316625493	0.026623662	0.039966066	0.053345159	0.066773
Ω=0.3	0.00837323427753044	0.016754781	0.025152953	0.033576062	0.042032
Ω=0.4	0.00193719010683629	0.003880198	0.00583484	0.007806935	0.009802
Ω=0.5	-0.0055706684049075	-0.01113718	-0.01669539	-0.02224114	-0.02777

Based on the above tabulated values of the angular velocity, we compute the required sums of the squares needed for ANOVA analysis and these are listed here-under:

Source of variation	**Degrees of freedom**	**Sum of squares**	**Mean square**	**Calculated F ratio**
Frequencies	4	0.014268734	0.003567183	18.54034
Amplitudes	4	0.00232	0.00058006	3.014845
Error	16	0.003078419	0.000192401	
Total	24			

The calculated F Ratio values are compared with tabulated F values for (4, 4) degrees of freedom at 0.05 level of significance. The critical values of F-ratio are calculated by using linear interpolation.

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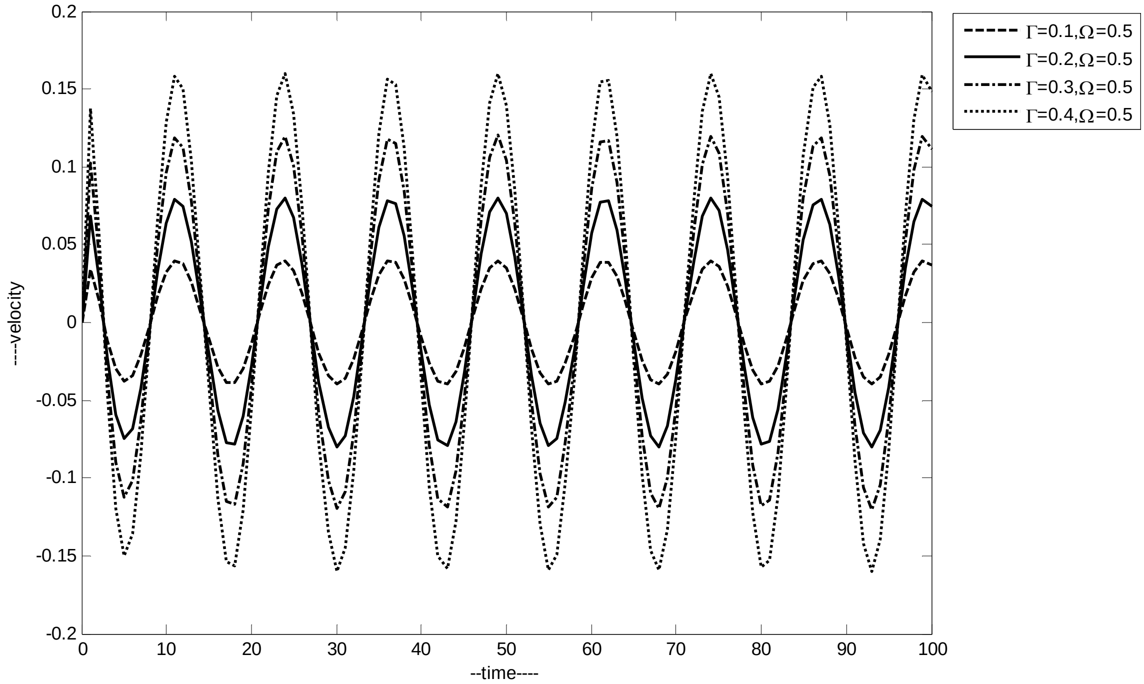

Figure-1. Variation of the velocity versus time for the frequency parameter (Ω = 0.5) and for different amplitudes (Γ).

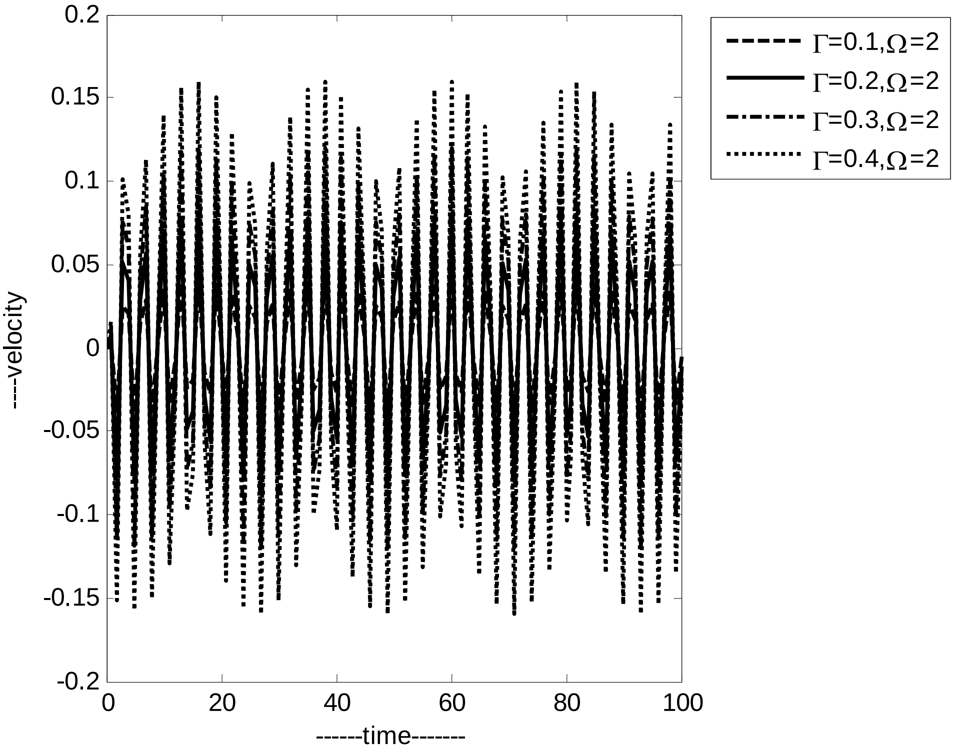

Figure-2. Variation of the velocity versus time for the frequency parameter (Ω = 2) and for different amplitudes (Γ).

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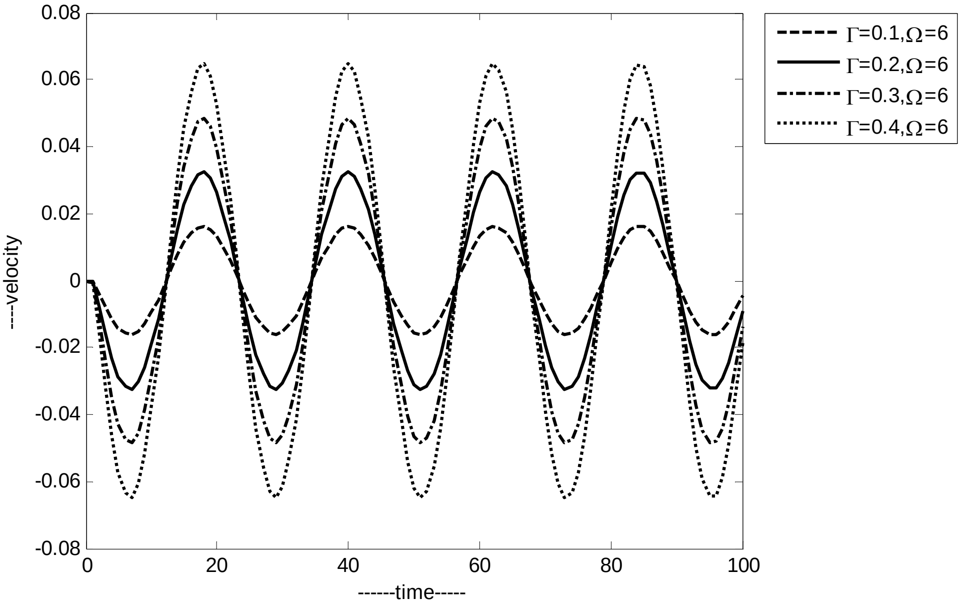

Figure-3. Variation of the velocity versus time for the frequency parameter (Ω = 6) and for different amplitudes (Γ).

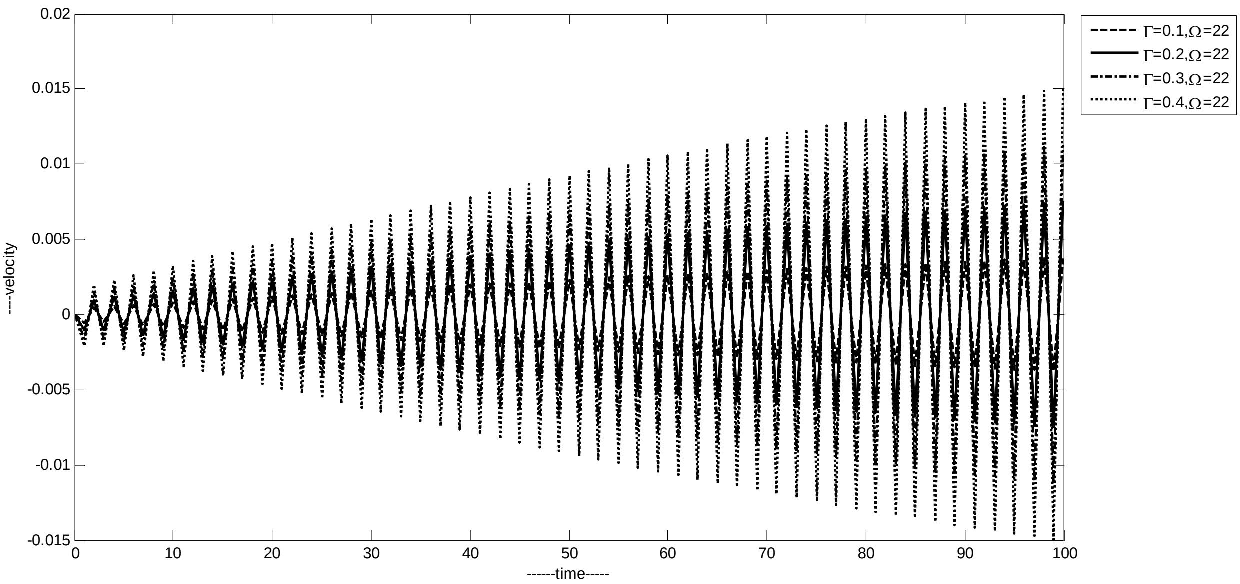

Figure-4. Variation of the velocity versus time for the frequency parameter (Ω = 22) and for different amplitudes (Γ).

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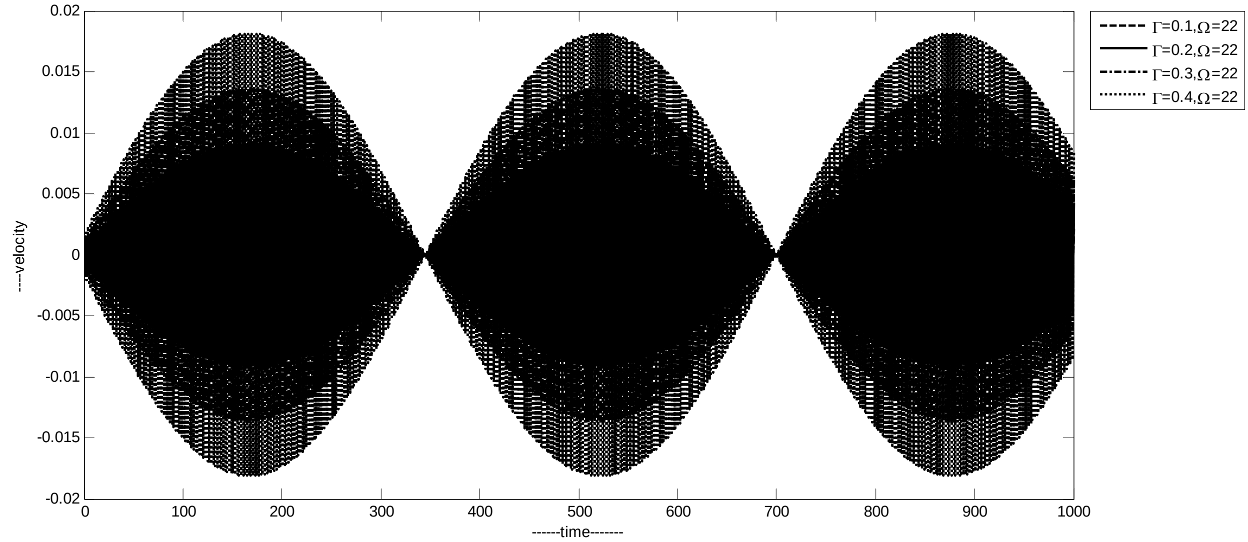

Figure-5. Variation of the velocity versus time for the frequency parameter (Ω = 22) and for different amplitudes (Γ). The time scale is taken up to τ = 1000sec. (phenomena of beats observed).

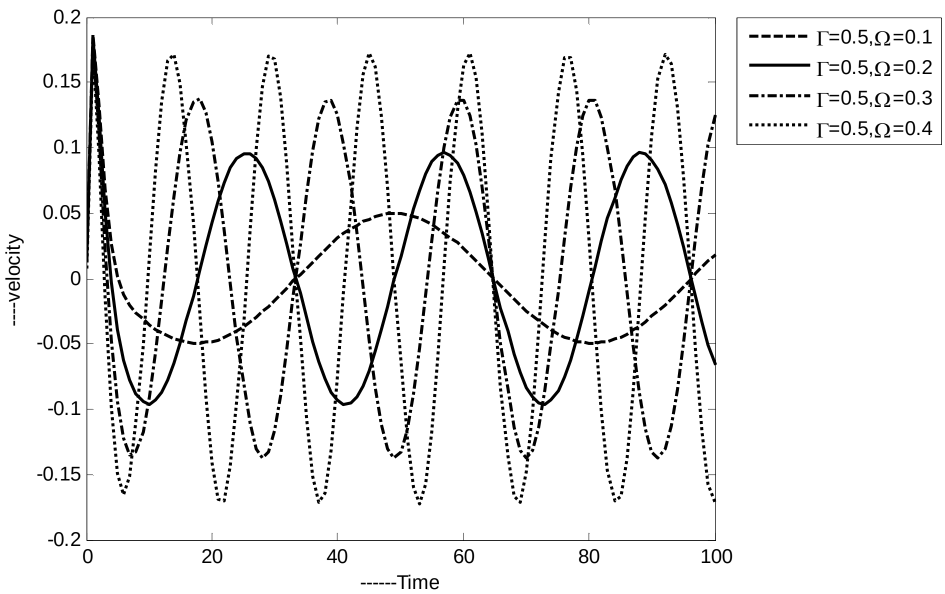

Figure-6. Variation of velocity versus time for the amplitude parameter (Γ = 0.5) and for different angular frequencies (Ω).

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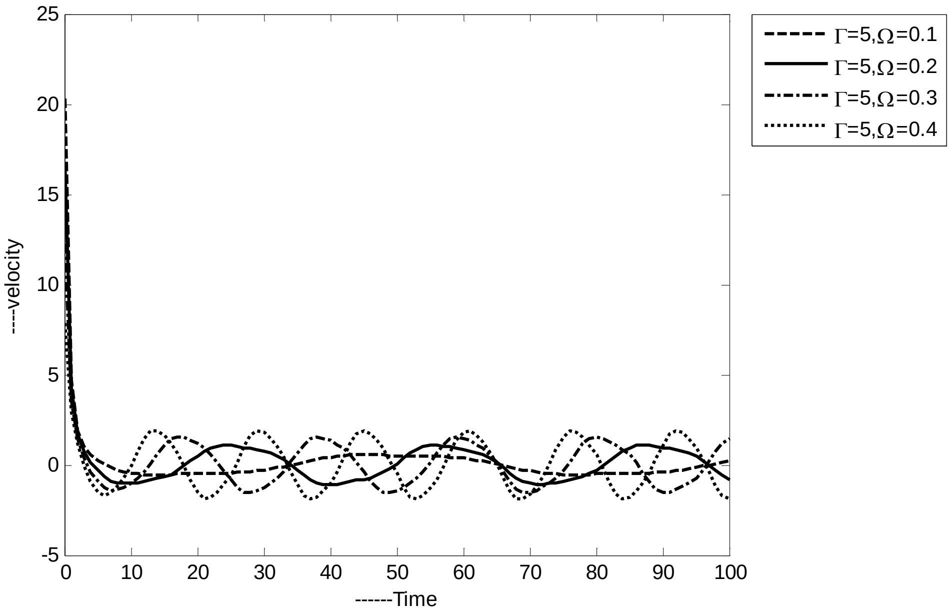

Figure-7. Variation of the velocity versus time for the amplitude parameter (Γ=5) and for different angular frequencies (Ω).

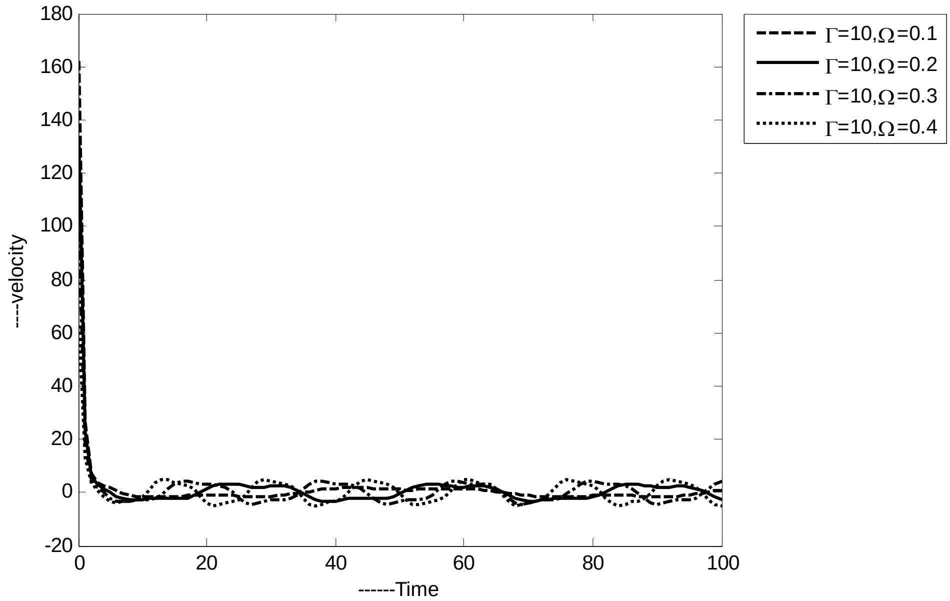

Figure-8. Variation of the velocity versus time for the amplitude parameter (Γ=10) and for different angular frequencies (Ω).

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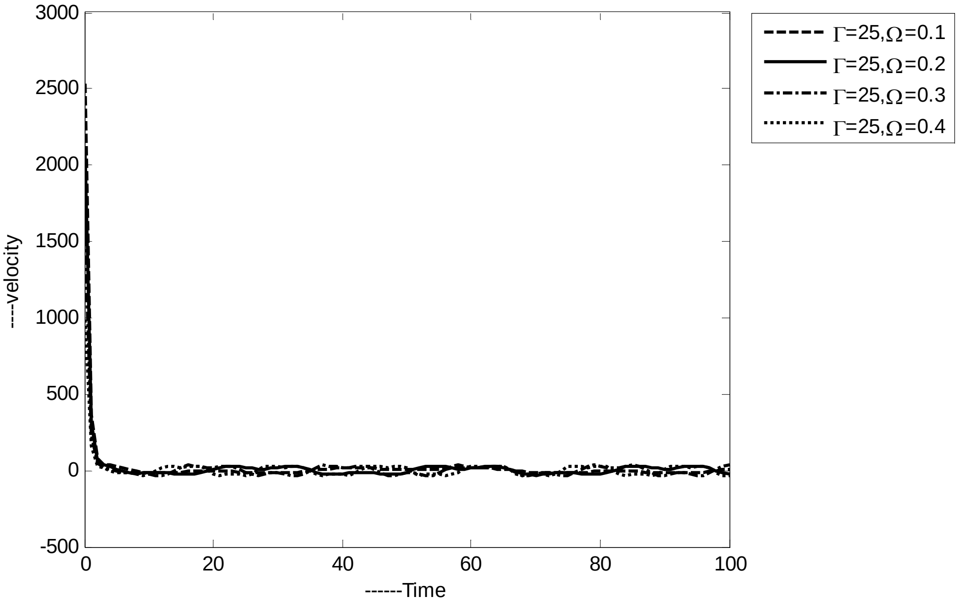

Figure-9. Variation of the velocity versus time for the amplitude parameter (Γ= 25) and for different angular frequencies (Ω).

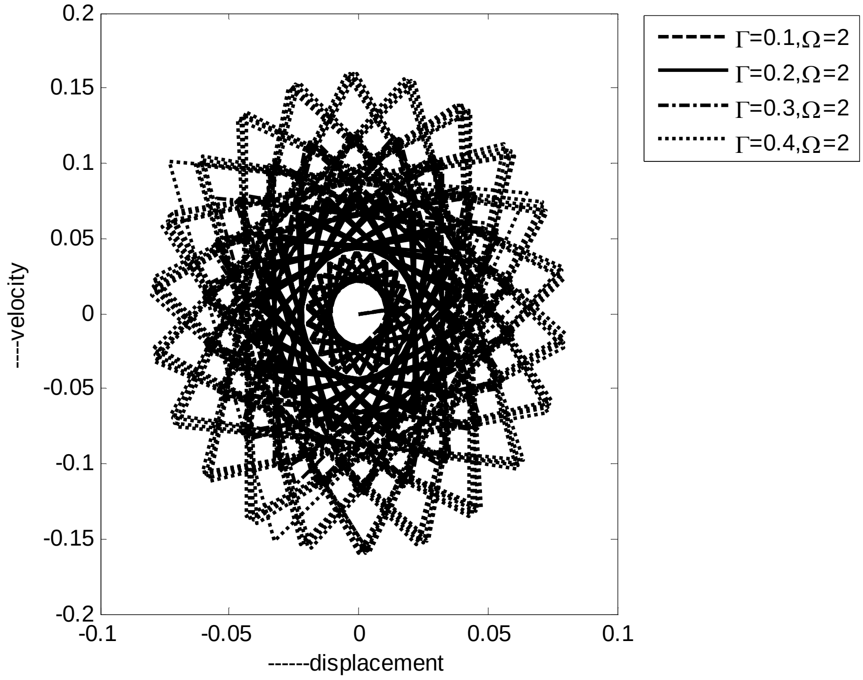

Figure-10. Variation of the displacement versus velocity for the angular frequency parameter (Ω = 2) and for different amplitudes (Γ).

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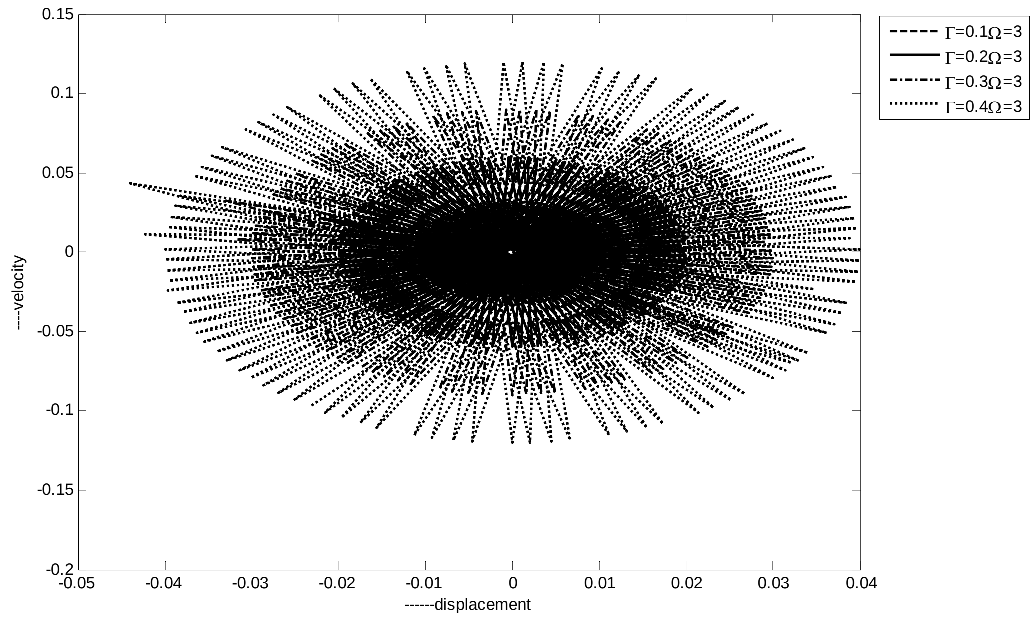

Figure-11. Variation of the displacement versus velocity for the angular frequency parameter (Ω = 3) and for different amplitudes (Γ).

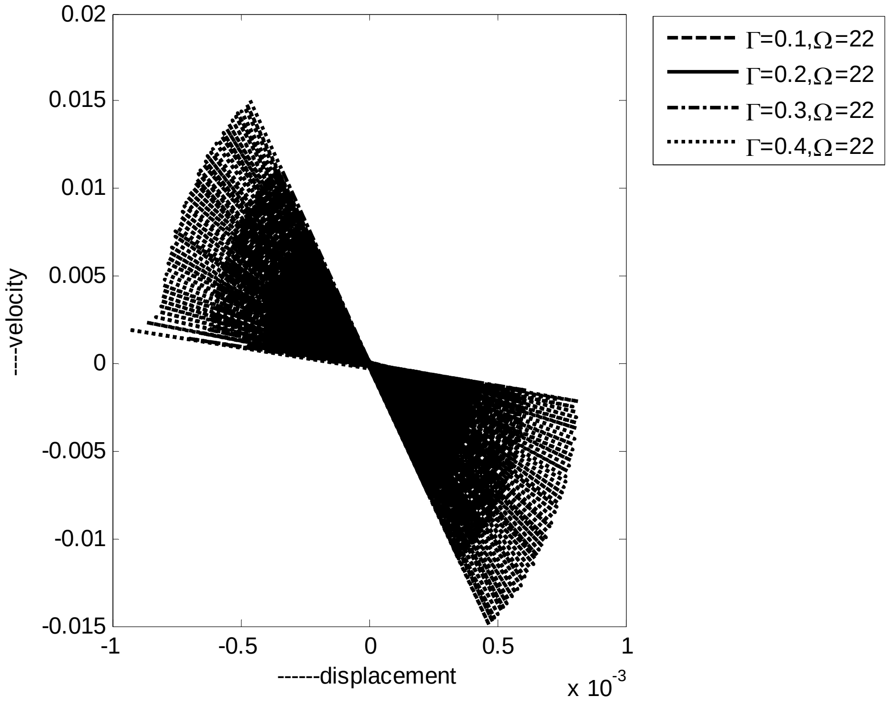

Figure-12. Variation of the displacement versus velocity for the angular frequency Parameter (Ω = 22) and for different amplitudes (Γ).

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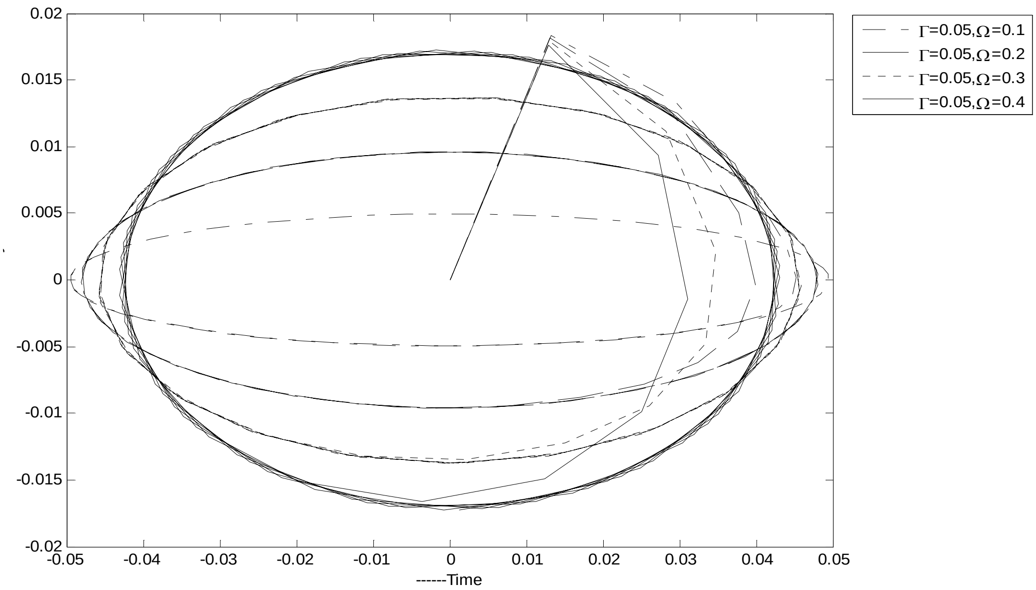

Figure-13. Variation of the displacement versus velocity for the angular displacement parameter (Γ= 0.05) and for different angular frequencies (Ω).

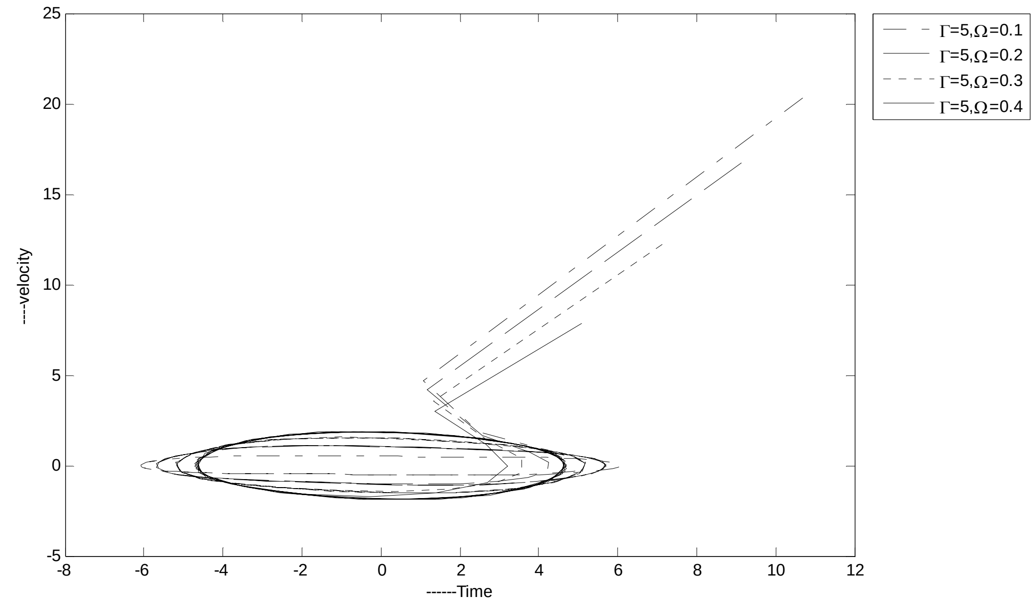

Figure-14. Variation of the displacement versus velocity for the angular displacement parameter (Γ=5) and for different angular frequencies (Ω).

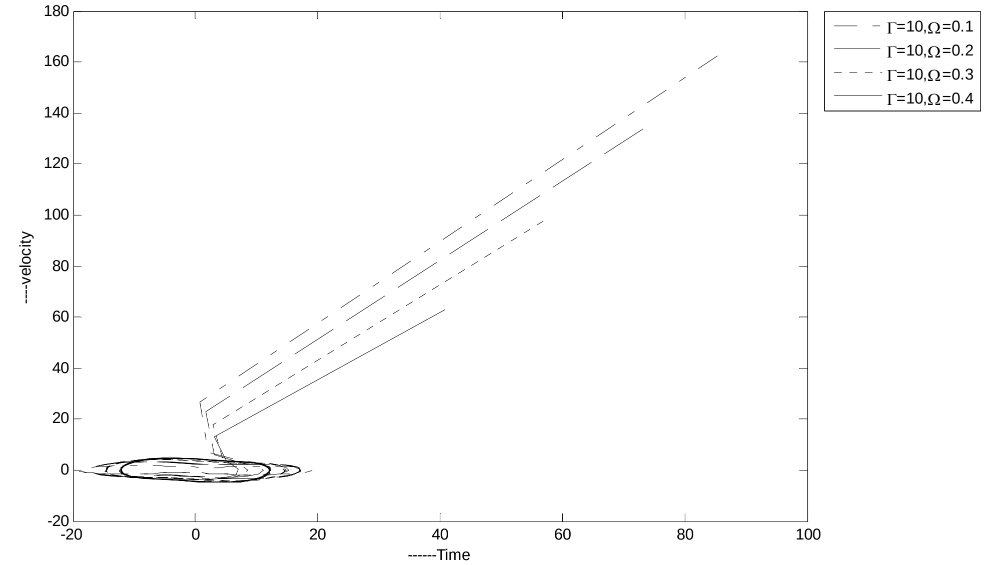

Figure-15. Variation of the displacement versus velocity for the angular displacement parameter (Γ=10) and for different angular frequencies (Ω).

Based on the numerical computations carried out (the details of which are not shown here due to space limitations) the following conclusions are drawn.

CONCLUSIONS

a) At different levels of time (τ) for fixed amplitude (Γ) and frequency Ω)

The variation in angular velocity of the eye due to amplitude- variation is significant up to the time instant τ = 2.718sec, there after no appreciable change in angular velocity of the eye due to amplitude variations would be observed.

b) At different levels of frequency (Ω) for fixed amplitude (Γ) and time(τ)

The variation in the angular velocity of the eye as time progresses is significant up to Ω=0.181149375beyond which there would not be any significance in such variations.

c) At different levels of amplitude (Γ) for fixed frequency (Ω) and time (τ)

The difference in angular velocity of the eye due to amplitude- variations is more significant compared to that due to the frequency variation. The illustrations exhibit the erratic variations of the angular velocity with the increase of Γ, Ω and τ of schizophrenia patient. Beyond the critical values, mentioned above, there would not be any variation leading to the arrest of the movements of the eye ball. The state of starring vision of a mentally retarded patient would begin at the time instant of the happening of the above events (1), (2) and (3) whichever is earlier. This would naturally depend upon movement of the target object and the constitution of the individual patient and the state of disease severity.

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