EFFECT OF SLIP ON HEAT TRANSFER TO PERISTALTIC TRANSPORT IN THE PRESENCE OF MAGNETIC FIELD WITH WALL EFFECTS

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ABSTRACT

The effects of slip and elasticity of flexible walls on peristaltic transport of an incompressible viscous fluid in a two dimensional uniform channel, with heat transfer in the presence of magnetic field is investigated. Using long wavelength approximation, a perturbation solution has been obtained in terms of wall slope parameter and closed form expressions are derived for average velocity, temperature and heat transfer. The effects of various pertinent parameters on average velocity and heat transfer coefficient have been studied. The time average velocity increase with slip but decrease with permeability. Trapping phenomenon is more significant for lower values of slip parameter, higher permeability and almost disappears for higher values of magnetic parameter. Further, the heat transfer coefficient increases with slip but decreases with permeability.

Keywords: peristalsis, slip, heat transfer, magnetic field, dynamic boundary conditions.

INTRODUCTION

Peristaltic transport is a form of material transport induced by progressive waves of area contraction and expansion traveling along the length of a distensible tube containing fluid. Peristalsis plays an important role in transporting many physiological fluids in human body in various situations such as urine transport from kidney to the bladder, movement of chyme in the gastrointestinal tract, swallowing of food through oesophagus and vasomotion of small blood vessels. Mechanical devices like finger and roller pumps operate on this mechanism. This mechanism is also used in transporting sensitive or corrosive fluids, sanitary fluids and noxious fluids in nuclear industry. Peristaltic transport has been studied under various conditions by using different assumptions like long wavelength or small amplitude ratio. Shapiro *et al*., [1] studied peristaltic pumping with long wavelength at low Reynolds number. Manton [2] considered an asymptotic expansion for flow in an axisymmetric pipe with long peristaltic waves of arbitrary shape. Radhakrishnamacharya [3] has investigated long wavelength approximation to peristaltic motion of a power law fluid. Muthu *et al*., [4] studied the influence of wall properties on the peristaltic motion of micro polar fluid. Hyat *et al*., [5] investigated peristaltic flow of a micro polar fluid in a channel with different wave forms.

Magneto hydrodynamics (MHD) is the dynamics of electrically conducting fluids. The mutual interaction between the fluid motion and magnetic field is the essential feature of the physical situation in the MHD fluid flow problems. MHD principles are employed in the design of heat exchangers, pumps, flow meters, radar systems, power generation etc. It is realized that the principles of magneto hydrodynamics find extensive applications in bioengineering and medical sciences. They include the development of magnetic devices for cell separation, targeted transport of drugs using magnetic particles as drug carriers, reduction of bleeding during surgeries and development of magnetic tracers. In recent years, the magneto hydrodynamic flow of a fluid in a channel with elastic and rhythmically contracting walls has received much attention. Mishra *et al*., [6] have studied pumping action on blood flow by a magnetic field. Mekheimer [7] investigated the peristaltic flow of blood under the effect of magnetic field in a non-uniform channel. Elshahed *et al*., [8] studied the influence of magnetic field on peristaltic transport of a Johnson-Segalman fluid. Kothandapani *et al*., [9] have studied peristaltic transport of a Jeffrey fluid under the effect of magnetic field in an asymmetric channel.

However, the interaction of peristalsis with heat transfer has not received much attention. The thermo dynamical aspects of blood may not be important when blood is inside the body but they become significant when it is drawn out of the body. Keeping in view the significance of heat transfer in blood flow, Victor and Shah [10] studied the thermo dynamical aspects of blood flowing in a tube treating blood as Casson fluid. Agrawal [11] analyzed the heat transfer to pulsatile flow of a conducting fluid through a porous channel in the presence of magnetic field. Tang *et al*., [12] investigated the peristaltic flow of a heat conducting fluid subject to a prescribed pressure drop and Newton's law of cooling at the boundary. Radhakrishnamacharya and Srinivasulu [13] studied the influence of wall properties on peristaltic transport with heat transfer. Srinivas *et al*., [14] investigated heat and mass transfer for peristaltic flow in the presence of magnetic field through a porous space with compliant wall.

All the above investigations on peristaltic transport have been done taking into account the classical no slip boundary condition. However, in several applications, the flow pattern corresponds to a slip flow and the fluid presents a loss of adhesion at the wetted wall making the fluid slide along the wall. Beaver and Joseph [15] were the first to investigate the fluid flow at the interface between a porous medium and fluid layer in an experimental study and proposed a slip boundary condition at the interface. Flows with slip would be useful for problems in chemical engineering, for example flows through pipes in which chemical reactions occur at the walls, two-phase flows in porous slider bearings. Saffman [16] proposed an improved slip boundary condition. Vajravelu *et al.*, [17] discussed the peristaltic transport of a micro polar fluid in a channel with permeable walls. Terrill [18] investigated laminar flow through a porous pipe with slip.

In view of the above discussion, in this paper, the interaction of peristaltic transport with heat transfer for a Newtonian fluid with slip boundary condition is studied by including the effects of magnetic field and wall properties. A perturbation method of solution has been obtained in terms of wall slope parameter and analytical solutions have been obtained for stream function, average velocity, temperature and heat transfer coefficient. The effects of various pertinent parameters on average velocity and heat transfer coefficient have been studied.

MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

Consider the two dimensional flow of a Newtonian incompressible viscous fluid in a uniform channel with heat transfer. It is assumed that the fluid is electrically conducting and a uniform magnetic field B_0 is applied in the transverse direction. Induced magnetic field is neglected compared to the applied magnetic field so that the magnetic Reynolds number can be neglected. The walls of the channel are flexible and are taken as stretched membrane, on which traveling sinusoidal waves are imposed. Cartesian coordinate system is chosen in such a way that the *x*-axis lies along the centre line of the channel and *y*-axis normal to it. The geometry of the wall surface is described by:

$$\eta(x,t) = d + a\sin\frac{2\pi}{\lambda}(x-ct) \qquad (1)$$

where a is the amplitude, λ is the wavelength, c is the velocity of the peristaltic wave, d is the mean half width of the channel and t is the time.

The equations which govern the MHD flow for the present problem are

$$\nabla\cdot\bar{V} = 0 \qquad (2)$$

$$\rho\frac{d\bar{V}}{dt} = -\nabla p + \mu\ \nabla^2\bar{V} + \bar{J}\times\bar{B} \qquad (3)$$

Where

$\bar{V}$ = velocity of the fluid
p = pressure
ρ = density
μ = coefficient of viscosity
$\bar{J}$ = current density
$\bar{B} = (\bar{B}_0 + \bar{B}_1)$ is the total magnetic field
$\bar{B}_1$ = induced magnetic field
$\bar{J}\times\bar{B}$ = Lorentz's force which is the body force acting on the fluid and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

Neglecting the displacement currents, the Maxwell equations and Ohm's law are:

$$\nabla\cdot\bar{B} = 0, \quad \nabla\times\bar{B} = \mu_m\bar{J}, \quad \nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}, \quad \bar{J} = \sigma(\bar{E} + \bar{V}\times\bar{B}) \qquad (4)$$

Where

σ = electric conductivity of the fluid
μ_m = magnetic permeability and is constant through out the flow field
$\bar{E}$ = electric field

The imposed and induced electric fields are assumed negligible. Under low magnetic Reynolds number approximation, the force $\bar{J}\times\bar{B}$ simplifies to

$$\bar{J}\times\bar{B} = -\sigma B_0^2\, u \qquad (5)$$

The governing equations of motion of the flexible wall, following Mittra and Prasad [19], may be expressed as:

$$L(\eta) = p - p_0 \qquad (6)$$

Where L is an operator which is used to represent the motion of stretched membrane with viscous damping force such that:

$$L = -T\frac{\partial^2}{\partial x^2} + m\frac{\partial^2}{\partial t^2} + C\frac{\partial}{\partial t} \qquad (7)$$

Where

T = tension in the membrane
m = mass per unit area
C = coefficient of viscous damping force
p_0 = pressure on the outside surface of the wall due to tension in the muscles.

For simplicity, we assume $p_0 = 0$. The equations governing the flow, in cartesian form for the present problem are:

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Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (8)$$

Momentum equation:

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\mu\ \nabla^2 u-\sigma B_0^2 u \qquad (9)$$

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial y}+\mu\ \nabla^2 v \qquad (10)$$

Energy Equation:

$$G\left[\frac{\partial T^*}{\partial t}+u\frac{\partial T^*}{\partial x}+v\frac{\partial T^*}{\partial y}\right]=\frac{k}{\rho}\ \nabla^2 T^*+\upsilon\,\Phi \qquad (11)$$

Where $\Phi=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2$

where $u(x,y,t), v(x,y,t)$ are the velocity components in the x and y directions, respectively, Φ is the dissipation function and it represents the time rate at which energy is being dissipated per unit volume through the action of viscosity, T^* is the temperature of the fluid, k is the thermal conductivity, G is the specific heat at constant pressure, υ is the kinematic coefficient of viscosity.

The slip boundary condition (Saffman slip condition) at the wall is:

$$d\ \frac{\partial u}{\partial y}=-\frac{\alpha}{\sqrt{Da}}u \text{ at } y=\pm\eta=\pm[d+a\sin\frac{2\pi}{\lambda}(x-ct)] \qquad (12)$$

Where

$Da\left(=\frac{k'}{d^2}\right)$ = Darcy's parameter

α = slip parameter

k' = permeability of the wall

The dynamic boundary conditions that are imposed on the fluid by the symmetric motion of the flexible walls at $y=\pm\eta(x,t)$ given by:

$$\frac{\partial}{\partial x}L(\eta)=\frac{\partial p}{\partial x}=\rho\upsilon\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]-\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]-\sigma B_0^2 u \qquad (13)$$

Further, the temperature of the lower wall is maintained at T_0^* and the upper wall at T_1^*, i.e.,

$$T^*=T_0^* \text{ at } y=-\eta(x,t) \qquad (14)$$

$$T^*=T_1^* \text{ at } y=\eta(x,t) \qquad (15)$$

Introducing stream function ψ such that

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x} \qquad (16)$$

and the following non-dimensional quantities

$$u'=\frac{u}{c},\ x'=\frac{x}{\lambda},\quad y'=\frac{y}{d},\ \eta'=\frac{\eta}{d},\quad \psi'=\frac{\psi}{cd},\quad t'=\frac{ct}{\lambda},\quad \theta=\frac{T^*-T_0^*}{T_1^*-T_0^*} \qquad (17)$$

and eliminating pressure between (9) and (10), the equations (9-15), become (after dropping the primes)

$$\delta R\left[\nabla_1^2\left(\frac{\partial\psi}{\partial t}\right)+\frac{\partial\psi}{\partial y}\nabla_1^2\left(\frac{\partial\psi}{\partial x}\right)-\frac{\partial\psi}{\partial x}\nabla_1^2\left(\frac{\partial\psi}{\partial y}\right)\right]=\nabla_1^4\psi-M^2\frac{\partial^2\psi}{\partial y^2} \qquad (18)$$

$$R\ \delta\left[\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right]=\frac{1}{\Pr}\left[\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right]+E\left[4\delta^2\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2+\left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)\right] \qquad (19)$$

$$\frac{\partial^2\psi}{\partial y^2}=-\frac{\alpha}{\sqrt{Da}}\frac{\partial\psi}{\partial y} \text{ at } y=\pm\eta(x,t)=\pm\left[1+\varepsilon\sin 2\pi(x-t)\right] \qquad (20)$$

$$\delta^2\frac{\partial^3\psi}{\partial x^2\partial y}+\frac{\partial^3\psi}{\partial y^3}-R\delta\left[\frac{\partial^2\psi}{\partial t\partial y}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right]-M^2\frac{\partial\psi}{\partial y}=E_1\frac{\partial^3\eta}{\partial x^3}+E_2\frac{\partial^3\eta}{\partial x\partial t^2}+E_3\frac{\partial^2\eta}{\partial t\partial x} \text{ at } y=\pm\eta(x,t) \qquad (21)$$

Further, $\theta=0$ on $y=-\eta$ (22)

$\theta=1$ on $y=\eta$ (23)

Here $\varepsilon = \left(\frac{a}{d}\right)$ is the amplitude ratio, $\delta = \left(\frac{d}{\lambda}\right)$ is the wave number, $R = \left(\frac{\rho c d}{\mu}\right)$ is the Reynolds number, $M = \left(\sqrt{\frac{\sigma}{\mu}} B_0 d\right)$ is the Hartman number, $\Pr = \left(\frac{G\rho\upsilon}{k}\right)$ is the Prandtl number, $E = \left(\frac{c^2}{G(\phi_1 - \phi_0)}\right)$ is the Eckert number and $\nabla_1^2 = \delta^2 \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$. Further, $E_1 = \left(\frac{-T\delta^3}{\rho\upsilon c}\right)$ and $E_2 = \left(\frac{mc\delta^3}{\rho\upsilon}\right)$ respectively represent the rigidity and stiffness of the wall and the viscous damping force in the wall is represented by $E_3 = \left(C\frac{\delta^2 d}{\rho\upsilon}\right)$. In particular, when $E_3 = 0$, the wall moves up and down with no damping force on them and hence indicates the case of elastic wall $(E_3 = 0)$.

We seek a perturbation solution in terms of wall slope parameter δ (δ <<1) as follows

$$S = S_0 + S_1\delta + S_2\delta^2 + \quad .... \tag{24}$$

where S represents any flow variable.

Substituting (24) in (18) to (23) and collecting the coefficients of various like powers of δ, we get the following sets of equations:

Zeroth order equations:

$$\frac{\partial^4\psi_0}{\partial y^4} - M^2\frac{\partial^2\psi_0}{\partial y^2} = 0. \tag{25}$$

$$E\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2 + \frac{1}{\Pr}\left[\frac{\partial^2\theta_0}{\partial y^2}\right] = 0 \tag{26}$$

Boundary conditions:

$$\frac{\partial^2\psi_0}{\partial y^2} = -\frac{\alpha}{\sqrt{Da}}\frac{\partial\psi_0}{\partial y} \quad \text{at} \quad y = \pm\eta(x,t) \tag{27}$$

$$\frac{\partial^3\psi_0}{\partial y^3} - M^2\frac{\partial\psi_0}{\partial y} = E_1\frac{\partial^3\eta}{\partial x^3} + E_2\frac{\partial^3\eta}{\partial x\partial t^2} + E_3\frac{\partial^2\eta}{\partial x\partial t} \quad \text{at} \quad y = \pm\eta(x,t) \tag{28}$$

$$\theta_0 = 0 \text{ on } y = -\eta(x,t) \tag{29}$$

$$\theta_0 = 1 \text{ on } y = \eta(x,t) \tag{30}$$

First order equations:

$$R\left[\frac{\partial^3\psi_0}{\partial y^2\partial t} + \frac{\partial\psi_0}{\partial y}\frac{\partial^3\psi_0}{\partial y^2\partial x} - \frac{\partial\psi_0}{\partial x}\frac{\partial^3\psi_0}{\partial y^3}\right] = \frac{\partial^4\psi_1}{\partial y^4} - M^2\frac{\partial^2\psi_1}{\partial y^2} \tag{31}$$

$$R\left[\frac{\partial\theta_0}{\partial t} + \frac{\partial\psi_0}{\partial y}\frac{\partial\theta_0}{\partial x} - \frac{\partial\psi_0}{\partial x}\frac{\partial\theta_0}{\partial y}\right] = \frac{1}{\Pr}\left[\frac{\partial^2\theta_1}{\partial y^2}\right] + 2\delta E\left[\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_1}{\partial y^2}\right] \tag{32}$$

Boundary conditions:

$$\frac{\partial^2\psi_1}{\partial y^2} = -\frac{\alpha}{\sqrt{Da}}\frac{\partial\psi_1}{\partial y} \quad \text{at} \quad y = \pm\eta(x,t) \tag{33}$$

$$M^2\frac{\partial\psi_1}{\partial y} = -R\left[\frac{\partial^2\psi_0}{\partial y\partial t} + \frac{\partial\psi_0}{\partial y}\frac{\partial^2\psi_0}{\partial y\partial x} - \frac{\partial\psi_0}{\partial x}\frac{\partial^2\psi_0}{\partial y^2}\right] \text{ at } y = \pm\eta(x,t) \tag{34}$$

$$\theta_1 = 0 \text{ on } y = -\eta(x,t) \tag{35}$$

$$\theta_1 = 0 \text{ on } y = +\eta(x,t) \tag{36}$$

Solving the equations (25), (26), (31) and (32) subject to the relevant boundary conditions (27) - (30) and (33) - (36), we will finally get the expressions for ψ_0, θ_0, ψ_1 and θ_1 as

$$\psi_0(x,y,t) = -\left[\frac{\Omega}{M^2}\right]y - \sinh(My)\left[\frac{E_4\Omega}{(M^4\sinh(M\eta) - M^3E_4\cosh(M\eta))}\right] \tag{37}$$

$$\theta_0(x,y,t) = \frac{-2E\Pr E_4^2\Omega^2}{\left[M^2\sinh[M\eta] - E_4M\cosh[M\eta]\right]^2}\left[\frac{\cosh[2My]}{8M^2} - \frac{y^2}{4}\right] + B_1y + B_2 \tag{38}$$

$$\psi_1(x,y,t) = C_1(x,t)\quad y + C_2(x,t)\quad \sinh[My] + \left[\frac{y\cosh[My]}{2M^3}\right]\ A_1(x,t) + \left[\frac{y^2}{4M^3}\sinh[My] - \frac{5y}{4M^4}\cosh[My]\right]\ A_2(x,t) + \left[\frac{2}{24M^4}\sinh[2My]\right]\ A_3(x,t) \tag{39}$$

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$$
\begin{aligned}
\theta_1(x,y,t) = {} & Z_1 \frac{\cosh[2My]}{4M^2} + Z_2 \frac{y^4}{12} + Z_3 \frac{y^3}{6} + Z_4\left[\frac{\cosh[3My]}{18M^2} + \frac{\cosh[My]}{2M^2}\right] \\
& + Z_5\left[\frac{y^2}{M^2}\cosh[My] - \frac{4}{M^3} y\sinh[My] + \frac{4}{M^4}\cosh[My] + \frac{2}{M^5}\sinh[My]\right] \\
& + Z_6\left[\frac{y^2}{M^2}\sinh[My] + \frac{4}{M^4}\sinh[My] + \frac{2}{M^5}\cosh[My]\right] \\
& + Z_7\left[\frac{1}{M^2} y\cosh[My] - \frac{1}{M^3}\sinh[My] - \frac{1}{M^5}\sinh[My]\right] + Z_8\left[\frac{1}{M}\cosh[My]\right] \\
& + Z_9\left[\frac{1}{4M^2} y\sinh[2My] + \frac{1}{4M^2}\sinh[2My] - \frac{1}{16M^4}\sinh[2My]\right] \\
& + Z_{10}\left[\frac{1}{2M^2}\cosh[My] - \frac{1}{18M^2}\cosh[3My]\right] \\
& + Z_{11}\left[\frac{1}{M^2} y\sinh[My] - \frac{2}{M^3}\cosh[My]\right] + Z_{12}\left[\frac{1}{M^2}\sinh[\mathrm{My}]\right] + Z_{13}\frac{\mathrm{y}^2}{2} \\
& + Z_{14}\left[\frac{y}{8M^2}\cosh[2My] - \frac{1}{8M^3}\sinh[2My] + \frac{y^3}{12}\right] \\
& + Z_{15}\left[\frac{\cosh[2My]}{8M^2} + \frac{y^2}{2}\right] + F_{13}\left[\frac{\cosh[2My]}{8M^2} - \frac{y^2}{4}\right] \\
& + F_2\left[\frac{1}{8M^2} y\sinh[2My] - \frac{3}{16}\cosh[2My]\right] \\
& + F_4\left[\frac{1}{8M^2} y^2\cosh[2My] - \frac{3}{8} y\sinh[2My] + \frac{5}{4M^4}\cosh[2My]\right] \\
& - F_4\frac{y^5}{120} + F_5\left[\frac{\cosh[My]}{8M^2} - \frac{\cosh[3My]}{9M^2}\right] + X_1(x,t)y + \mathrm{X}_2(x,t)
\end{aligned}
\tag{40}
$$

Where

$$\Omega(x,t) = E_1\frac{\partial^3\eta}{\partial x^3} + E_2\frac{\partial^3\eta}{\partial x\partial t^2} + E_3\frac{\partial^2\eta}{\partial x\partial t} \quad E_4 = -\frac{\sqrt{Da}}{\alpha},\ B_1 = \frac{1}{2\eta}$$

$$B_2 = \frac{1}{2} + \frac{2E\Pr E_4^2\Omega^2}{\left[M^2\sinh[M\eta] - E_4 M\cosh[M\eta]\right]^2}\left[\frac{\cosh[2M\eta]}{8M^2} - \frac{\eta^2}{4}\right]$$

$$N = M^6[M\sinh[M\eta] - E_4\cosh[M\eta]]^3$$

$$F_{13} = F_1 + F_3$$

Using (16) in (37) and (39), we get the expressions for velocity as:

$$u_0 = -\frac{\Omega}{M^2} - \frac{\Omega E_4\cosh[My]}{\left[M^3\sinh[M\eta] - E_4 M^2\cosh[M\eta]\right]} \tag{41}$$

$$
\begin{aligned}
u_1 = {} & C_1 + MC_2\cosh[\mathrm{My}] + y\sinh[My]\left[\frac{A_1}{2M^2} - \frac{3A_2}{4M^3}\right] + \cosh[My]\left[\frac{A_1}{2M^3} - \frac{5A_2}{4M^4}\right] \\
& + y^2\cosh[My]\left[\frac{A_2}{4M^2}\right] + \cosh[2My]\frac{A_3}{12M^3}
\end{aligned}
\tag{42}
$$

The average velocity $\bar{u}$ over one period of the motion, is given by:

$$\bar{u} = \int_0^1 u dt = u_0 + \delta u_1 + ... \quad (43)$$

The heat transfer coefficient H is given

$$H = \frac{\partial \eta}{\partial x}\frac{\partial \theta_0}{\partial y} + \delta\left[\frac{\partial \theta_0}{\partial x} + \frac{\partial \eta}{\partial x}\frac{\partial \theta_1}{\partial y}\right] \quad (44)$$

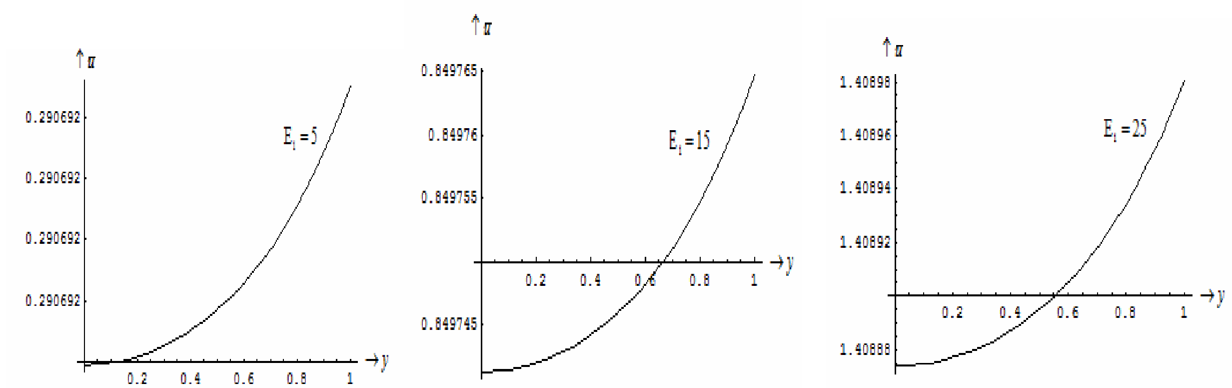
Substituting u_0 and u_1 from (41) and (42) in (43), we get the expression for the average velocity.
Using (38) and (40) in (44), the expression for heat transfer coefficient can be obtained.
The expressions for C_1, C_2, X_1, X_2, A_1 to A_3, Z_1 to Z_{15} and F_1 to F_5 are not given for the sake of brevity.

RESULTS AND DISCUSSIONS

The time average velocity $\bar{u}$ given by Equation (43) has been evaluated using numerical integration and MATHEMATICA software for different values of various parameters with $z(= x - t) = 0.7$. Further, the absolute values of $\bar{u}$ are graphically presented in Figures 1-6.
The Equation (21) shows that E_1, E_2 and E_3 cannot be taken as zero simultaneously.

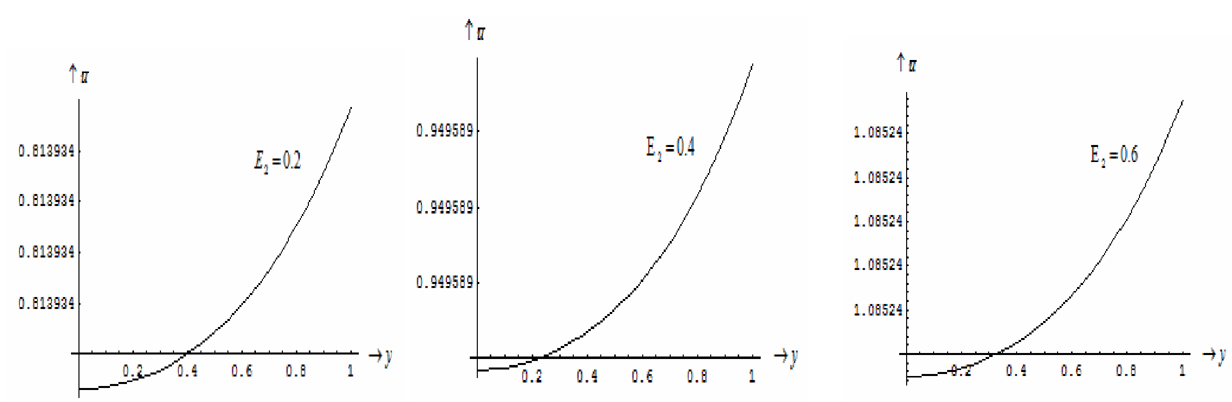
We consider the particular interesting case of stiffness in the wall $(E_2 \neq 0)$ and ------------------------- ------------- inelastic wall $(E_3 \neq 0)$. --------------------------- ---

It can be observed that average velocity increases with rigidity (E_1) [Figures 1(a)-1(c)], stiffness (E_2) [Figures 2(a)-2(c)], viscous damping (E_3) [Figures 3(a)-3(c)] and slip parameter (α) Figures [4(a)-4(c)] but decreases with permeability parameter (Da) [Figures 5(a)-5(c)]. Further, the average velocity increases with magnetic parameter (M) [6(a))-6(c)].



1(a) 1(b) 1(c)

Figure-1. Effect of E_1 on average velocity $\bar{u}$ when $E_2 = 0.2, E_3 = 0.2$ (M = 2, R =10, Da = 0.003, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03$).



2(a) 2(b) 2(c)

Figure-2. Effect of E_2 on average velocity $\bar{u}$ when $E_1 = 1, E_3 = 0.4$ (M = 2, R =10, Da = 0.003, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03$).

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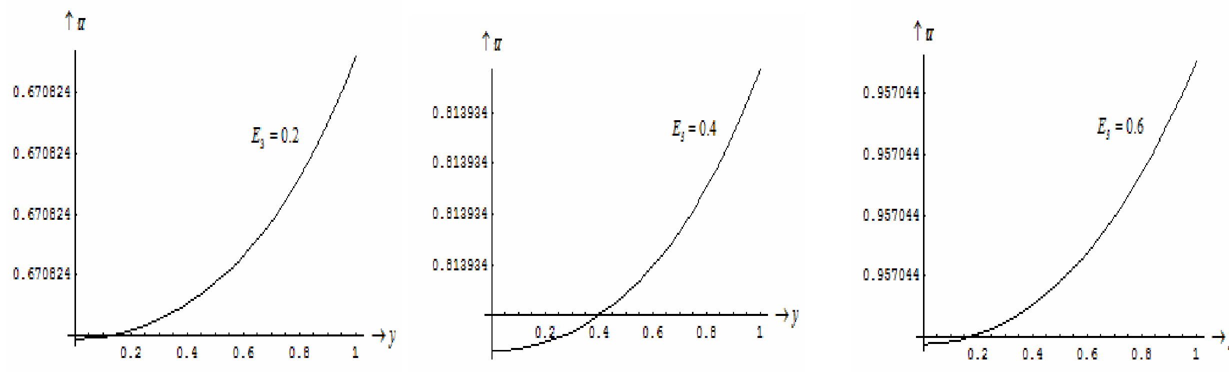
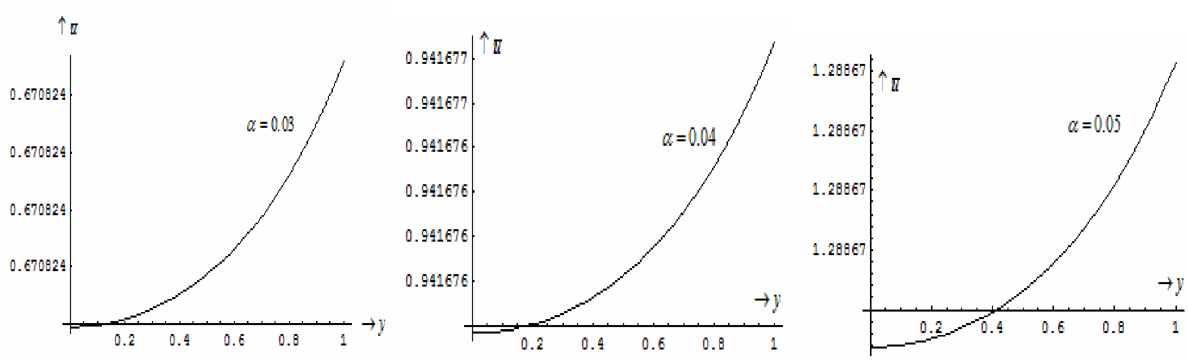


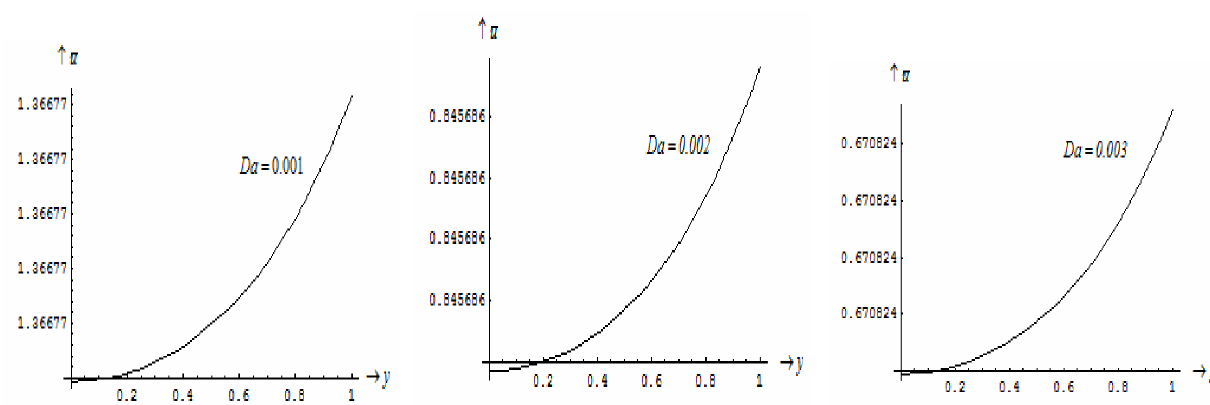
3(a) **3(b)** **3(c)**

Figure-3. Effect of E_3 on average velocity $\bar{u}$ when $E_1 = 1, E_2 = 0.2$ (M = 2, R =10, Da = 0.003, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03$).



4(a) **4(b)** **4(c)**

Figure-4. Effect of α on average velocity $\bar{u}$ when $E_2 = 0.2, E_3 = 0.2$ ($E_1 = 1$, M = 2, R = 10, Da =0.003, $\delta = 0.2, \varepsilon = 0.2$).



5(a) **5(b)** **5(c)**

Figure-5. Effect of *Da* on average velocity $\bar{u}$ when $E_2 = 0.2, E_3 = 0.2$ ($E_1 = 1$, M = 2, R=10, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03$).

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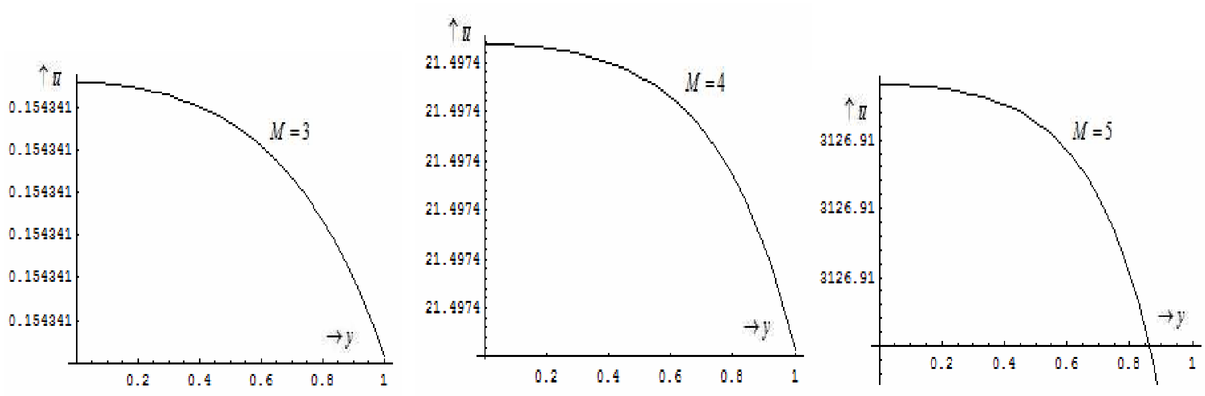
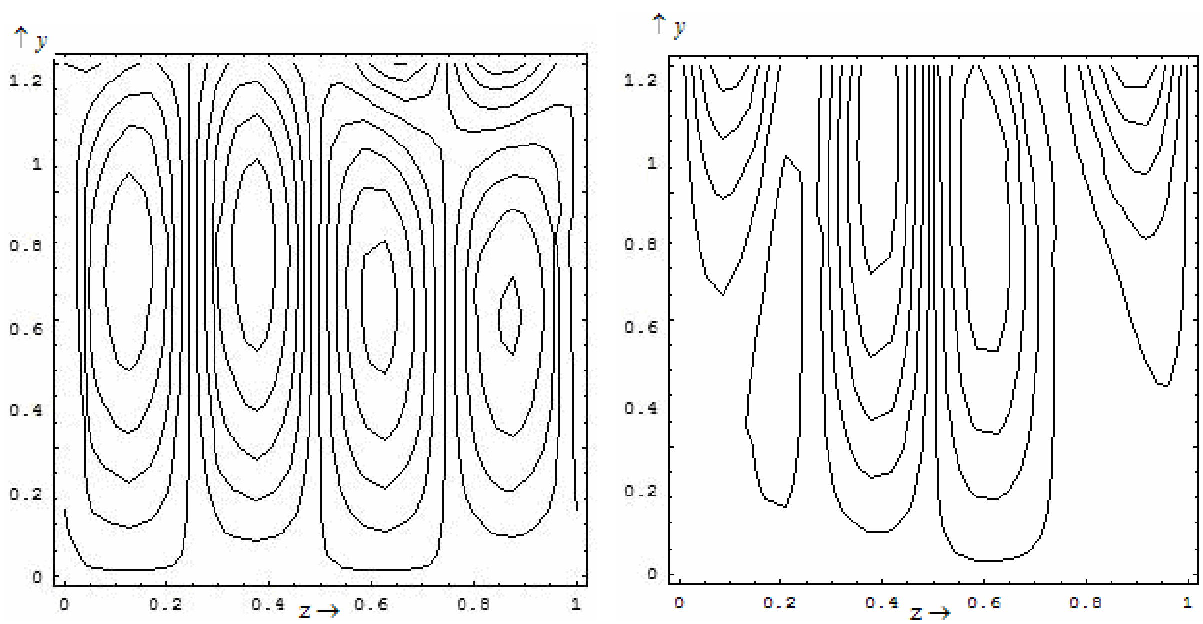


6(a) **6(b)** **6(c)**

Figure-6. Effect of M on average velocity $\bar{u}$ when $E_2 = 0.2, E_3 = 0.4$ ($E_1 = 1$, R = 10, Da = 0.003, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03$).

The effects of various parameters on the stream line pattern are shown in Figures 7-12. It is interesting to note that trapping, an important phenomenon of peristalsis, is observed in all the cases. Further, the bolus formation is more predominant for lower values of rigidity (E_1) [Figures 7(a) and 7(b)] and stiffness parameter (E_2) [Figures 8(a) and 8(b)] but the effect of viscous damping (E_3) on trapping is not very significant [Figures 9(a) and 9(b)]. Figures 10(a) and 10(b) show that trapping phenomenon is more significant for lower values of slip parameter (α) and higher permeability (Da) [Figures 11(a) and 11(b)]. Further, trapping almost disappears as magnetic parameter (M) increases [Figures 12(a) and 12(b)].



7(a) $E_1 = 1$ **7(b)** $E_1 = 20$

Figure-7. Effect of E_1 on stream line pattern ($E_2 = 0.2, E_3 = 0.4, M = 0.2, \varepsilon = 0.2, \delta = 0.2, \alpha = 0.03, Da = 0.003$).

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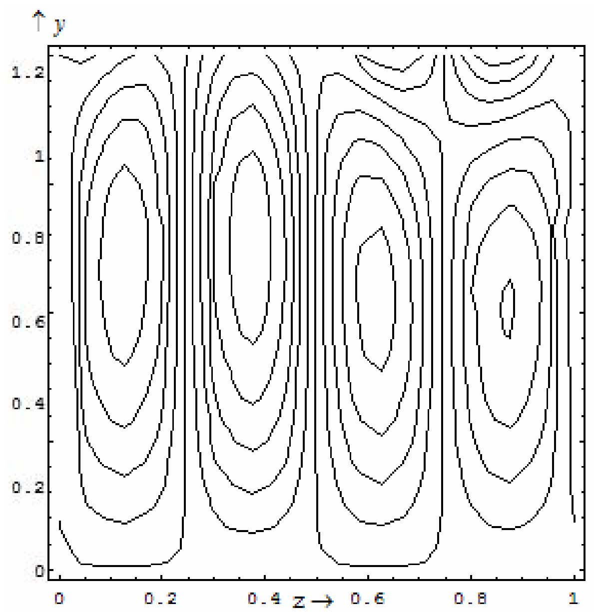
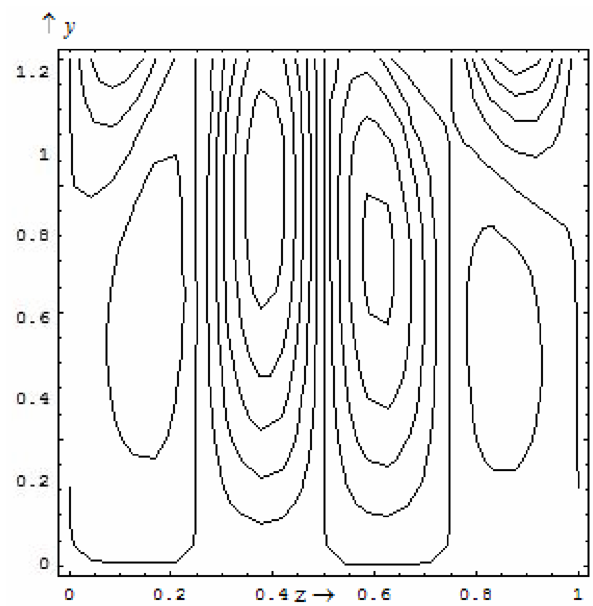
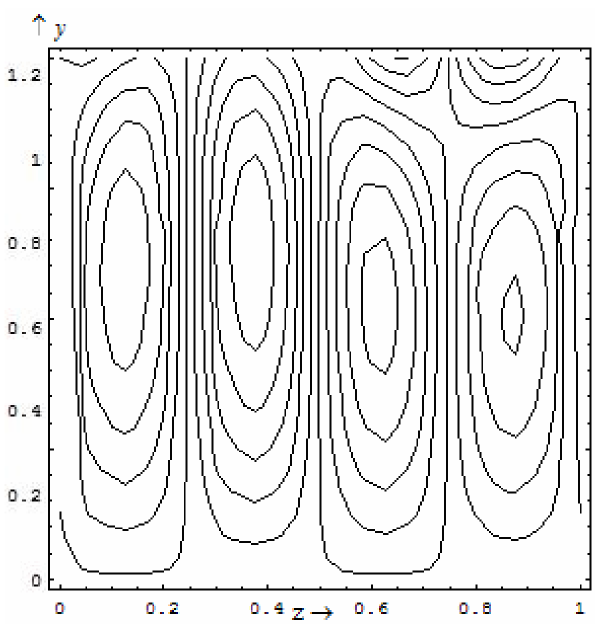


8(a). $E_2 = 1$

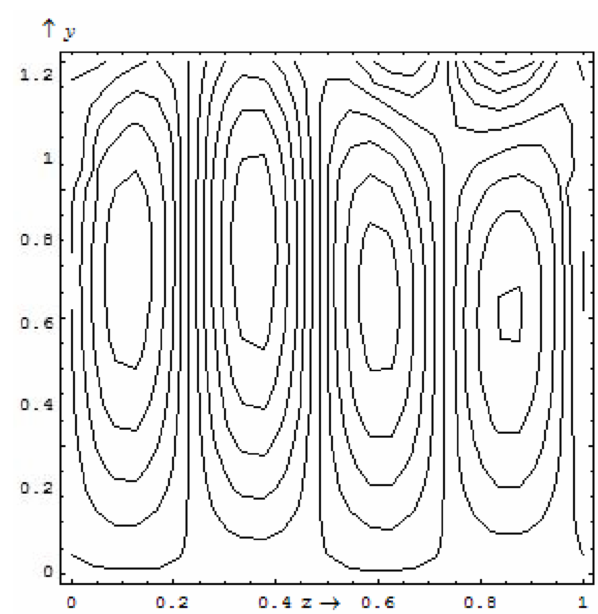


8(b). $E_2 = 8$

Figure-8. Effect of E_2 on stream line pattern ($E_1 = 1, E_3 = 0.4, M = 0.2, \varepsilon = 0.2, \delta = 0.2, \alpha = 0.03, Da = 0.003$).



9(a). $E_3 = 0.2$



9(b). $E_3 = 0.8$

Figure-9. Effect of E_3 on stream line pattern ($E_1 = 1, E_2 = 0.2, M = 0.2, \varepsilon = 0.2, \delta = 0.2, \alpha = 0.03, Da = 0.003$).

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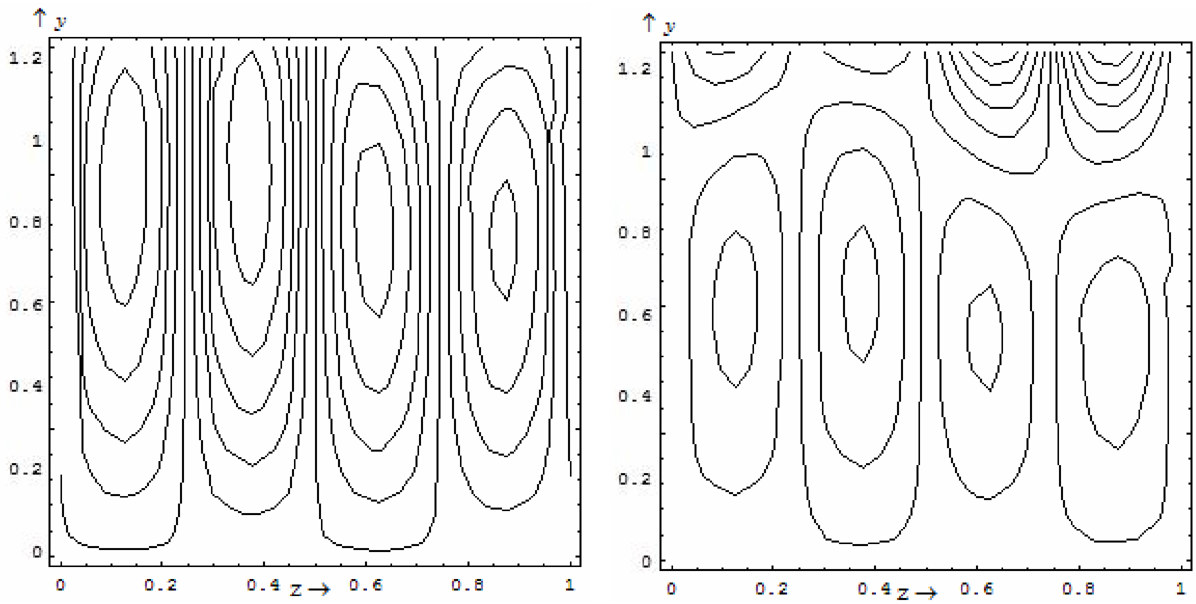
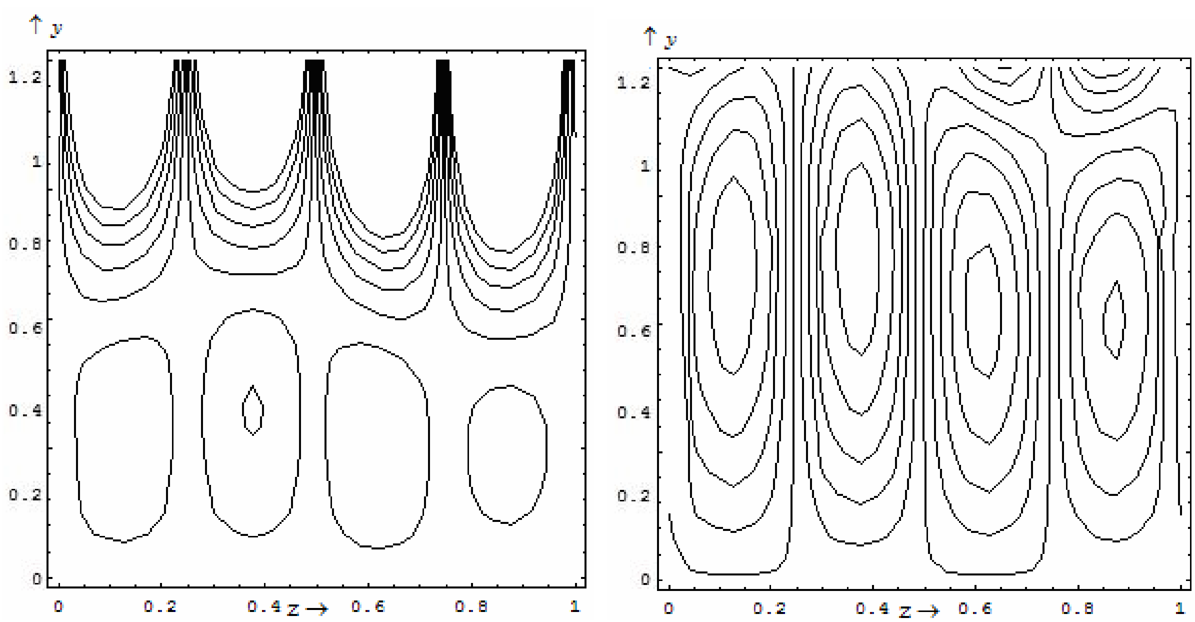


10(a). $\alpha = 0.02$ **10(b).** $\alpha = 0.04$

Figure-10. Effect of α on stream line pattern ($E_1 = 1, E_2 = 0.1, E_3 = 0.2, M = 0.2, \varepsilon = 0.2, \delta = 0.2, Da = 0.003$).



11(a). $Da = 0.001$ **11(b).** $Da = 0.003$

Figure-11. Effect of Da on stream line pattern ($E_1 = 1, E_2 = 0.2, E_3 = 0.2, M = 0.2, \varepsilon = 0.2, \delta = 0.2, \alpha = 0.03$).

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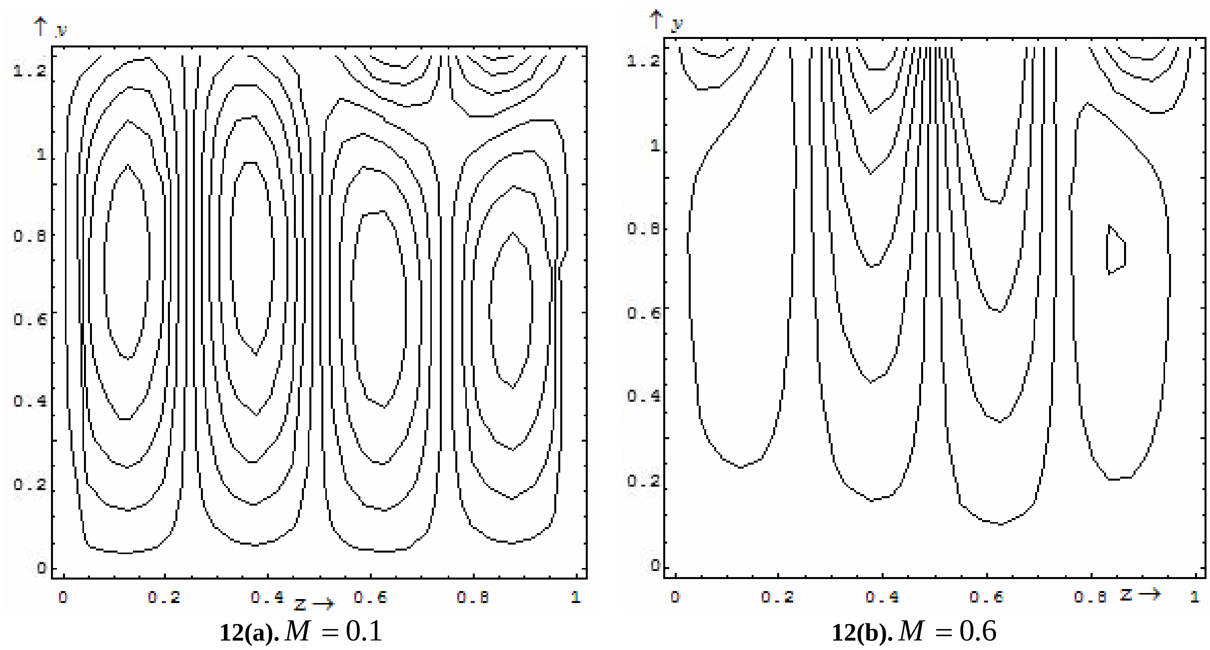

Figure-12. Effect of M on stream line pattern ($E_1 = 1, E_2 = 0.1, E_3 = 0.2, \alpha = 0.03, Da = 0.003$).

It can be noticed that heat transfer coefficient increases with rigidity (E_1) [Figure-13], stiffness (E_2) [Figure-14] and slip parameter (α) [Figure-16] but decreases with viscous damping (E_3) [Figure-15] and permeability parameter (Da) [Figure-17]. Further, heat transfer coefficient increases with magnetic parameter (M) [Figures 18.2(a)-18.2(c)], Prandtl number $(\Pr)$ [Figure-19] and Eckert number (E) [Figure-20]. However, this increase is more significant in the region nearer to the boundary of wall. It may be noted that the heat transfer is negative for lower values of magnetic parameter $(M < 1)$ [Figure-18.1].

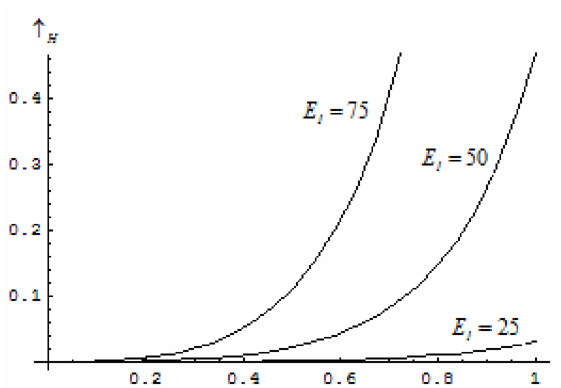

Figure-13. Effect of E_1 on heat transfer coefficient H when $E_2 = 0.15, E_3 = 0.15$ (*M=2, R=10, Da=0.003*, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03, \Pr = 1, E = 1$).

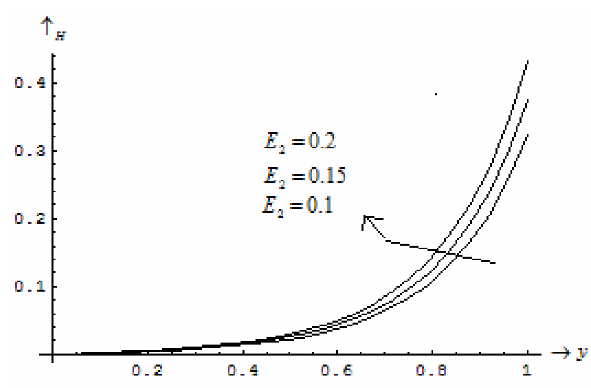

Figure-14. Effect of E_2 on heat transfer coefficient H when $E_1 = 1, E_3 = 0.15$ (*M=2, R=10, Da=0.003*, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03, \Pr = 1, E = 1$).

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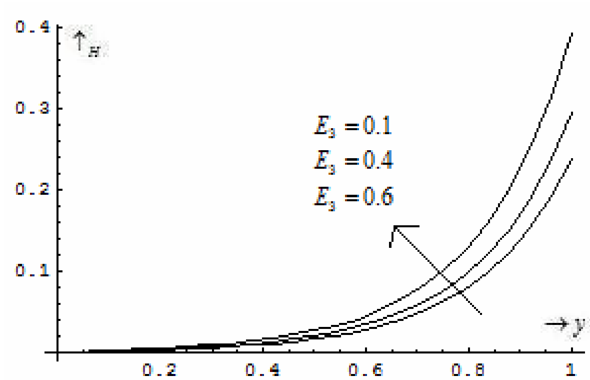


Figure-15. Effect of E_3 on heat transfer coefficient *H* when $E_1 = 1$, $E_2 = 0.15$ (*M=2, R=10, Da=0.003*, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03, \Pr = 1, E = 1$).

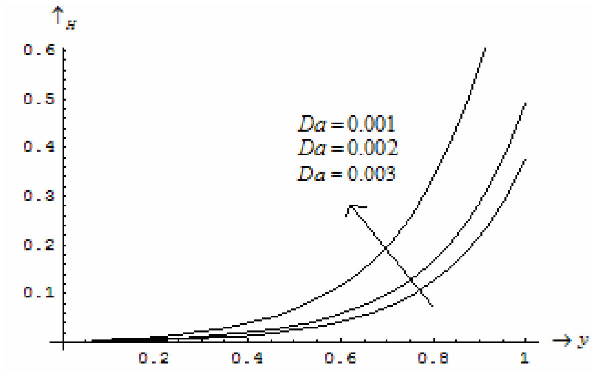


Figure-17. Effect of *Da* on heat transfer coefficient *H* when $E_2 = 0.15$, $E_3 = 0.15$ ($E_1 = 1$, *M=2, R=10*, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03, \Pr = 1, E = 1$).

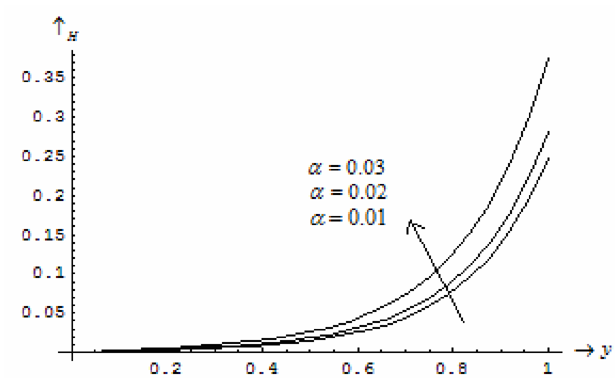


Figure-16. Effect of α on heat transfer coefficient *H* when $E_2 = 0.15$, $E_3 = 0.15$ ($E_1 = 1$ M=2, R=10, Da=0.003, $\delta = 0.2, \varepsilon = 0.2, \Pr = 1, E = 1$).

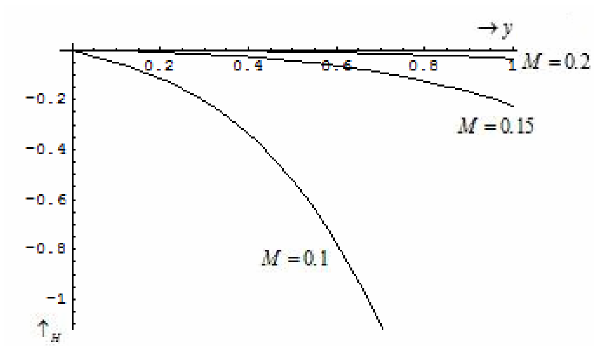
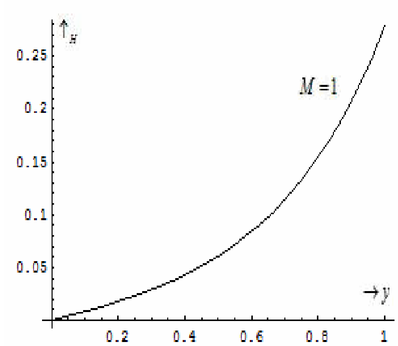
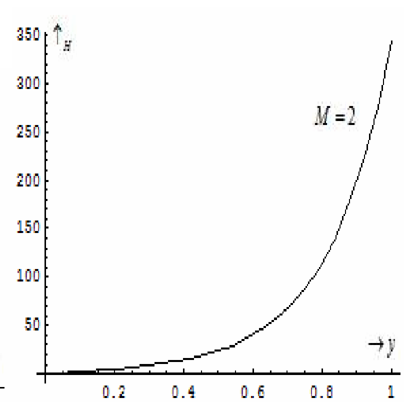


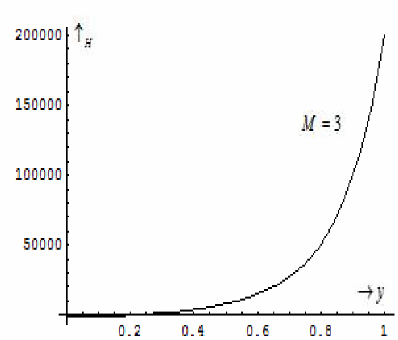
Figure-18.1. Effect of M on heat transfer coefficient *H* when $E_2 = 0.2$, $E_3 = 0.4$ (E_1=1, *R=10, Da=0.003*, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03, \Pr = 1, E = 1$).



18.2(a)



18.2(b)



18.2(c)

Figure- 18.2. Effect of M on heat transfer coefficient *H* when $E_2 = 0.2$, $E_3 = 0.4$ (E_1=1, *R=10, Da=0.003*, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03, \Pr = 1, E = 1$).

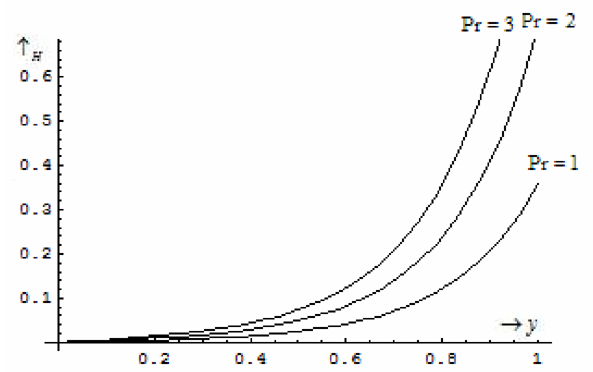


Figure-19. Effect of Pr on heat transfer coefficient *H* when $E_2 = 0.15$, $E_3 = 0.2$ (*M=2*, $E_1 = 1$, *R=10*, *Da=0.003*, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03, E = 1$).

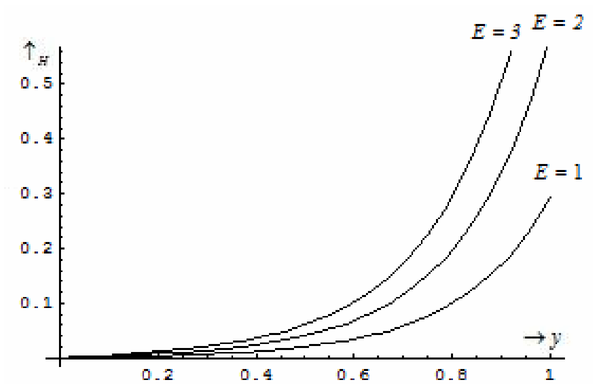


Figure-20. Effect of E on heat transfer coefficient *H* when $E_2 = 0.15$, $E_3 = 0.4$ (*M=2*, $E_1 = 1$, *R=10*, *Da=0.003*, $\delta = 0.2, \varepsilon = 0.2, \alpha = 0.03, \Pr = 1$).

CONCLUSIONS

The effect of slip on peristaltic transport of an incompressible Newtonian fluid in a two dimensional channel with heat transfer under the influence of magnetic field with wall effects has been analyzed. The governing equations have been linearized under long wavelength approximation and the analytical expressions for the average velocity, temperature and heat transfer have been derived.

The time average velocity increases with slip but decreases with permeability. Trapping an important phenomenon of peristalsis has been observed in all the cases. Trapping phenomenon is more significant for lower values of slip parameter and higher permeability but almost disappears for higher values of magnetic parameter. Further, the heat transfer coefficient increases with slip, Prandtl number and Eckert number but decreases with permeability and magnetic parameter.

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