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NODAL ANALYSIS MODELS OF LOOPED WATER DISTRIBUTION NETWORKS

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ABSTRACT

There are three methods for analyzing flow and pressure distribution in looped water supply networks (the loop method, the node method, the element method) taking into consideration hydraulic parameters chosen as unknown. For all these methods, the nonlinear system of equations can be solved by iterative procedures (Hardy–Cross, Newton–Raphson, linear theory). In the case of extending or rehability distribution networks the unknown parameters being the piezometric heads at nodes, the node method for network analysis is preferred. In this paper a generalized classic model for the nodal analysis of complex looped systems with nonstandard network components is formulated and the solvability of new problems, alongside the determination of pressure state in the system. Also, this paper shows a different approach to this problem by using the method of variational formulations for the development of an improved model based on the unconditional optimization procedures. This model has the advantage that it uses a specialized optimization algorithm which minimizes directly an objective multivariable function without constraints, implemented in a computer program. The paper compares proposed models to the classic Hardy-Cross method, and shows the good performance of these models. Based on these models a study regarding implications of pipe network longtime operation on energy consumption is performed.

Keywords: water supply, looped networks, hydraulic analysis, node method, variational formulation, computational model.

1. INTRODUCTION

Water supply of large urban and industrial centers is made by distribution networks sized bigger and bigger, being necessary that, in order to ensure greater uniformity and stability of pressure lines with favorable economic and energy effects, to be achieved in a more complex structure (looped networks, several supply sources, bos-ster pmps, inner potential elements etc.). Also, network extension design or redesign of networks for energy optimization of their operation lead to complicating the general scheme of the system and thus increase the difficulties of calculation it.

Formulation of appropriate mathematical models, which allow the determination of discharge and pressure distribution in looped networks with nonstandard components is essential both for accurate and efficient resolution of design stage, and network analysis in different operating modes (normal or emergency regime). These cases occur especially in the design of network expansions or the redesign of any networks for operational and energy optimization.

There are three methods for analyzing flow and pressure distribution in looped water supply networks (the loop method, the node method, the element method) taking into consideration hydraulic parameters chosen as unknown. For all these methods the nonlinear system of equations can be solved by iterative procedures: Hardy – Cross method [1], [2], [3], [4], Newton-Raphson method [5], [6], [7] and linear theory method [8], [9].

Urban water distribution network has a known configuration, resulted of its design, and service pressures set according to adopted construction types. In time, to an existing network, could be added consumers and potential elements that alter the original pressure distribution and, therefore is necessary an analysis to find solutions to ensure service pressures in all consumer nodes.

Using a sufficient number of simulations may be fixed piezometric head (heads) of supply nod (nodes) and other necessary measures for service pressure ensures and also energy optimization of network. For that purpose, is efficient use of nodal analysis, in which the unknowns are generally piezometric heads.

Although in the node method the equations number is greater than in the loop method, density of nonzero elements of the node equations matrix is less than that of the loop equations [10]. Nodal equations system is easier to be formulated, forming a "rare" matrix of coefficients.

In this paper is formulated a generalized classic model for the nodal analysis of complex looped systems with nonstandard network components and the solvability of new problems, alongside the determination of pressure state in the system, on which base was elaborated a computer program. Also, this paper shows a different approach to this problem by using the method of variational formulations for the development of an improved model based on the unconditional optimization procedures. This model has the advantage that it uses a specialized optimization algorithm which minimizes directly an objective multivariable function without constraints, implemented in a computer program. Based on these models a study regarding implications of pipe network longtime operation on energy consumption is performed.

2. BASIS OF HYDRAULIC ANALYSIS

In the case of a complex topology for a looped network, with reservoirs and pumps at the nodes, the total

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number of independent loops (closed-loops, possibly containing booster pumps installed in the pipes, and pseudoloops) M is given by the following formula:

$$M = T - N + N_{RP} \quad (1)$$

where:

T = number of pipes in network
N = number of nodes
N_{RP} = number of pressure generating facilities, equal to the number of nodes with know hydraulic grade.

Each open-loop (pseudoloop) makes the connection between a node with a known piezometric head (reservoir) or with a determined relation discharge – piezometric head (pump station), and another node with a known piezometric head or a determined relation discharge-piezometric head.

In classical analysis of looped networks, fundamental equations of the computational model express:

Discharge continuity at nodes:

$$\sum_{\substack{i=1 \\ i\neq j}}^{N} Q_{ij} + q_j = 0 \quad (j = 1,...,N - N_{RP}) \quad (2)$$

where:

Q_{ij} = discharge through pipe ij, with the sign (+) when entering node j and (–) when leaving it
q_j = consumption discharge (demand) at node j with the sign (+) for node inflow and (–) for node outflow.

Energy conservation in loops:

$$\sum_{\substack{ij\in m \\ ij=1}}^{T} \varepsilon_{ij} h_{ij} - f_m = 0 \quad (m = 1,...,M) \quad (3)$$

where:

h_{ij} = head loss of the pipe ij
ε_{ij} = orientation of flow through the pipe, having the values (+1) or (–1) as the water flow sense is the same or opposite to the path sense of the loop m, and (0) value if $ij \notin m$
f_m = pressure head introduced by the potential elements of the loop m, given by the relations:

- simple closed–loops:

$$f_m = 0 \quad (4)$$

- closed-loops containing bosster pmps installed in the pipes:

$$f_m = \sum_{\substack{ij\in m \\ ij=1}}^{T} \varepsilon_{ij} H_{p,ij} \quad (5)$$

- open-loops with pumps and/or reservoirs at nodes:

$$f_m = Z_I - Z_E \quad (6)$$

where:

Z_I, Z_E = piezometric heads at pressure devices at the entrance or exit from the loop
$H_{p,ij}$ = the pumping head of the bosster pump integrated on the pipe ij, for the discharge Q_{ij}, approximated by parabolic interpolation on the pump curve given by points:

$$H_{p,ij} = AQ_{ij}^2 + B\left|Q_{ij}\right| + C \quad (7)$$

the coefficients A, B, C can be determined from three points of operating data [11].

The head loss is given by the Darcy-Weisbach equation:

$$h_{ij} = \frac{8}{\pi^2 g} \lambda_{ij} \frac{L_{ij}}{D_{ij}^r} Q_{ij}^2 \quad (8)$$

where:

g = gravitational acceleration
λ_{ij} = friction factor of pipe ij which can be calculated using the Colebrook-White formula or the explicit equation proposed in [12] for the transitory turbulence flow
D_{ij}, L_{ij} = diameter and the length of pipe ij
r = exponent having the value 5.0.

Equation (8) is difficult to use in the case of pipe networks and therefore it is convenient to write it in the following general form:

$$h_{ij} = R_{ij} Q_{ij}^{\beta} \quad (9)$$

where,

R_{ij} is the hydraulic resistance of pipe ij, having the succeeding expression:

$$R_{ij} = \frac{8\lambda_*}{\pi^2 g} \frac{L_{ij}}{D_{ij}^r} \quad (10)$$

The variation of hydraulic parameters λ_* and β has been determined for different pipe materials and water temperatures θ (Figure-1), using a computer program.

Specific consumption of energy for water distribution w_{sd}, in kWh/m^3, is obtained by referring the hydraulic power dissipated in pipes to the sum of node discharges:

$$w_{sd} = 0.00272 \frac{\sum_{ij=1}^{T} R_{ij} \left|Q_{ij}\right|^{\beta+1}}{\sum_{\substack{j=1 \\ q<0}}^{N} \left|q_j'\right|} \quad (11)$$

where,

q_j' is the outflow at the node j.

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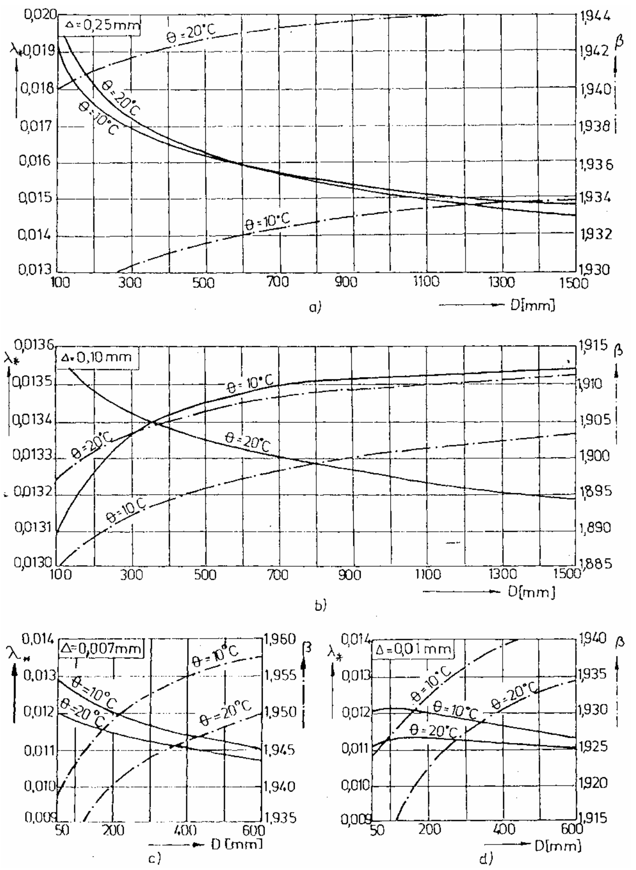

Figure-1. Variation diagram of hydraulic parameters λ and β a) reinforced concret and cast iron; b) steel; c) PE-HD; d) PVC.

3. PRINCIPLE OF NODE METHOD

In case that is required analysis of pressure state in a distribution network or network contains several potential elements and fittings is suitable, the use of piezometric heads as unknown, i.e., the nodal equations. Equation (9) can be written as follows:

$$h_{ij} = Z_i - Z_j + \Pi_{ij} = R_{ij} Q_{ij} \left| Q_{ij} \right|^{\beta-1} \quad (12)$$

or:

$$Q_{ij} = R_{ij}^{-\frac{1}{\beta}} h_{ij}^{\frac{1}{\beta}} = R_{ij}^{-\frac{1}{\beta}} \left(Z_i - Z_j + \Pi_{ij} \right) \left| Z_i - Z_j + \Pi_{ij} \right|^{\frac{1-\beta}{\beta}} \quad (13)$$

in which: Z_i and Z_j are the piezometric heads at nodes i and j; Π_{ij} – active pressure introduced by the intermediate pump on the pipe ij.

Substituting equation (13) in equation (2) one gets a system of N–N_{RP} equations at nodes with N–N_{RP} unknown:

$$f_j = \sum_{\substack{i \neq j \\ i=1}}^{N} R_{ij}^{-\frac{1}{\beta}} \left(Z_i - Z_j + \Pi_{ij} \right) \left| Z_i - Z_j + \Pi_{ij} \right|^{\frac{1-\beta}{\beta}} + q_j = 0$$

$$(j = 1, \ldots, N - N_{RP}) \quad (14)$$

By solving linear algebric equation system (14) piezometric heads in N–N_{RP} nodes are determined because in other N_{RP} nodes piezometric heads must be known (at least N_{RP} = 1), and then, with equations (12) and (13) the head losses and discharges in the pipes are calculated.

The node method principle consists in assumption a set of initial piezometric heads in nodes as known, which are corrected successively until residue of discharge in nodes f_j becomes as small as possible. That is why this principle is named “discharges equalization principle” [1]. To achieve this objective, is suitable use of Newton-Raphson numerical algorithm, with some precautions to avoid singular points, taking into account ease of construction and implementation in a computer program of it.

4. NODAL ANALYSIS MODEL IN THE CLASSIC FORMULATION

Nodal analysis model calls the following basic data: network topology, lengths, diameters and roughness of pipes, elevation heads and concentrated discharges for each node; active pressure on pipes; piezometric heads in one or several nodes of network (critical points, pressure generating facilities).

- Numerically solving of the nonlinear algebric equation system generated by equation (14) is performed with Newton–Raphson algorithm in following main steps:

a) *Establishment of an initial approximation* (k = 0) of piezometric heads $Z^{(0)} = \{ Z_1^{(0)} \; Z_2^{(0)} \; \ldots \; Z_N^{(0)} \}$, which are made in equation (14), admitting for relation (13) a linear form:

$$\sum_{\substack{i \neq j \\ i=1}}^{N} R_{ij}^{-1} (Z_j^{(0)} - Z_i^{(0)} - \Pi_{ij}) = q_j^{\beta} \;\; (j = 1, \ldots, N - N_{RP}) \quad (15)$$

b) *Determination of the correction vector* $\Delta Z = \{\delta Z_1 \; \delta Z_2 \; \ldots \; \delta Z_N\}$ at some iteration (k+1) is made solving the following linear algebric equation system:

$$\begin{bmatrix} \frac{\partial f_1}{\partial Z_1} \cdots\cdots \frac{\partial f_1}{\partial Z_N} \\ \vdots \qquad\quad \vdots \\ \frac{\partial f_N}{\partial Z_1} \cdots\cdots \frac{\partial f_N}{\partial Z_N} \end{bmatrix} \begin{Bmatrix} \delta Z_1 \\ \vdots \\ \delta Z_N \end{Bmatrix} = \begin{Bmatrix} -f_1 \\ \vdots \\ -f_N \end{Bmatrix} \quad (16)$$

in which partial derivatives are obtained from (14).

Because N_{RP} referential piezometric heads must be imposed, system (14) is solved for N–N_{RP} unknowns;

c) *Change of the unknowns vector Z* according to the equation:

$$Z^{(k+1)} = Z^{(k)} + \theta \Delta Z^{(k+1)} \quad (17)$$

in which weight factor $\theta \in (0, 1]$, and the correction vector from iteration (k +1) is the term $\Delta Z^{(k+1)}$;

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d) Steps *b* and *c* are performed iteratively until is *achieved calculation accuracy* ε established by condition:

$$\left|-f_j^{(k)}\right| \le \varepsilon \quad (j = 1,...,N - N_{RP}), \tag{18}$$

or the maximum number of iterations allowed.

After the determination of piezometric heads Z_j, could be easy calculated also discharges in pipes with relation (13), also other hydraulic parameters characteristic for network (available pressure, velocities etc.)

- Although node equations are easily generated, they are accompanied by some calculation difficulties such occurrence oscillations around the solution [6] and existence of singular points ($Z_i+\Pi_{ij} \cong Z_j$) of the Jacobian, resulted from pipes with small head losses.

To eliminate difficulties due to instability in singular points, proceed to a cubic spline–type regularization for function having form sgn $(Z_i–Z_j+\Pi_{ij})|Z_i–Z_j+\Pi_{ij}|^x$, replacing functions f_j $(Z_i–Z_j)$ expressed by (14) with functions:

$$f_j = \begin{cases} \left|Z_i - Z_j + \Pi_{ij}\right| > \omega : \\ \sum\limits_{\substack{i \ne j \\ i=1}}^{N} R_{ij}^{-1}\left(Z_i - Z_j + \Pi_{ij}\right)\left|Z_i - Z_j + \Pi_{ij}\right|^{x-1} + q_j = 0; \\ \left|Z_i - Z_j + \Pi_{ij}\right| \le \omega : \\ \sum\limits_{\substack{i \ne j \\ i=1}}^{N} \omega^x R_{ij}^{-x} \dfrac{Z_i - Z_j + \Pi_{ij}}{\omega}\left[(x-1)\left(\dfrac{Z_i - Z_j + \Pi_{ij}}{\omega}\right)^2 + \right. \\ \left. \qquad + 3 - x\right] + q_j = 0; \end{cases} \tag{19}$$

where, $x = 1/\beta$ and ω is chosen conveniently ($10^{-4}...10^{-5}$). Partial derivatives are obtained from (19) as follows:

$$\frac{\partial f_j}{\partial Z_i} = \frac{\partial f_i}{\partial Z_j} = \begin{cases} \left|Z_i - Z_j + \Pi_{ij}\right| > \omega : \\ xR_{ij}^{-x}\left|Z_i - Z_j + \Pi_{ij}\right|^{x-1}; \\ \left|Z_i - Z_j + \Pi_{ij}\right| \le \omega : \\ \omega^x R_{ij}^{-x}\left[\dfrac{3}{\omega}(x-1)\left(\dfrac{Z_i - Z_j + \Pi_{ij}}{\omega}\right)^2 + \dfrac{3-x}{\omega}\right] \end{cases} \tag{20}$$

$$\frac{\partial f_j}{\partial Z_j} = -\sum_{\substack{i \ne j \\ i=1}}^{N} \frac{\partial f_j}{\partial Z_i} \tag{21}$$

• After examination of the relation (14), it is observed that fulfilment of discharge continuity at nodes could be accomplished admitting as variables not only piezometric heads Z_j, but also hydraulic resistances R_{ij} and concentrated discharges in nodes q_j, conditioned that the sum of all those unknown to be $N–N_{RP}$, so the use of the model could be extended to new problems solving.

Notting the unknowns piezometric heads with $\overline{Z} = \{Z_1...Z_w\}$, hydraulic resistances with $\overline{R} = \{R_{ij}...R_{pr}\}$ and concentrated discharges at nodes with $\overline{q} = \{q_j...q_n\}$, and starting from initial vector $X^{(0)} = \{Z_1^{(0)}...Z_w^{(0)} R_{ij}^{(0)}...R_{pr}^{(0)} q_j^{(0)}...q_n^{(0)}\}$ could be determined corrections for each iteration from linear system:

$$\begin{bmatrix} \dfrac{\partial f_1}{\partial Z_1} \cdots\cdots \dfrac{\partial f_1}{\partial q_n} \\ \vdots\ \vdots \\ \dfrac{\partial f_{w+1}}{\partial Z_1} \cdots\cdots \dfrac{\partial f_{w+1}}{\partial q_n} \\ \vdots\ \vdots \\ \dfrac{\partial f_N}{\partial Z_1} \cdots\cdots \dfrac{\partial f_N}{\partial q_n} \end{bmatrix} \begin{Bmatrix} \delta Z_1 \\ \vdots \\ \delta R_{ij} \\ \vdots \\ \delta q_n \end{Bmatrix} = \begin{Bmatrix} -f_1 \\ \vdots \\ -f_{w+1} \\ \vdots \\ -f_N \end{Bmatrix} \tag{22}$$

in which partial derivatives on R_{ij} and q_j have expressions:

$$\frac{\partial f_j}{\partial R_{ij}} = \begin{cases} \left|Z_i - Z_j + \Pi_{ij}\right| > \omega : \\ -xR_{ij}^{-(x+1)}(Z_i - Z_j + \Pi_{ij})\left|Z_i - Z_j + \Pi_{ij}\right|^{x-1}; \\ \left|Z_i - Z_j + \Pi_{ij}\right| \le \omega : \\ -x\omega^{-x} R_{ij}^{-(x+1)} \dfrac{Z_i - Z_j + \Pi_{ij}}{\omega}\left[(x-1)\left(\dfrac{Z_i - Z_j + \Pi_{ij}}{\omega}\right)^2 + \right. \\ \left. +3 - x\right] \end{cases} \tag{23}$$

$$\frac{\partial f_j}{\partial q_j} = 1 \tag{24}$$

Algebric equation system (22) has no solution for those combinations of unknown that lead to the existence of lines or columns in the matrix of system with all the terms null. Therefore the choice of these combinations must respect some rules:

- for each node there must exist at least one of following unknown: concentrated discharge at node, piezometric head for that node or any adjacent node, hydraulic resistance of any pipe that meet in the node;
- a node that has concentrated discharge unknown shall be connected to at least one other node with known discharge;
- a pipe with hydraulic resistance unknown must not have more than one unknown at the nodes that define it (either piezometric head or used discharge in the node).

Using Newton-Raphson algorithm for solving system of linear equations (19) has following advantages:

- Jacobian matrix contains at most $N+2T$ non-zero elements [10], which gives the property to be "rare";
- In most cases this matrix is symmetric, irreducible and weakly dominant diagonal, which ensures the existence of inverse matrix;

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- Moreover, the matrix inverse is a positive matrix, property that gives qualities of numerical stability in solving linear algebric system (22) for each iteration.

Based on this nodal analysis model, a computer program ANOREC [13] was developed in the FORTRAN programming language for IBM-PC compatible micro systems.

5. NODAL ANALYSIS MODEL IN VARIATIONAL FORMULATION

If instead of classical equations (14) using relations (3), (13) and a performance function that express energy consumption in network, nodal analysis of looped water distribution networks may be made with an unconditional optimization model.

Thus, in variational formulation of nodal analysis of looped networks, determination of piezometric heads Z_j is performed on the criterion of minimizing energy consumption in network per time unit (power), expressed analytical by objective function:

$$F_{\mathrm{e}} = \sum_{ij=1}^{T} \left(\int_{0}^{h_{ij}} Q_{ij}\, \mathrm{d}h_{ij} \right) - \sum_{j=1}^{N} \left(\int_{0}^{Z_j} q_j\, \mathrm{d}Z_j \right) \rightarrow \min \qquad (25)$$

subject to constraints (3) of energy conservation in loops. After substituting the equation (13) in (25) and after the integrals have been calculated the constraints can be eliminated and the problem can be simplified to the finding of the minimum of a function with $N–N_{RP}$ variables (Z_i, Z_j) without constraints:

$$F_{\mathrm{e}} = \frac{\beta}{\beta+1} \sum_{ij=1}^{T} R_{ij}^{-\frac{1}{\beta}} \left| Z_i - Z_j + \Pi_{ij} \right|^{\frac{\beta+1}{\beta}} - \sum_{j=1}^{N} q_j Z_j \rightarrow \min, \qquad (26)$$

Using the extremum requirements $\partial F_e/\partial Z_j = 0$ (j = 1... $N–N_{RP}$) one gets the system of node equations (14).

Variational formulation reduces considerably the magnitude of the problem, reaching out from a system with $N–N_{RP}$ independent nonlinear equations with $N–N_{RP}$ unknowns and M constraints to only one function with $N–N_{RP}$ unknowns, without constraints. This formulation becomes advantageous using a specific algorithm for direct minimization of function (26), such as conjugate gradient algorithm recommended in the literature [14], [10], [15]. Permissible error in calculation ε is considered equal to 10^{-5}.

Having determined the piezometric head at nodes, the available pressure head H_j at nodes is calculated using equation:

$$H_j = Z_j - ZT_j \qquad (27)$$

where, ZT_j is the elevation head at node j.

Then the discharges Q_{ij} through pipes are determined with functional equation (13) and also other hydraulic parameters of the network.

Based on nodal analysis model in variational formulation a computer program ANOREV [13] was developed in FORTRAN programming language for IBM-PC compatible micro systems.

6. APPLICATIONS

6.1 Determination of discharge and pressure distribution in a looped network

The looped distribution network with the topology from Figure-2 is considered. It is made of cast iron and is supplied with a discharge of 0.50 m^3/s.

The following data are known: pipe length L_{ij}, in m, pipe diameter D_{ij}, in m, elevation head ZT_j, in m, industrial concentrated discharges in nodes q_j, in m^3/s, piezometric head at the "critical node" Z_1 = 124 m, and the exponent β = 1.936.

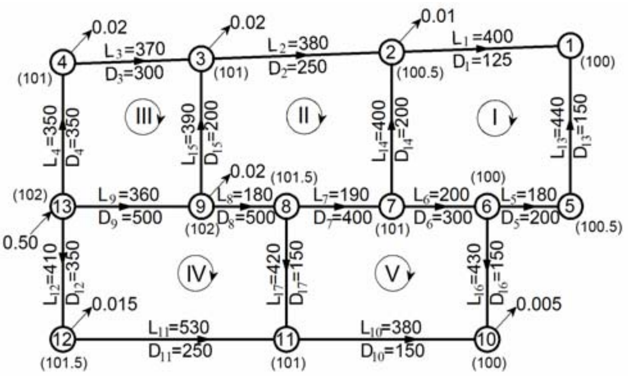

Figure-2. Scheme of analyzed distribution network.

It is required to determine the pumping head, discharges and pressures distribution using the classic HARDY–CROSS procedure and the two nodal analysis models (ANOREC, ANOREV) developed above. Results of the numerical solution performed by means of an IBM computer, referring to the hydraulic characteristics of the pipes and nodes are presented in Tables 1 and 2.

Table-1 shows the discharges and head losses through pipes established by using the three mentioned models of computation (the iterative tolerance imposed is 10^{-5}). It can be seen that the results are very close. The difference between the discharges obtained with HARDY–CROSS and these given by ANOREC vary between 0.01 % (pipe 9-8) to 1.8% (pipe 8–11), and the difference between discharges obtained with HARDY–CROSS and ANOREV varies from 0 % (pipe 7-6) to 2.8% (pipe 9-8). Specific consumption of energy for water distribution is 0.00705 kWh/m^3 for all three computational models used.

Table-2 presents the values for the piezometric head Z_j and the residual pressure head H_j at nodes determined by using the classic procedure and the two new models of computation. The piezometric head at the node 13 has the following values: 131.435 m, 131.356 m and 131.363 m which give a residual pressure head of 29.435 m, 29.356 m and 29.363 m that is sufficient for the supply water to the consumers. The divergence of piezometric line on network contour is 0.216 m for HARDY-CROSS, 0.001 m for ANOREV and only 0.0 m for ANOREC.

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Table-1. The discharges and head losses trough pipes.

Pipe $i-j$	Computational model					
	HARDY– CROSS		ANOREC		ANOREV	
	Q [m^3/s]	h [m]	Q [m^3/s]	h [m]	Q [m^3/s]	h [m]
0	1	2	3	4	5	6
2–1	0.01210	3.955	0.01219	3.754	0.01204	3.915
3–2	0.03618	0.895	0.03616	0.915	0.03605	0.889
4–3	0.06957	1.217	0.06902	1.251	0.06916	1.203
13–4	0.11413	1.369	0.11358	1.437	0.11371	1.356
6–5	0.03770	1.443	0.03762	1.428	0.03776	1.447
7–6	0.08206	0.907	0.08174	0.938	0.08206	0.907
8–7	0.13518	0.521	0.13496	0.559	0.13531	0.521
9–8	0.17542	0.261	0.17544	0.288	0.18034	0.276
13–9	0.25264	1.064	0.25318	1.172	0.25266	1.065
11–10	0.01589	2.495	0.01614	2.469	0.01610	2.558
12–11	0.04797	2.156	0.04798	2.206	0.04806	2.163
13–12	0.09503	1.122	0.09504	1.192	0.09501	1.122
5–1	0.01655	3.124	0.01647	2.972	0.01661	3.147
7–2	0.02618	1.578	0.02628	1.584	0.02625	1.587
9–3	0.02549	1.461	0.02603	1.516	0.02579	1.495
6–10	0.01673	3.120	0.01649	2.291	0.01661	3.075
8–11	0.01329	1.951	0.01353	1.938	0.01327	1.945

Table-2. The piezometric head and the available pressure head at nodes.

Node j	Computational model					
	HARDY – CROSS		ANOREC		ANOREV	
	Z_j [m]	H_j [m]	Z_j [m]	H_j [m]	Z_j [m]	H_j [m]
0	1	2	3	4	5	6
1	124.000	24.000	124.000	24.000	124.000	24.000
2	127.955	27.455	127.754	27.254	127.915	27.415
3	128.850	27.850	128.6681	27.668	128.804	27.804
4	130.066	29.066	129.920	28.920	130.007	29.007
5	127.124	26.624	126.972	26.472	127.147	26.647
6	128.566	28.566	128.399	28.399	128.594	28.594
7	129.533	28.533	129.338	28.338	129.502	28.502
8	130.053	28.553	129.896	28.396	130.023	28.523
9	130.371	28.371	130.184	28.184	130.299	28.299
10	125.446	25.446	125.490	25.490	125.520	25.520
11	128.157	27.157	127.958	26.958	128.078	27.078
12	130.313	28.813	130.165	28.665	130.241	28.741
13	131.435	29.435	131.356	29.356	131.363	29.363

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6.2 Implications of pipes longtime operation on energy consumption for water pumping

Presented computational models, together with those existing in literature, mostly solved the problem of distribution networks analysis or design. Computers removes the calculation difficulties created by complexity and different network operating assumptions, remaining that specialists focus their efforts to establish accurate basic data, which conditiones the precision of results.

Special attention should be paid to sufficiently accurate assessment of pipe roughness influencing the calculation of head losses, because they have a significant influence on energy consumption and over optimal network solution in a water supply system.

In distribution network design new, clean and properly mounted pipes are considered and their absolute roughness is adopted according to the pipe material.

Many measurements made by different researchers [16], [17] show an increase in pipe roughness due to corrosion, depositions of material and aging which has the consequence of their reduced capacity of transport up to 50%. To achieve discharge established in design calculation must be increased the hydraulic slope and thus proportional pumping energy.

Also, residues of iron and minerals deposited in pipes and their corrosion products lead to changes in water quality from network.

For water pipes, taking into account properties of the water in connection with the formation of deposits in pipes, Kamerstein [17] propose division into five groups of natural waters, which correspond to the same number of average growth rates of roughness, each of them determining the reduction percentage of pipe transport capacity.

Reduction of the transported flow by pipe with its operating duration can be expressed by the equation:

$$Q_{ij} = Q_{ij}^{(0)}\left(1 - 0{,}01 n_0 \tau^{m_0}\right) \qquad (28)$$

where:

$Q_{ij}^{(0)}$ = calculated transport capacity of pipe

τ = operating duration, in years

n_0, m_0 = addiction parameters of physic–chemical properties of water, having the most likely mean values given in Table-3.

Absolute roughness variation, depending on the number of operating years, could be expressed with equation obtained based on determinations by Kamerstein:

Table-3. Values of parameters ω, n_0, m_0 depending on physic–chemical properties of water.

Group	Water properties	ω [mm/year]	n_0	m_0
0	1	2	3	4
I	Poorly mineralized water, non–corrosive. Water containing little organic matter and dissolved iron.	0.025	2.3	0.50
II	Poorly mineralized water, corrosive. Water containing dissolved organic matter and iron less than 3mg/dm^3.	0.070	2.3	0.50
III	Corrosive water containing little amount of chlorides and sulfates Water with iron content of over 32 mg/dm^3.	0.200	6.4	0.50
IV	Corrosive water with high content of chlorides and sulfates (over 500 ... 700 mg/dm^3). Untreated water with high content of organic substances.	0.510	11.6	0.40
V	Highly mineralized water (fixed mineral residue over 2000 mg/dm^3) and corrosive, with high carbonate hardness and permanent hardness reduced.	0.800	18.0	0.35

$$\Delta = \Delta_0 + \omega\tau \qquad (29)$$

where:

Δ_0 = initial value of absolute roughness

ω = average growth rates of roughness (Table-3).

Because growth rates of roughness in time depend on several factors, laboratory carrying out of real rough is very difficult, being necessary direct determinations on networks in operation for precise appreciation of pipe absolute roughness.

In the absence of reliable data from tests under realistic conditions, calculations can be performed with values given by equation (29). Based on this equation has been determined variation of roughness coefficient λ_* and discharge exponent β in function of ω for pipes manufactured by different materials (reinforced concrete, cast iron, steel, PVC and PE–HD) with operating durations of 10, 25, 50 years, for average water temperature of 15^{o}C. The results are represented in the diagrams from Figure-3.

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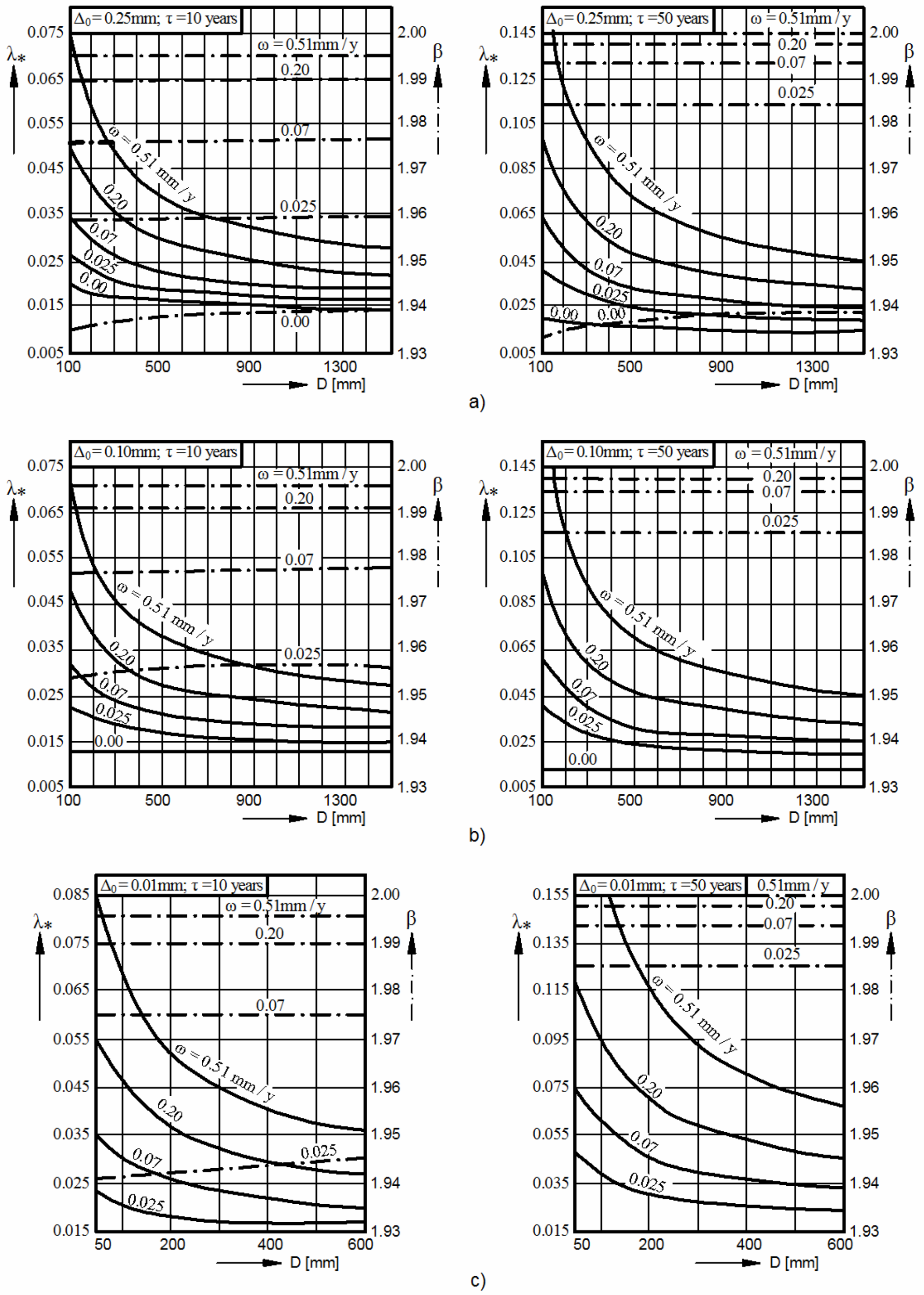

Figure-3. Variation of hydraulic parameters λ_* and β for pipes
a–reinforced concrete and cast iron; *b*–steel; *c* – PVC and PE-HD.

The main phenomena occurred in operation, leading to increased pipe roughness are deposits of material (clogging) and corrosion or erosion of pipe material.

Very low or zero velocity of the water for longer periods of time facilitate clogging and deposits consolidation in the presence of carbonate. In addition of mechanical clogging occurs and biological clogging.

The main causes leading to the formation of deposits in pipes are:

- suspended substances in water, for untreated industrial water networks;
- temporary hardness of water;

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- corrosive action of transported water, that lead to iron oxide deposits formation;
- biological action due to ferruginous bacteria and some protozoa.

Both corrosion and clogging of pipes by increasing roughness increase energy losses, with negative effects on pressure and flow distribution in network. In this case, result a preferential water supply of some points over others.

Defining *pressure distribution stability* in network as ratio between the minimum pressure (at the maximum consumption in critical point) and maximum pressure (for zero consumption):

$$\sigma_H = \frac{H_{\min}}{H_{\max}} \tag{30}$$

and taking into account the similitude relations of centrifugal pumps, is defined and *discharge distribution stability*:

$$\sigma_Q = \frac{Q_{\min}}{Q_{\max}} = \sqrt{\frac{H_{\min}}{H_{\max}}} \tag{31}$$

For illustration of the above mentioned the looped distribution network is considered in Figure-2. It has been calculated using the computer program ANOREV for the available pressure head at nodes after 10, 25 and 50 years of operation with two growth rates of roughness, and also pressure and discharge stability. The numerical results are presented in Table-4.

Table-4. Available pressure head at nodes, H_j [m].

Node *j*	ω [mm/year]						
	0.00	**0.025**			**0.200**		
	Project	τ [years]			τ [years]		
		10	**25**	**50**	**10**	**25**	**50**
0	1	2	3	4	5	6	7
1	24.000	22.859	22.371	20.453	18.202	13.364	7.284
2	27.455	26.502	26.295	25.485	24.548	22.587	20.213
3	27.850	26.528	26.374	25.776	25.087	23.651	21.920
4	29.066	28.855	28.770	28.443	28.069	27.293	26.364
5	26.624	26.021	25.754	24.719	23.525	21.024	17.986
6	28.566	28.283	28.122	27.498	26.784	25.312	23.568
7	28.533	28.224	28.231	27.831	27.375	26.443	25.355
8	28.553	28.415	28.338	28.043	27.709	27.034	26.248
9	28.371	28.213	28.151	27.912	27.643	27.099	26.467
10	25.446	24.731	24.353	22.870	21.133	17.416	12.773
11	27.157	26.645	26.438	25.631	24.695	22.724	20.315
12	28.813	28.644	28.576	28.312	28.008	27.379	26.824
13	29.435	29.435	29.435	29.435	29.435	29.435	29.435
σ_H	0.815	0.777	0.760	0.695	0.618	0.454	0.247
σ_Q	0.903	0.881	0.872	0.834	0.786	0.674	0.497
w_{sd} [kWh/m^3]	0.0071	0.0082	0.0086	0.0103	0.0123	0.0165	0.0216
Pumping energy increase [%]		3.9	5.5	12.0	19.7	36.1	56.7

After operating time of 10 years, with a growth rate of roughness of 0.025 mm/year, a relative little growth of hydraulic slope is obtained from 4.1% (pipe 9-8) to 17.3% (pipe 2-1), but after a operating time of 50 years and a growth rate of roughness of 0.2 mm/year, hydraulic slopes increase very much, reaching 175% (pipe 9-8) until 259% (pipe 5-1).

Pressure distribution stability decreases with 4.7... 69.7%, and discharge distribution stability worsens with 2.4...45%.

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In order to maintain transport capacity of pipes, in case of analyzed network, increased pumping head is needed and thus pumping electricity consumption from 3.9% (ω = 0.025 mm/year, τ = 10 years) to 56.7% (ω = 0.2 mm/year, τ = 50 years), and accordingly increase the specific energy for water distribution in network from 15.6% to 206%.

Effect of pipe roughness increase in time on pressure distribution consists in the more pronounced increase in hydraulic slopes than those expected from design, higher the network operating time is, with the following implications:

- significant reduction of discharge for consumption points, leading to difficulties in water use and often to necessity of over equipping of pump stations and also to wrong design of extensions or rehabilitations of networks where the pipes have already operating duration;
- increase of pressure in network for achievement of same transported flow, having as consequences a greater energy consumption and disturb of optimal calculation;
- generation of an additional water loss in network, which could be as greater as the number of damages increase, to the end of material lifetime.

If for new pipes, the absolute roughness could be considered unchanged for a period of 10…12 years, after this operating period it is absolute necessary to consider change of roughness stage for inner pipe wall.

7. CONCLUSIONS

The three analysis methods of looped networks (loop method, nodal method, element method) are theoretically equivalent. The mathematical model for all is based on conservation equations of discharges in nodes and energy in loops and on functional equation head loss - discharge in component elements of the network.

In case in which unknowns of a network are nodal piezometric heads, concentrated consumption in nodes and/or hydraulic resistances the nodal method is preferred as hydraulic analysis mean.

By the possibility to introduce as unknown consumptions at nodes and hydraulic resistances of some pipes, nodal analysis model ANOREC offers greater elasticity compared to loop analysis and extend its use for new problems. Such problems could be: study of existing network for establishing the possibilities of connection of new consumers or identifying hydraulic resistance, determination of pressure stage in network in order of ensure service pressure.

The mathematical model expressed by the objective functions (26) constitutes a new way of hydraulic analysis of complex looped networks based on unconditioned optimization techniques. This model replaces the solving of the nonlinear system of equations (2), (3), (9) with the direct minimization of a multivariable function, without constraints that express the energy consumption across the network.

The computer program ANOREV includes this particular aspect and contain the conjugate gradient algorithm, which give it efficiency especially in operational analysis of complex distribution networks. This new method is computationally more efficient and consequently helps the designer to get the best design of water distribution systems with fewer efforts.

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