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BIT ERROR RATE, PERFORMANCE ANALYSIS AND COMPARISION OF M x N EQUALIZER BASED MAXIMUM LIKELIHOOD AND MINIMUM MEAN SQUARE ERROR MIMO RECEIVER FOR WIRELESS CHANNEL

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ABSTRACT

This paper investigates on the performance comparison of m x n equalizer based maximum likelihood (ML) and minimum mean square error (MMSE) MIMO receiver for wireless channel. The effect of fading and interference effects can be combated with equalizer. This paper also analyses the performance of MMSE equalizer based receiver for MIMO wireless channel. The bit error rate (BER) characteristics for the various transmitting and receiveing antennna are simulated in Matlab tool box and many advantages and disadvantages of the system are described. The simulation carried out in signal processing lab show that the MMSE equalizer based receiver is a good choice for removing some inter symbol interference (ISI) and minimizes the total noise power. However the simulation results show ML equalizer performance better. The simulation tool box used is Matlab V.7.0 and the results were obtained at RF signal processing lab. The results show that the BER decreases as the m x n antenna configurations is increased for both ML and MMSE equalizer based detector. ML equalizer based detector out performs the MMSE receiver in terms of BER characteristics.

Keywords: wireless channel, MIMO, MMSE, ISI, bit error rate, signal to noise ratio.

1. INTRODUCTION

Wireless communication technology has shown that when multiple antennas at both transmitter and receiver are employed it provides the possibility of higher data rates compared to single antenna systems [1, 2]. The system with multiple antennas at the transmitter and receiver is commonly referred as multiple input multiple output (MIMO) systems. The multiple antennas are thus used to increase data rates through multiplexing or to improve performance through diversity. This method offers higher capacity to wireless systems and the capacity increases linearly with the number of antennas and link range with out additional bandwidth and power requirements. MIMO channel model [4, 5] is depicted in Figure-1 with M transmitter and N receiver antennas. It can be achieved by higher spectral efficiency and link reliability or diversity (reduced fading).
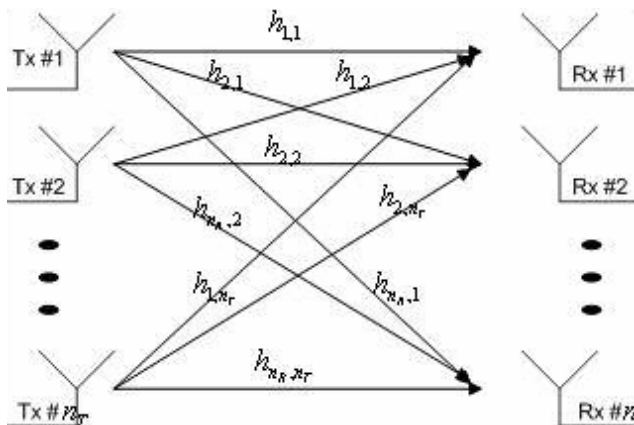

Figure-1. MIMO model.

2. ML EQUALIZER BASED RECEIVER

It is a preferred method of parameter estimation in statistics. It is an important tool for many statistical non-linear modeling techniques. MLE estimate is obtained by maximizing the log-likelihood function since because the functions are monotonically related to each other; hence MLE estimate is obtained by maximizing either one. For determining the maximum likelihood estimators of the log-normal distribution parameters μ and σ, can be determined as follows,

$$f_L(x; \mu, \sigma) = \prod_{i=1}^{n} \{\frac{1}{x_i}\} f_N(\ln x; \mu, \sigma)$$

(1)

where, f_L denote the probability density function of the log-normal distribution and f_N is the normal distribution. Therefore, using the same indices to denote distributions, we can write the log-likelihood function as

$$l_L(\mu, \sigma | x1, x2 \dots, xn) = -\sum_k \ln x_x + l_N(\mu, \sigma | \ln x1, \ln x2, \dots, \ln xn)$$

$$= \text{Constant} + l_N(\mu, \sigma | \ln x1, \ln x2, \dots, \ln xn)$$

(2)

Since the first term is constant with regard to μ and σ, both logarithmic likelihood functions, ℓ_L and ℓ_N, reach their maximum with the same μ and σ. Hence, using the formulas for the normal distribution maximum likelihood parameter estimators and the equality above, we deduce that for the log-normal distribution as

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$$\hat{\mu} = \frac{\sum_k ln\, x_k}{n}, \qquad \hat{\sigma}^2 = \frac{\sum_k (ln\, x_k - \hat{\mu})^2}{n} \qquad (3)$$

The Maximum Likelihood receiver tries to find, which

$$J = |y - H\hat{x}|^2$$

minimizes
J

$$= \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} \right\|^2 \qquad (4)$$

Since the modulation is BPSK, the possible values are +1 or -1. To find the Maximum Likelihood solution, we need to find the minimum from the all four combinations of x_1 and x_2

$$J_{+1,+1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} +1 \\ +1 \end{bmatrix} \right\|^2$$

$$J_{+1,-1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} +1 \\ -1 \end{bmatrix} \right\|^2$$

$$J_{-1,+1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} +1 \\ +1 \end{bmatrix} \right\|^2$$

$$J_{-1,-1} = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\|^2 \qquad (5)$$

3. MMSE EQUALIZER

In MIMO wireless communication, an equalizer is employed which is a network that makes an attempt to recover a signal that suffers with an Inter symbol Interference (ISI) and improves the BER characteristics and maintains a good SNR. A minimum mean square error (MMSE) estimator is a method in which it minimizes the mean square error (MSE), which is a common measure of estimator quality. Minimum mean-square error equalizer, which does not usually eliminate ISI completely but instead, minimizes the total power of the noise and ISI components in the output.

A minimum mean square error (MMSE) estimator describes the approach which minimizes the mean square error (MSE), which is a common measure of estimator quality. The main feature of MMSE equalizer is that it does not eliminate ISI completely but, minimizes the total power of the noise and ISI components in the output. Let x be an unknown random variable, and let y be a known random variable. An estimator x^ (y) is any function of the measurement y, and its mean square error is given by

$$MSE = E\{(X^ - X^2)\} \qquad (6)$$

where the expectation is taken over both x and y.

The MMSE estimator is then defined as the estimator achieving minimal MSE. In many cases, it is not possible to determine a closed form for the MMSE estimator. In these cases, one possibility is to seek the technique minimizing the MSE within a particular class, such as the class of linear estimators. The linear MMSE estimator is the estimator achieving minimum MSE among all estimators of the form $AY + b$. If the measurement Y is a random vector, A is a matrix and b is a vector.

Let us now try to understand the math for extracting the two symbols which interfered with each other. In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}\, x_1 + h_{1,2} x_2 + n_1 = [h_{1,1}\ h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \qquad (7)$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1}\, x_1 + h_{2,2} x_2 + n_2 = [h_{2,1}\ h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \qquad (8)$$

Where

y_1, y_2 are the received symbol on the first and second antenna, respectively.
$h_{1,1}$ is the channel from 1st transmit antenna to 1st receive antenna,
$h_{1,2}$ is the channel from 2nd transmit antenna to 1st receive antenna,
$h_{2,1}$ is the channel from 1st transmit antenna to 2nd receive antenna,
$h_{2,2}$ is the channel from 2nd transmit antenna to 2nd receive antenna,
x_1, x_2 are the transmitted symbols and
n_1, n_2 are the noise on 1st and 2nd receive antennas.

The equation can be represented in matrix notation as follows:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \qquad (9)$$

Equivalently,

$$y = Hx + n \qquad (10)$$

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient W which minimizes the

$$E\{[W_{y-x}][W_{y-x}]^H\}$$

Criterion,

Where

W = Equalization Matrix

H = Channel Matrix and

n = Channel noise

y = Received signal.

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To solve for x, we need to find a matrix W which satisfies $WH = I$. The Minimum Mean Square Error (MMSE) detector for meeting this constraint is given by,

$$W = [H^H H + N_o I]^{-1} H^H \qquad (11)$$

This matrix is known as the pseudo inverse for a general m x n matrix

Where

$$H^H H = \begin{pmatrix} h^*_{1,1} & h^*_{2,1} \\ h^*_{1,2} & h^*_{2,2} \end{pmatrix} \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} =$$

$$\begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h^*_{1,1}h_{1,2} + h^*_{2,1}h_{2,2} \\ h^*_{1,2}h_{1,1} + h^*_{2,2}h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \qquad (12)$$
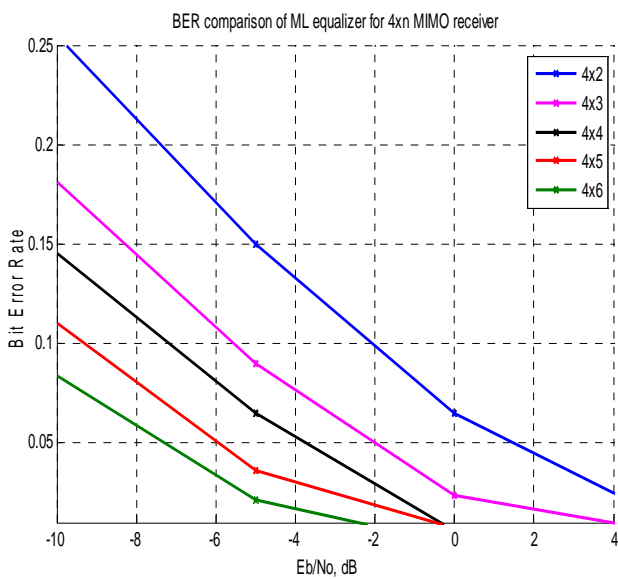
4. SIMULATION RESULTS AND DISCUSSIONS

The simulations were carried out at RF Signal Processing Lab. Figure-2 and Table-1 shows that as the number of receivers (n) is increased keeping the number of transmitters (m = 2, 3 and 4) as constant it is observed that the Bit Error Rate (BER) decreases in ML equalizer.
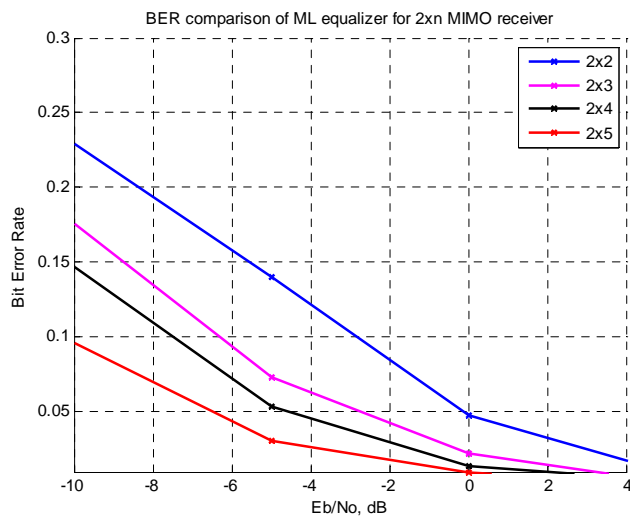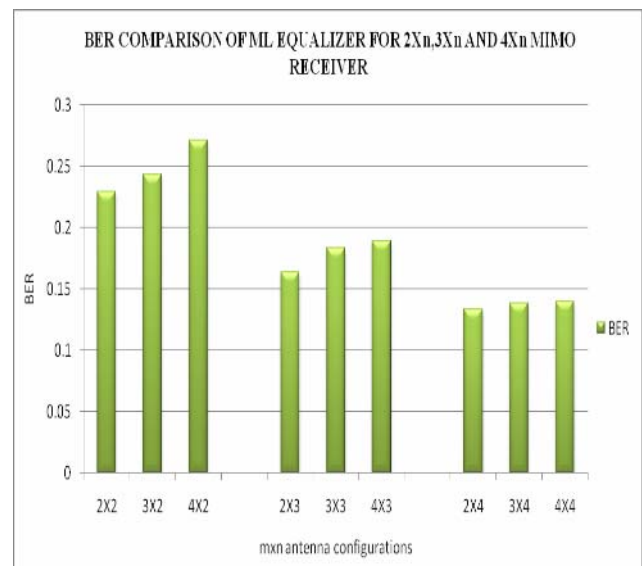

2(a)
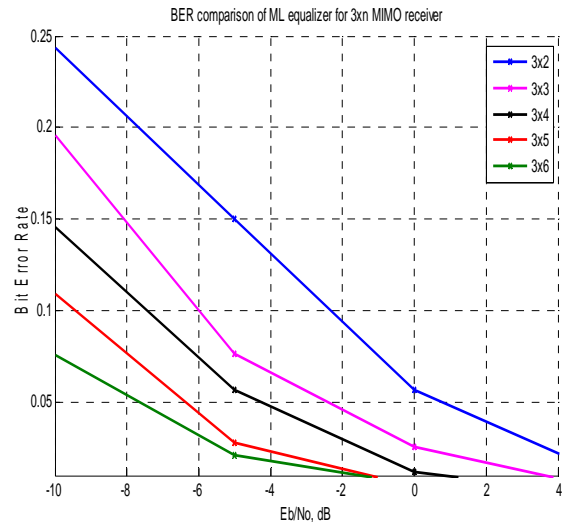
2(b)

2(c)

2(d)

Figure-2. Comparison of BER analysis for m x n antenna configurations of ML equalizer.
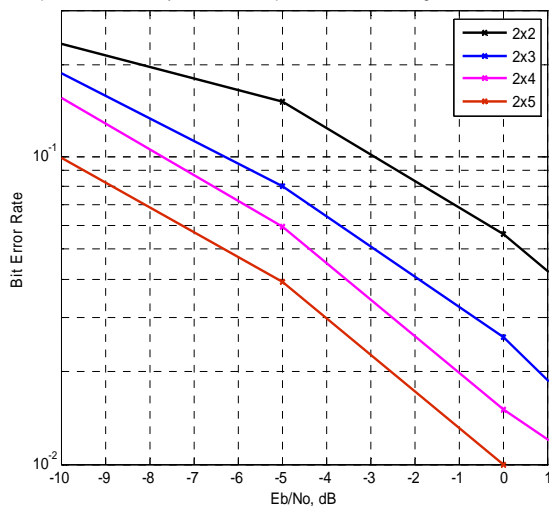
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Table-1.

S. No.	Bit error rate value for E_b/N_o = -10dB					
	m x n	Value	m x n	Value	m x n	Value
1	2x2	0.229	3x2	0.2427	4x2	0.271
2	2x3	0.1633	3x3	0.1833	4x3	0.189
3	2x4	0.133	3x4	0.1386	4x4	0.14
4	2x5	0.096	3x5	0.1003	4x5	0.113
5	-	-	3x6	0.07089	4x6	0.074

Now let us consider the simulation analysis of MMSE equalizer for 2 x n antenna configurations which means keeping the transmitter antenna as two and vary the number of antennas in the receiver side. From the matlab figure window shown in Figure-2, it's evident that the BER decreases as the receiver antenna increases for the zero forcing equalizer. For a better clarity the data from the figure window is taken and plotted in the form of bar chart as shown in Figure-3.
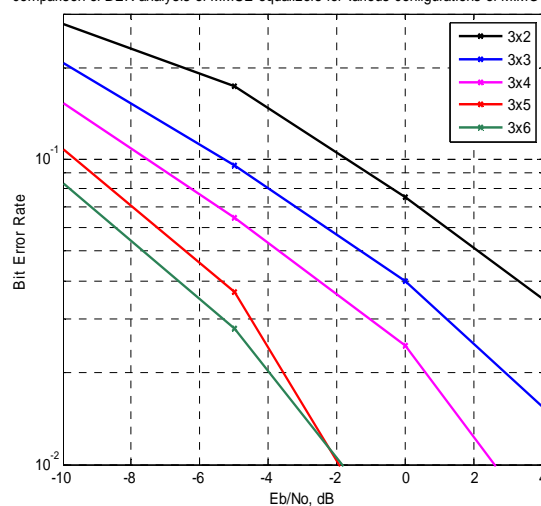
Figure-3 and Table-2 shows that as the no. of receivers (n) is increased keeping the no. of transmitters (m = 2, 3 and 4) as constant it is evident that the Bit Error Rate (BER) decreases in MMSE equalizer and also when the number of transmitters is less than the number of receivers BER decreases and vice versa.

3(a)

3(b)

3(c)

3(d)

Figure-3. Comparison of BER analysis for m x n antenna configurations of MMSE equalizer.

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Table-2. Bit error rate values for m x n antenna configurations of MMSE equalizer.

S. No.	Bit error rate value for $E_b/N_o = -10dB$					
	m x n	Value	m x n	Value	m x n	Value
1	2x2	0.233	3x2	0.2633	4x2	0.275
2	2x3	0.1727	3x3	0.1883	4x3	0.193
3	2x4	0.1385	3x4	0.1448	4x4	0.152
4	2x5	0.099	3x5	0.1077	4x5	0.121
5	-	-	3x6	0.0791	4x6	0.087

CONCLUSIONS

This paper is a simulation study on the performance comparison analysis of M x N Equalizer based ML and MMSE receiver for MIMO wireless receiver. Hence a more balanced linear equalizer is the Minimum mean square error equalizer, which however is not eliminate ISI completely but instead minimizes the total power of the noise and ISI components in the output. From the above the following observations are made.ML is low computational complexity. It minimizes the probability of a sequence error. Hence from the above graphs it is evident that the BER decreases as the number of receiving antenna increases with respect to number of transmitting antenna in ML equalizer based MIMO receiver. This equalizer is used for mobile communication link. ML equalizer outperforms MMSE based equalizer for MIMO Wireless channel.

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