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THE NUMERICAL SOLUTION OF THE MOTION OF A SPHERE IN A DUSTY GAS

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ABSTRACT

The paper considers the effect on the dusty gas otherwise at rest at infinity due to uniform motion of a sphere. The dust particles are assumed to have small relaxation time. Using the potential solution of gas flow at large Reynolds number R, an equation for the concentration of dust near the sphere is derived and solved numerically. It is also shown that particles do not collide with the sphere until the Stokes number σ is greater than 1/12 if we assume the gas flow unchanged by the presence of the dust particles and also graphically represented concentration of dust about sphere along radius vector.

Keywords: dusty gas, sphere, viscous fluid, finite deference techniques, perturbation, stokes number, Reynolds number.

INTRODUCTION

Interest in the problem of mechanics of systems with more than one phase has devolved rapidly in the recent years. Situations which occur frequently are concerned with motion of liquid or gas which contains a distribution of solid particles. Such type of situation occurs for example, in the movement of dust laden air, in the problem of fluidization, in the use of dust in gas cooling systems to enhance heat transfer processes, and in the process by which rain drops formed by coalescence of small droplets which might be considered as solid particles for purpose of examining their movement prior to coalescence.

Carrier [1] Rudinger [2] Marble [3] did an extensive work on the models of dusty gas flows and shock waves in dusty gas. Later Saffman [4] formulated equations for small disturbance in plane parallel flow of a dusty gas. Following his model Michael[5] and Michael and Norrey [6] studied the steady motion of a dusty gas past a fixed surface and arrived at approximate solutions. Nirmala [7] studied the effect of fine and coarse dust particles on transport of in the trachea. Sanchita Ghosh [8] studied on hydro magnetic pulsatile of a dusty fluid. Here in this paper Saffman [4] model is employed to study motion of a sphere in dusty gas by Finite difference technique. The dust is represented by a large number density N of small dust particles whose volume concentration is small, but mass concentration is appreciable. It is assumed that the individual particles of dust are so small that Stokes flow approximation to their motion relative to the gas, is appreciable. The equations of motion gives rise to two additional independent parameters due to the presence of Dust, viz. f, the mass concentration of the dust and τ, relaxation which is representative of the time scale on which velocity of the dust adjusts itself to changes in neighboring gas velocity. When $\tau = 0$, this adjustment is instantaneous, and we have a limiting case in which the dust moves with gas at each point. The motion in this case is closely related to flow of a clean gas. We consider here the flow of a dusty gas for small non-zero values of τ by a perturbation of the solution at $\tau = 0$. Here Reynolds number is assumed to be large, and as a first step towards the solution, the problem considers in detail the perturbation of the un-separated potential flow for a sphere. The analysis shows that when a nonsingular perturbation of a potential flow is assumed, the concentration of dust particles becomes logarithmically infinite to the front stagnation point of the sphere. It also found that dust particles cannot reach the sphere except at the front stagnation point, there being a dust streamline emanating from the point which delaminates a thin dust free layer adjacent to the sphere whose thickness is of the order 'σ a' where σ is Stokes number, $\tau\, U/a$ and U, the velocity of sphere and a, its radius.

MATHEMATICAL FORMULATION AND SOLUTION OF PROBLEM

The equations governing the motion of dusty gas as given by Saffman [4] are:

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}.\nabla\vec{u}\right) = -gradp + \mu\nabla^2\vec{u} + KN(\vec{v} - \vec{u}) \quad (1)$$

$$div\,\vec{u} = 0 \quad (2)$$

$$m\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}.\nabla\vec{v}\right) = k(\vec{u} - \vec{v}) \quad (3)$$

$$\frac{\partial N}{\partial t} + div(N\,\vec{v}) = 0 \quad (4)$$

Where $\bar{u}$ *and* $\bar{v}$ are velocities of gas and dust particles. N is the number density of dust particles, each of mass m. K is the Stokes coefficient of resistance, p, ρ, μ, being the pressure, density and viscosity of the gas. The time relaxation parameter τ is given from (3) by $\tau = \frac{m}{K}$. *When* $\tau \to 0$ equation (3) shows that $\bar{u} \to \bar{v}$.

Substituting for $\bar{u} - \bar{v}$ in (1), from (3) we have

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$$\rho\left(\frac{\partial \vec{u}}{\partial t}+\vec{u}.\nabla \vec{u}\right)=-\text{grad}\,p+\mu\nabla^2\vec{u}-Nm\left(\frac{\partial \vec{v}}{\partial t}+\vec{v}.\nabla\vec{v}\right) \quad (5)$$

When $\tau \rightarrow 0$ equation (5) becomes:

$$(1+f)\left(\frac{\partial \vec{u}}{\partial t}+\vec{u}.\nabla \vec{u}\right)=-\frac{1}{\rho}\,grad\;p+\upsilon\,\nabla^2\vec{u} \quad (6)$$

Where the mass concentration of dust, $f=\frac{mN}{\rho}$ and $\upsilon=\mu/\rho$. In this limiting case when we put $\bar{u}=\bar{v}$ in (4) and using (2) we obtain

$$\frac{\partial N}{\partial t}+\vec{u}.\nabla N=0$$

This indicates N remains constant in the neighborhood of any given dust or gas particle. A simple case in which N is uniform and equals N_0 everywhere in the incident flow, then we have f = f_0 a constant. Equation (6) then represents flow of a clean gas with uniform density $\rho(1+f_0)$ and viscosity μ. The solution for dusty gas flow at Reynolds number R is then equivalent to the solution for a clean gas at the increased Reynolds number $R(1+f_0)$. For the motion of a sphere with velocity U, the gas velocity changes on the length scale of the radius 'a' of the sphere, a perturbation on the solution for τ = 0 can be obtained in terms of small dimensionless parameter $\sigma=\tau U/a$. For spherical dust particles of radius 'd' and density ρ d, condition $\rho\le 1$ becomes

$$\frac{2}{3}R\left(\frac{(\rho d)}{\rho}\right)\left(\frac{d}{a}\right)^2\le 1$$

Now consider the potential flow of liquid due to the motion of the sphere in the limiting case when τ = 0, neglecting for the present viscous boundary layer and separation effects. The solution $\nabla^2\vec{u}$ = 0 in equation (6) and the effect of the dust is simply to scale up the pressure variations over the sphere by the factor $(1+f_0)$.

Let $\vec{u}_0$ represent the unperturbed velocity of the dust and gas, where

$$\vec{u}_0=grad\;\varphi$$

$$\varphi=\frac{Ua^3}{2r^2}\cos\theta \quad (7)$$

r,θ being spherical polar coordinates from the centre of the sphere, with $\theta=0$ as the downstream direction and U the velocity of sphere.

In the perturbation let $\vec{u}=\vec{u}_0+\vec{u}^1$; $\vec{v}=\vec{u}_0+\vec{v}^1$ represent gas and dust velocities for a small non zero value of τ where $\vec{u}^1$, $\vec{v}^1$ represent small perturbation velocities of order τ. Also we suppose $N=N_0+N^1$; $f=f_0+f^1$ and $p=p_0+p^1$. Neglecting the internal effect of viscosity in the gas and taking only the first order terms we have from equation(5):

$$\vec{u}_0.\nabla\vec{u}^1+\vec{u}^1.\nabla\vec{u}_0+f_0\left\{\vec{v}^1.\nabla\vec{u}_0+\vec{u}_0.\nabla\vec{v}^1\right\}+f^1\vec{u}_0.\nabla\vec{u}_0=-\frac{1}{\rho}\,grad\;p^1 \quad (8)$$

Similarly the linearised form of equation (3) for the dust flow is

$$\tau\;\vec{u}_0\;.\nabla\vec{u}_0=\vec{u}^1-\vec{v}^1 \quad (9)$$

Neglecting higher order Equation (4) becomes

$$f_0\;div\;\vec{v}^1+\vec{u}_0\;.\nabla f^1=0 \quad (10)$$

Eliminating $\vec{v}^1$ from equation (9) and (10), we have

$$\vec{u}_0\;.\nabla f^1=f_0\;\tau\;div\;\vec{u}_0.\nabla\vec{u}_0$$

Since

$$\vec{u}_0.\nabla\vec{u}_0=grad\;\frac{\vec{u}_0^2}{2}$$

The above equation can be written as:

$$\vec{u}_0.\nabla f^1=f_0\,\tau\,\nabla^2\frac{\vec{u}_0^2}{2} \quad (11)$$

Using equation (7) and Laplacian in spherical polar coordinates, we have

$$\vec{u}_0.\nabla f^1=f_0\,\tau\,\frac{9u^2a^6}{2r^8}\left(3\cos^2\theta+\sin^2\theta\right),$$

The right hand side being always positive, and an even function about the plane $\theta=\pi/2$, the left hand side being $\left|\vec{u}_0\right|.\frac{\partial f^1}{\partial s}$; which shows that f^1 increases monotonically along a streamline and the rate of increase is symmetric about $\theta=\pi/2$.

Writing in terms of r and θ, equation (11) becomes

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$$\frac{\partial f^1}{\partial r}\cos\theta + \frac{\sin\theta}{2r}\frac{\partial f^1}{\partial\theta} = \frac{9 f_0\ \tau U a^3}{2 r^5}(3\cos^2\theta + \sin^2\theta) \quad (12)$$

Substituting $f^1(r,\theta) = \dfrac{F(\theta)}{r^4}$

The equation reduces to

$$\frac{Sin\theta}{2} F'(\theta) + 4F(\theta)Cos\theta = 9 f_0 \frac{\tau U a^3}{2}(3Cos^2\theta + Sin^2\theta) \quad (13)$$

Applying explicit finite difference formula to equation (13), we get

$$F_{i+1} = F_i\left(-\frac{8 Cos\ \theta i}{Sin\ \theta i} + 1\right) + 9\tau U a^3 f_0 (3 Cos\ \theta_i\ Cot\ \theta_i + Sin\ \theta_i)\delta\theta \quad (14)$$

With $F\left(\dfrac{\pi}{2}\right) = 0$ $F(\theta)$) values are evaluated for

$\theta_1 = \dfrac{\pi}{16}, \theta_2 = \dfrac{\pi}{8}, \theta_3 = \dfrac{3\pi}{16}, \theta_4 = \dfrac{\pi}{4}, \theta_5 = \dfrac{5\pi}{16}, \theta_6 = \dfrac{6\pi}{16}, \theta_7 = \dfrac{7\pi}{16}, \theta_8 = \dfrac{\pi}{2}$, and shown graphically in Figure-1.

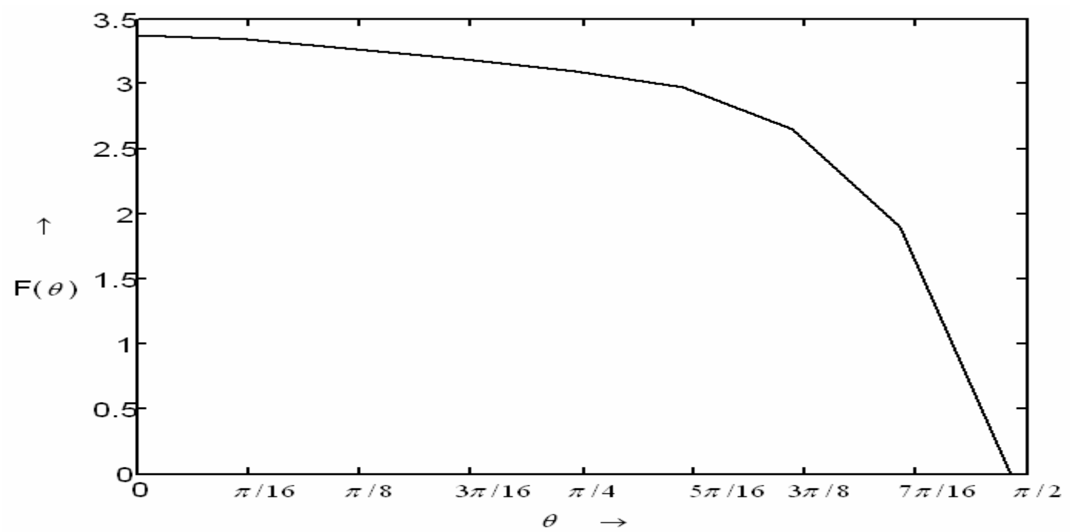

Fig (1): Concentration of dust about sphere along radius vector

The stagnation points for the motion of a sphere are given $\theta = 0$ *and* $\theta = \pi$. Thus f^1, the concentration of dust particles as the sphere moves at stagnation points, is given by:

Put $\theta = 0$ in equation (12) and integrating

$$f^1 = \frac{27\ f_0\ \tau U}{8a}$$

In the dimensionless from the unperturbed streamlines are given by

$$r = c\sin^2\theta \quad (15)$$

and (11) can be written as:

$$u_0 \frac{\partial f^1}{\partial s} = f_0 \tau\ \nabla^2 \frac{u_0^2}{2}$$

Where $\dfrac{\partial f^1}{\partial s}$ *represents the rate of* f^1 with length along a streamline. Using (15) to eliminate θ we may deduce the following expressions for $\dfrac{\partial \bar{f}}{\partial r}$ *and* $\dfrac{\partial \bar{f}}{\partial \theta}$ on the streamline K, where $\bar{f} = \dfrac{2\ a\ f^1}{9\ f_0\ \tau U}$

The Lagrangian form of equation (12) and (15) using we get.

$$\frac{\partial \bar{f}}{\partial r} = \pm \frac{\left(3 - \frac{2r}{c}\right)}{r^5(1 - r/c)^{\frac{1}{2}}};\ \frac{\partial \bar{f}}{\partial \theta} = \frac{c^{\frac{1}{2}}}{2}\ \frac{\left(3 - \frac{r}{c}\right)}{r^{\frac{9}{2}}} \quad (16)$$

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In the expression for $\frac{\partial \bar{f}}{\partial r}$ the - ve sign is taken for $\theta = \pi \ \ to\ \theta = \pi/2$

And $+\ ve\ sign\ from\ \theta = \pi/2\ to\ \theta = 0$.

SMALL VALUES OF f

If f_0 is small, it follows from equation (10) that f^1 *is* small for the second order and to the first order equation (8) tells us that $\vec{u}^1 = p^1 = 0$ and equation (9) gives

$$\vec{v}^1 = -\tau\,(\vec{u}_0\ .\ \nabla\,\vec{u}_0)$$

So that

$$\vec{v} = \vec{u}_0 + \vec{v}^1$$
$$= grad\ \left\{\varphi - \frac{\tau}{2}\,(grad\ \ \varphi)^2\right\}$$

Hence $\vec{v}$ remains a potential field in this case with potential

$$\phi = U\ a\left[\frac{\cos\theta}{2\,r^2} - \frac{\sigma}{2\,r^6}\left(\cos^2\theta + \frac{\sin^2\theta}{4}\right)\right] \quad (17)$$

The equation for dust streamlines is given by:

$$\frac{d\,r}{r\,d\theta} = \frac{-\cos\theta + \frac{3\sigma}{r^4}\left(\cos^2\theta + \frac{\sin^2\theta}{4}\right)}{-\frac{\sin\theta}{2} + \frac{3\sigma}{r^4}(\sin\theta\ \cos\theta)} \quad (18)$$

It is interesting to observe that when dust particles are clean, that is when $\sigma = 0$, its streamlines coincide with those of fluid particles. It is intersecting to trace the divergence of the gas particles from the path given by equation (15). In order to do so, we write the equation of the streamline in the form.

$$\frac{\sin^2\theta}{r} = \bar{c} + \bar{c}(\theta) \quad (19)$$

Where $\bar{c}$ is a small change in c of order σ, representing the displacement of the particles at an angle θ, we then have

$$\frac{d\,\bar{c}}{d\,\theta} = -\frac{\sin^2\theta}{r^2}\frac{dr}{d\,\theta} + \frac{2\sin\theta\,\cos\theta}{r} \quad (20)$$

Eliminating $\frac{d\,r}{d\,\theta}$ from equation (18) and (20) we have

$$\frac{d\,\bar{c}}{d\,\theta} = \frac{6\sigma}{r^5}\left(\frac{\cos^2\theta}{2} + \frac{\sin^2\theta}{4}\right)\sin\theta \quad (21)$$

This shows that dust path lines coincide with gas path line at a far away distance from the sphere. Since at $\theta = 0,\ we\ have\ \frac{d\bar{c}}{d\,\theta} = 0,$ it follows that path line of dust and gas particles coincide along the direction $\theta = 0$.

Integrating (21) along the streamline $r = \frac{\sin^2\theta}{c}$, we have

$$\frac{2\bar{c}}{3\,\sigma} = \left[\frac{3\cot\ \theta}{4}\left\{\frac{\cos\ ec^7\theta}{3} + \frac{\cos\ ec^5\theta}{6} + \frac{5\cos\ ec^3\theta}{24} + \frac{5\cos\ ec\ \ \theta}{16}\right\} + \frac{15}{128}\log\ \tan^2\theta/2\right] \quad (22)$$

DUST SEPARATION STREAMLINE

Ruling out the case in which the sphere acts as a steady source of dust, we must conclude that there is a separating streamline for the dust which starts at the first stagnation point. In the first approximation, the position of this separation line will be given by the equation.

$$\vec{v}\ .\ \vec{n} = 0 \quad (23)$$

Assuming f to be small and if we write $r = a + \sigma\,a\,\delta(\theta)$, as the equation for separation line, we have to the first order from equation (16) and (19) and (21) we have

$$\delta(\theta) = \frac{1}{3\sigma} + \frac{c}{\sin^6\theta} - \frac{2}{\sin^6\theta}\left[\frac{\cos^3\theta}{3} - \frac{2\cos^5\theta}{5} + \frac{\cos^7\theta}{7}\right]$$
$$-\frac{1}{2\sin^6\theta}\left[\cos\theta - \cos^7\theta\ \ - \cos^3\theta + \frac{3}{5}\cos^5\theta\right] \quad (24)$$

CRITICAL VALUE OF σ

Although the main discussion of this paper is based on small values of σ, it is worthwhile to study the critical value of σ at which particles begin to collide with the sphere. This can be done on the assumption that the gas velocity is unchanged by the dust and that head-on collisions with the sphere by the particles on the upstream axis will be the first to occur.

The equation of motion for a particle on this axis in dimensionless form is:

$$\frac{dv}{dr} = \frac{v - \frac{1}{r^3}}{\sigma v} \tag{25}$$

We have to solve this equation with boundary condition $v = 0 \ at \ r = \alpha.$ Let us investigate the behavior of solution of equation (23) at the it's stagnation point $r = 1.$ Writing r =1 + *h* where *h* is small equation (25), becomes:

$$\frac{dv}{dh} = -\left\{\frac{v-(1-3h)}{\sigma v}\right\} \tag{26}$$

This may be written in parametric form with parameter proportional to the time.

$$\frac{dv}{dt} = 1-3h - v$$
$$\frac{dh}{dt} = \sigma v \tag{27}$$

Thus *v* and *h* have the form $e^{\lambda t}$

Where $\lambda^2 + \lambda + 3\sigma = 0$

When $\sigma \le 1/12$ the roots λ_1 and λ_2 are real and –ve and the time taken for particles to come to stagnation point approaches infinitely like log *h* as $h \to 0$. When $\sigma > 1/12$ we find $v,$ non-zero at h = 0 and the particles collide with the sphere in a finite time. This result agrees with that of Michael [7].

CONCLUSIONS

$f'(r,\theta) = \frac{F(\theta)}{r^4}$ It follows for a given value of θ $f'(r,\theta) \propto \frac{1}{r^4}$ *i.e.* $f'(r,\theta)$ Decreases along line $\theta = \theta_c$ and $\to 0$ at long distance.

Mass concentration is found to be maximum at $\theta = 0$ and gradually decreases with the increase of θ obtaining 0 at $\theta = \frac{\pi}{2}$. The concentration is symmetrical about $\theta = 0$ and the equation for dust stream lines is obtained. The dust particles path lines are found to coincide with gas path lines at far away distance sphere.

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