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HIGHER ORDER THEORY FOR FREE VIBRATION ANALYSIS OF FUNCTIONALLY GRADED MATERIAL PLATES

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ABSTRACT

In this paper, an analytical solution is developed to study the free vibration analysis of functionally graded material (FGM) plate without enforcing zero transverse shear stress conditions on the top and bottom surfaces of the plate using higher order displacement model. The material properties are assumed to be varied continuously through the thickness direction according to a simple power law distribution in terms of volume fraction of material constituents. The effective material properties are obtained by applying linear rule of mixtures. The derivation of equations of motion for higher order displacement model is obtained using principle of virtual work. The governing equations of FGM plate are established by applying energy principles and are solved by Navier's method. The influence of side-to-thickness ratio, modulus ratio and aspect ratio on the nondimensional natural frequencies is studied. The obtained numerical results are compared with the results available in the literature.

Keywords: FGM plate, higher order theory, Navier's method, free vibration.

INTRODUCTION

Laminated composite plates are widely used because of high specific strength and high specific stiffness. However, laminated composite materials usually have an abrupt change in mechanical properties across the interface where two different materials are bonded together at some extreme loading conditions; this can result in cracking and large inter-laminar stresses leading to delamination. One way to solve these problems is to employ functionally graded materials. A functionally graded material (FGM) is a material in which the composition and structure gradually change resulting in a corresponding change in the properties of the material. This FGM concept can be applied to various materials for structural and functional uses [1-2]. The behavioral analysis of functionally graded composite materials is an important field of research owing to the interest for a wide range of applications: thermal barrier coatings for turbine blades (electricity production), armor protection for military applications, fusion energy devices, biomedical materials including bone and dental implants, space/aerospace (space vehicles, aircraft, aerospace engines, rocket heat shields) industries, automotive applications, etc., because of their superior advantages such as high resistance to temperature gradients, capability to withstand to high loads and high temperature fields and high durable properties, reduction in residual and thermal stresses, high wear resistance, and an increase in strength to weight ratio when compared to the other engineering materials. Hence, the non-linear behavior of functionally graded plates has to be understood for their optimum design. Chun-Sheng Chen *et al*., [3] studied the nonlinear behavior of laminated plate's at large vibration amplitudes. They concluded that, the higher-order shear deformation terms had a significant influence on the plate in a large amplitude vibration as thickness ratio decreases and the plate was stacked with fewer layers.

Pradhan *et al*., [4] studied the natural frequencies of the functionally graded cylindrical shell made up of stainless steel and zirconia. They observed that, natural frequencies of cylindrical shells are dependent on the constituent volume fractions and boundary conditions. Yang and Hui-Shen Shen [5-6] investigated the dynamic response of initially stressed functionally graded rectangular thin plates. In this, the plate is assumed to be clamped on two opposite edges and the remaining two edges may be simply supported or clamped or may have elastic rotational edge constraints. They were employed a one-dimensional differential quadrature approximation and the Galerkin procedure in the free vibration analysis. Free and forced vibration analyses for initially stressed functionally graded plates in thermal environment are also studied. Gopalakrishnan *et al*., [7] developed a new beam element based on the first-order shear deformation theory to study the thermo-elastic behavior of functionally graded beam structures. They considered both exponential and power-law variations of material property distribution to examine different stress variations. Alinaghizadeh and Isvandzibaei [8] studied the vibration analysis of cylindrical shells made of FGM composed of stainless steel and nickel. They were used third order shear deformation shell theory to derive the governing equations of motion. Hiroyuki Matsunaga [9] used a 2-D higher order theory for analyzing natural frequencies and buckling stresses of FG plates. Qian *et al*., [10, 11] analyzed free and forced vibrations of both homogeneous and FG thick plates with the higher order shear and normal deformable plate theory by using mesh less local Petrov-Galerkin method. Navazi *et al*., [12] investigated the effects of various boundary conditions, volume fraction index and beams length-to-height ratio on free vibration analysis of FGM beams using analytical method. The equations of motion of FGM beams are derived using first order shear deformation beam theory (FSDBT1) and Hamilton's principle.

The Present work is concerned with the free vibration analysis of functionally graded material plates without enforcing zero transverse shear stress conditions on the top and bottom surfaces of the plate using higher order displacement model, with different boundary conditions, aspect ratios and side to thickness ratios and modulus ratio.

2. HIGHER- ORDER THEORY FOR DISPLACEMENT MODEL

In formulating the higher-order shear deformation theory, a rectangular plate of total thickness *h,* side length *a* in the x-direction and *b* in the y-direction is considered and the location of the rectangular Cartesian coordinate axes used to describe deformations of the plate are given in Figure-1. It is assumed that a state of plane strain exists. Hence, in formulating the higher-order shear deformation theory, a rectangular plate of composed of functionally graded material through the thickness. It is assumed that the material is isotropic. In order to approximate 3D plate problem to a 2D one, the displacement components u (x, y, z, t), v (x, y, z, t) and w (x, y, z, t) at any point in the plate space are expanded in terms of thickness coordinate. The elasticity solution indicates that the transverse shear stress vary parabolically through the plate thickness. This requires the use of displacement field in which the in-plane displacements are expanded as cubic functions of the thickness coordinate in addition the transverse normal strain may vary in non linearly trough the plate thickness. The higher-order displacement field, which satisfies the above criteria, is assumed in the following form [13-18]:

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_o(x,y) + z\theta_z(x,y) + z^2 w_o^*(x,y) + z^3\theta_z^*(x,y) \end{aligned}\right\} \quad (1)$$

Where

u_0, v_0 are the in plane displacements of a point (x, y) on the mid plane.

w_o is the transverse displacement of a point (x, y) on the mid plane.

θ_x, θ_y , θ_z are rotations of the normal to the mid plane about y and x-axes.

u_0^*, v_0^*, w_0^*, θ_x^*, θ_y^*, and θ_z^* are the corresponding higher order deformation terms.

In the present work analytical formulation and solution were obtained without enforcing zero transverse shear stress conditions on the top and bottom surfaces of the plate using the displacement model in Eq. (1).

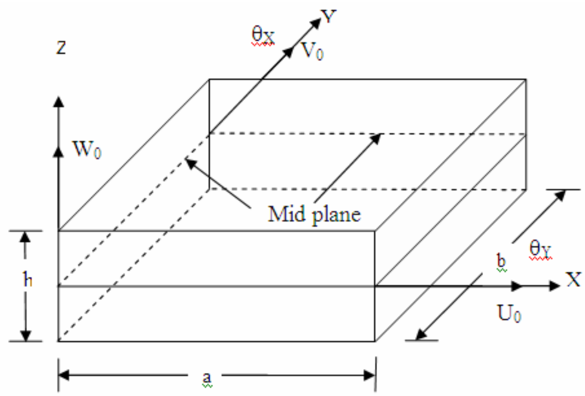

Figure-1. Geometry of FGM plate along with reference axis and displacement components.

Substitution of displacement relations from Eq. (1) into the strain displacement equations, the following relations are obtained as:

$$\begin{aligned} \varepsilon_x &= \varepsilon_{xo} + zk_x + z^2\varepsilon_{xo}^* + z^3 k_x^* \\ \varepsilon_y &= \varepsilon_{yo} + zk_y + z^2\varepsilon_{yo}^* + z^3 k_y^* \\ \varepsilon_z &= \varepsilon_{zo} + zk_z + z^2\varepsilon_{zo}^* \\ \gamma_{xy} &= \varepsilon_{xyo} + zk_{xy} + z^2\varepsilon_{xyo}^* + z^3 k_{xy}^* \\ \gamma_{yz} &= \phi_y + zK_{yz} + z^2\phi_y^* + z^3 k_{yz}^* \\ \gamma_{xz} &= \phi_x + zK_{xz} + z^2\phi_x^* + z^3 k_{xz}^* \end{aligned} \quad (2)$$

2.1. Constitutive relations

The variation of material properties of a FGM plate can be expressed as:

$$P(Z) = (P_t - P_b)V + P_b \quad (3)$$

Where P denotes a generic material property like modulus, P_t and P_b denotes the corresponding properties of the top and bottom faces of the plate, respectively, and n is a parameter that dictates the material variation profile through the thickness. Also V in Eq. (3) denotes the volume fraction of the top face constituent and follows a simple power-law as:

$$V = \left(\frac{Z}{h} + \frac{1}{2}\right)^n \quad (4)$$

Where h is the total thickness of the plate, z is the thickness coordinate and n is a parameter that dictates the material variation profile through the thickness. Here it is assumed that moduli E and G vary according to Eq. (3) and the Poisson's ratio v is assumed to be a constant. The linear constitutive relations are:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{5}$$

Where

$\sigma = (\sigma_x, \sigma_y, \sigma_Z\ \tau_{xy}, \tau_{yz}, \tau_{xz})^t$ are the stresses

$\varepsilon = (\varepsilon_X, \varepsilon_Y, \varepsilon_{Z,} \gamma_{XY}, \gamma_{YZ}, \gamma_{XZ})^t$ are the strains

Q_{ij}'s are the plane stress reduced elastic constants in the plate axis. The superscript t denotes the transpose of a matrix.

2. 2. Equations of motion

The governing equations of displacement model in Eq. (1) are derived using the dynamic version of the principle of virtual displacements. The dynamic version of the principle of virtual work or Hamilton's principle can be written in the analytical form as:

$$\int_0^T (\delta U + \delta V - \delta K)\ dt = 0 \tag{6}$$

Where

δU = virtual strain energy
δV = virtual work done by applied forces
δK = virtual kinetic energy
δU + δV = total potential energy.

The virtual strain energy, work done and kinetic energy are given by:

$$\delta U = \int_A \left\{ \int_{-h/2}^{h/2} \left[\sigma_x \delta \in_x + \sigma_y \delta \in_y + \tau_{xy}\delta\gamma_{xy} + \tau_{xz}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz}\right] dz \right\} dx\ dy \tag{7}$$

$$\delta V = - \int q \delta W_0\ dx\ dy \tag{8}$$

$$\begin{aligned} \delta K = \int_A \Big\{ \int_{-h/2}^{h/2} \rho_0 \Big[& \left(\dot{u}_0 + Z\dot{\theta}_x + Z^2 \dot{u}_0^* + Z^3 \dot{\theta}_x^*\right)\left(\delta\dot{u}_0 + Z\delta\dot{\theta}_x + Z^2 \delta\dot{u}_0^* + Z^3 \delta\dot{\theta}_x^*\right) + \\ & \left(\dot{v}_0 + Z\dot{\theta}_y + Z^2 \dot{v}_0^* + Z^3 \dot{\theta}_y^*\right)\left(\delta\dot{v}_0 + Z\delta\dot{\theta}_y + Z^2 \delta \dot{v}_0^* + Z^3 \delta \dot{\theta}_y^*\right) + \\ & \dot{w}_0 \delta\dot{w}_0 \ \Big] dz \ \Big\} dx\ dy \end{aligned} \tag{9}$$

Where

q = distributed load over the surface of the functionally graded plate

ρ_0 = density of plate material

$\dot{u}_0 = \partial u_0 / \partial t$, $\dot{v}_0 = \partial v_0 / \partial t$ etc. indicates the time derivatives.

Substituting for δu, δv and δk from Eq. (7-9) in to the virtual work statement in Eq. (6) and integrating through the thickness of the functionally graded plate, the in-plane, transverse force and moment resultant relations are obtained. Substituting Eq. (5) into force and moment resultants and upon integration the expressions obtained and written in a matrix form which defines the stress/strain relations of the FGM plate is given by:

$$\begin{Bmatrix} N \\ N^* \\ -- \\ M \\ M^* \\ -- \\ Q \\ Q^* \end{Bmatrix} = \begin{bmatrix} A & | & B & | & 0 \\ B^t & | & D_b & | & 0 \\ 0 & | & 0 & | & D_s \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \varepsilon_0^* \\ -- \\ K_s \\ K^* \\ -- \\ \phi \\ \phi^* \end{Bmatrix} \tag{10}$$

The principle of virtual work is used to derive the equilibrium equations and are expressed in terms of $u_{o,}$ $v_{o,}$ $w_{o,}$ θ_x, θ_y, θz, u_o^*, v_o^*, w_0^*, θ_x^*, θ_y^*, θ_z^* by substituting for the force and moment resultant from Eq.(10).

3. ANALYSIS OF FUNCTIONALLY GRADED MATERIAL PLATE USING DISPLACEMENT MODEL

The simply supported (SS) boundary conditions are considered for displacement model. The Navier solution procedure, displacement components that satisfy the equations of boundary conditions are considered for the analysis. The Solutions are obtained using Newton Raphson method.

3.1 Newton Raphson method for nonlinear analysis

The Newton Raphson iterative method is based on Taylor's series expansion. In the present work, the equation $[S(\Delta_{s+1})]\ \{\Delta\}_{s+1} = \{F\}$ is solved for generalized displacement vector $\{\Delta\}_{s+1}$ by Newton Raphson iterative method.

The iterative procedure is as follows:

$\{R\}\ \{\{\Delta\}_{s+1}\} = [S(\Delta_{s+1})]\ \{\Delta\}_{s+1} - \{F\}$

R is called Residual and $[S(\Delta_{s+1})]$ is the stiffness matrix, which is a function of the unknown deflections $\{\Delta\}_{s+1}$.

Expanding {R} in Taylor series about $\{\Delta\}^r_{s+1}$

$\{0\} = \{R\}_{s+1} + [K^T (\{\Delta\}^r_{s+1})] \{\delta\Delta\} + O(\{\delta\Delta\}^2)$

Where

O (.) denotes the higher-order terms in $\{\delta\Delta\}$
$[K^T]$ is known as the tangent stiffness matrix (geometric stiffness matrix).
$\{R\}^r_{s+1} = [K(\Delta^r_{s+1})] \{\Delta\}^r_{s+1} - \{F\}$

The assembled equations are then solved for incremental displacement vector after imposing the boundary and conditions of the problem.

$\{\delta\Delta\} = -[K^T(\{\Delta\}^r_{s+1})]^{-1} \{R\}^r_{s+1}$

$\{\Delta\}^{r+1}_{s+1} = \{\Delta\}^r_{s+1} + \{\delta\Delta\}$

Total displacement vector is obtained from the tangent stiffness matrix, using the latest known solution and the process will continue until the termination criteria with a pre-selected error tolerance is obtained.

4. RESULTS AND DISCUSSIONS

The Navier solutions are developed for rectangular plates with two sets of simply supported (SS) boundary conditions. The two types of boundary conditions are given below.

At edges x = 0 and x = a

$$v_0 = 0,\ w_o = 0,\ \theta_y = 0,\ \theta_z = 0,\ M_x = 0,\ v_0^* = 0,\ w_0^* = 0,$$
$$\theta_y^* = 0,\ \theta_z^* = 0,\ M_x^* = 0,\ N_x = 0,\ N_x^* = 0. \qquad (11)$$

At edges y = 0 and y = b

$$u_0 = 0,\ w_o = 0,\ \theta_x = 0,\ \theta_z = 0,\ M_y = 0,\ u_0^* = 0,\ w_0^* = 0,$$
$$\theta_x^* = 0,\ \theta_z^* = 0,\ M_y^* = 0,\ N_y = 0,\ N_y^* = 0.$$

The above simply supported boundary conditions are considered for solutions of the plates using higher-order shear deformation theory.

The Mechanical load is expanded in double Fourier sine series as:

$$q(x,y,) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Q_{mn} \sin\alpha x \sin\beta y \qquad (12)$$

Where

$$Q_{mn}(z) = \frac{4}{ab}\int_0^a \int_0^b q(x,y) \sin\alpha x \sin\beta y \, dxdy \qquad (13)$$

Where

$$\alpha = \frac{m\pi}{a} \text{ and } \beta = \frac{n\pi}{b} \qquad (14)$$

In order to verify the accuracy and efficiency of the developed theories results and to study the effects of transverse shear deformation, the following typical material properties are used for obtaining the numerical results.

Material 1: (Aluminium)

$E = 70GPa, \upsilon = 0.3, \rho = 2{,}707 Kg/m^3, \kappa = 204 W/mK, \alpha = 23X10^{-6}/^0C$

Material 2: (Zirconia)

$E = 151GPa, \upsilon = 0.3, \rho = 3{,}000 Kg/m^3, \kappa = 209 W/mK, \alpha = 10X10^{-6}/^0C$

The center deflection and load parameter are presented here in non-dimensional form using the following.

$$\overline{w} = w_0/h; \qquad P = q_0 a^4 / E_m h^4$$

For free vibration analysis, the Eigen values problem is given as:

$$([S] - \lambda [M]) = 0 \qquad (15)$$

Where

$\lambda = \omega^2$ is the eigen value.

The real positive roots of the Eq. (15) give the square of the natural frequency ω_{mn} associated with mode (m, n). The smallest of the equation is called the fundamental frequency.

The natural frequencies of functionally graded material plate are presented here in non-dimensional form using the following multiplier $\overline{\omega} = (\omega\, a^2/h)\sqrt{(\rho/E_2)}$

Simply supported square functionally gradient material plate under transverse load is considered for comparisons of fundamental natural frequencies for various material variation parameters (n). The solution procedures outlined in the previous section are applied to the above set of material properties of functionally graded simply supported square plates subjected to transverse load. The variation of non-dimensionalized fundamental frequencies against side to thickness ratio, modulus ratio and aspect ratio as a function of material variation parameter (n) for a simply supported FGM plates for displacement model is shown in Figures 2-5. The results obtained using higher order theory is compared with the available literature [19-20].

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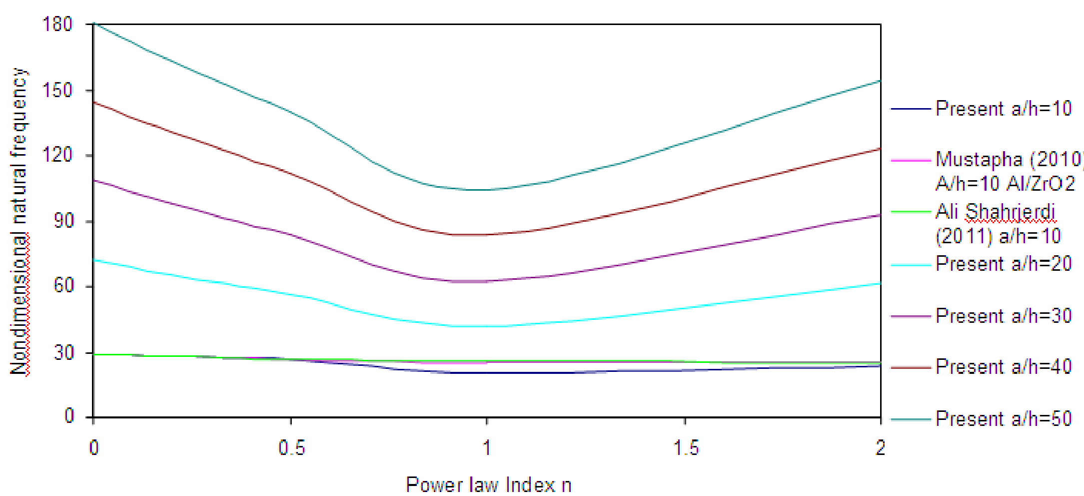

Figure-2. Non-dimensionalized natural frequencies Vs Power law index n with different side to thickness ratio's for a simply supported FGM plate.

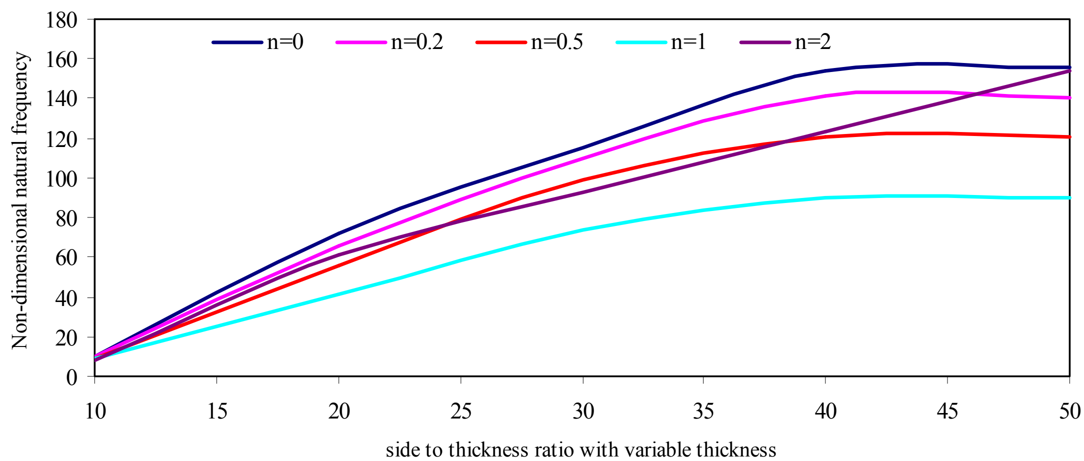

Figure-3. Non-dimensionalized natural frequencies Vs side to thickness ratio (a/h) for a simply supported FGM plate with variable thickness for model.

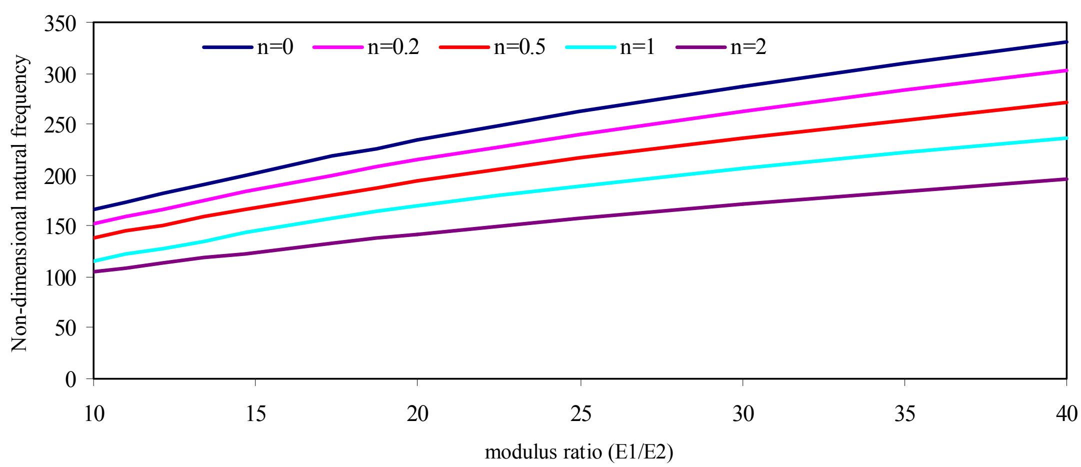

Figure-4. Non-dimensionalized natural frequencies Vs modulus ratio (E1/E2) for a simply supported FGM plate for model.

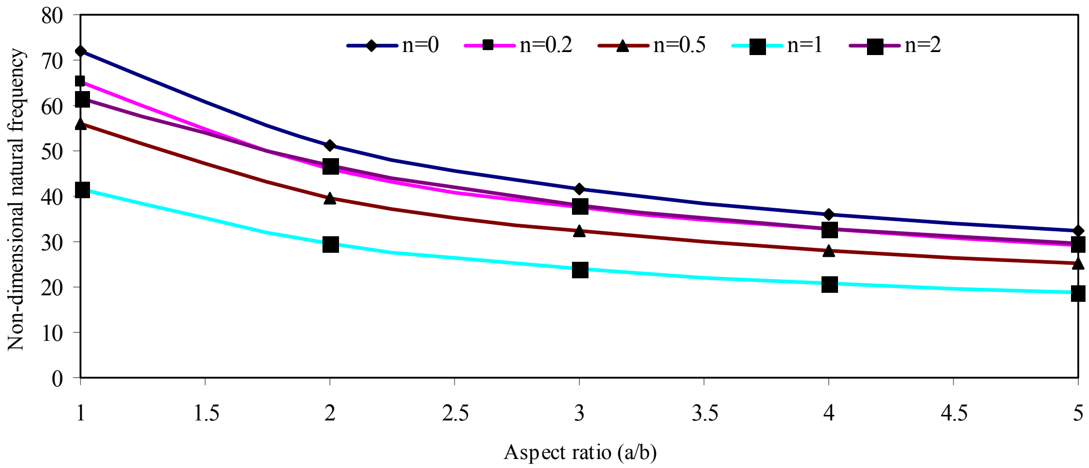

Figure-5. Non-dimensionalized natural frequencies Vs aspect ratio (a/b) for a simply supported FGM plate for model.

Figures 2 to 5 shows the non-dimensionalized fundamental frequencies with respect to side to thickness ratio, aspect ratio and modulus ratio for a simply supported FGM plate for displacement model respectively as a function of material variation parameter (n). It is noticed that the fundamental frequencies are increasing with increase in side to thickness ratio, modulus ratio and decreasing with the increase of aspect ratio. From Figure-2, it is seen that the present results are very close agreement with the Mustapha (2010) and Ali Shahrjerdi (2011) results.

5. CONCLUSIONS

The following conclusions are drawn for the free vibration analysis of FGM plates.

- It is to be concluded that the increasing of properties of ceramic to metal causes decreasing in natural frequency of plate;
- It is to be found that the natural frequencies of FGM plate with different constituents lies between those of natural frequencies of metal and ceramic;
- It can also be conclude that the natural frequencies increase with volume fraction of ceramic;
- It is to be concluded that the natural frequencies of homogeneous ceramic plate is maximum among those of all functionally graded material plates;
- It is to be found that the property of FGM with a small value of material variation parameter (n) approaches a ceramic plate, thus the frequency of FGM is high. On the other hand the property of FGM with a large value of n approaches metal plate and frequency of FGM is small; and
- It is to be concluded that the natural frequencies of FGM are decreased with increase of volume fraction index n in other words, the increasing of the proportion of ceramic to metal results in a decreasing in natural frequencies.

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