



CONTROL SYSTEM DESIGN OF GUIDED MISSILE

Muhammad A. R. Yass¹, Mohammed Khair Aldeen Abbas² and Ismail Ibrahim Shabib¹

¹Electro-Mechanical Engineering Department, University of Technology, Iraq

²Mechanical Engineering Department, Al-Nahrain University, Iraq

E-Mail: mkaskar79@yahoo.com

ABSTRACT

In the present work, analytical performance of dynamic motion and transfer function were calculated in roll mode of flying body. The analysis of the control circuit was made by using four methods, Routh Criterion to determine whether the system is stable or not, Root-Locus method used to determine the limitation of the stability for different value of rate gyro (0.1 to 0.3) and different value of gain (1 to 10), Frequency Response method used to find best transfer function which have shortest time setting and less amount of overshoot and last method used the compromising method which was done first with aileron deflection to determine the limited value of gain, secondly with capability actuator swelling rate. Final transfer function selected was with rate of gyro equal to 0.3 and gain equal to 8 for best steady state behavior. The method above is a perfect solution for flying body control system in all modes and gives an excellent result.

Keywords: guided missile, flying body control system, Routh criterion, Root-Locus method, frequency response method.

INTRODUCTION

It can determine whether our design of a control system meets the specification if the desired time response of the controlled variable determined. by deriving the differential equation for the system solving them an accurate solution of the system's performance can be obtained, but this approach is not feasible for other than simple system if the response doesn't meet the specifications 'it is not easy to determine from this solution just what physical parameters in the system should be changed to improve the response.

The ability of prediction the system's performance by an analysis that does not require the actual solution of the differential equation. Also, we would like to indicate this analysis readily the manner or method by which this system must be adjusted or compensated to produce the desired performance characteristics.

The first thing beat we want to know about a given is whether or not it is stable. This can be determined by examining the root obtained from the involved in determine the root can be tedious, a simpler approach is desirable. By applying Routh's criterion to the characteristic equation it is possible in short or unstable. Yes it doesn't satisfy us because it doesn't indicate the degree of stability of the system, i.e., the amount of overshoot must be maintained, within prescribed limit and transients must die out in a sufficiently short time. The graphical met holds to be described in this test not only indicate whether a system is stable or not but, for a stable system also a low degree of stability.

These are two basic methods available to us, we can choose to analyze and interpret the steady state suicidal response of the transfer function of the system obtains an idea of the system's response. This method is based upon the interpretation of a SyQuest plot. Although this frequency-response approach doesn't yield an exact quantitative prediction of the system's performance, i.e., the poles of the control ratio $c(s)/r(s)$ can not be determine, enough information can be obtained to indicate whether the system need to be the system should be compensated.

This deals with the record method that roots locus method, which incorporates the more desirable features of both, the classical method and the frequency - response method. The root locus is a plot of the root of the characteristic equation of the closed loop system as the function of the gain. This graphical approach yield a clear indication of the effect of gain adjustment with solving small effort compared with other method. The underlying principle is that the poles of $C(s)/R(s)$ (transient response) are released to the source and poles of the open loop transfer function $G(s) H(s)$ and also to the gain. An important advantage of the root locus method is that the root of the characteristic equation of the system can be obtained directly, this result in a complete and accurate solution of the transient and steady state response of the controlled variable. Another important feature is that an approximate solution can be obtained with a reduction of the work required. As with any other design technique, a person who has obtained a coefficient experience with this method is able to apply it and to synthesize a compensating network, if one is required, with relative ease.

MATHEMATICAL ANALYSIS

In the present work, the analytical performance of dynamic motion and transfer function were calculated in roll mode of flying body. The analysis of the control circuit was made by using four methods, Routh Criterion to determine whether the system stable or not, Root-Locus method used to determine the limitation of the stability for different value of rate gyro (0.1 to 0.3) and different value of gain (1 to 10), Frequency Response method used to find best transfer function which have shortest time setting and less amount of overshoot and last method used the compromising method which was done first with aileron deflection to determine the limited value of gain, secondly with capability actuator swelling rate. Final transfer function selected was with rate of gyro equal to 0.3 and gain equal to 8 for best steady state behavior. The method



above is a perfect solution for flying body control system in all modes and give an excellent result.

AERODYNAMIC ANALYSIS

Roll attitude control system

The non dimensional Lateral equation [4]

$$(I_x \ddot{\phi} + \frac{d}{2v} \frac{sqd}{I_x} \dot{\phi}) P = cl_a \frac{sqd}{I_x} \delta_a \quad (1)$$

A multi-loop feed-back control system applied to flying-body, a main feedback of roll angle an attitude gyro and an inner loop of roll rate feed-back [3], see Figure-1.

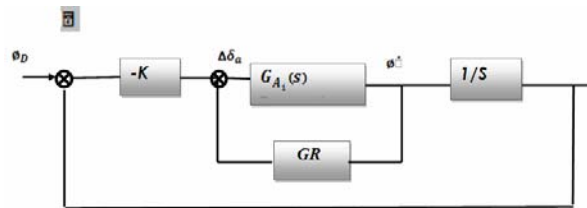


Figure-1. Roll attitude control system.

$$GA = \frac{\phi^*}{\Delta \delta_a} \quad (2)$$

Re-write eq. (1)

$$\left(D - \frac{d}{2v} \frac{sqd}{I_x} cl_p \right) P = cl_a \frac{sqd}{I_x} \delta_a$$

$$cl_a = cl_{\delta_a} \cdot \delta_a \quad \text{and}$$

$$P = \phi^*, \quad P^* = \phi^{**} \quad [4] \quad \text{Then}$$

$$\frac{\phi^*}{\delta_a}(s) = \frac{cl_{\delta_a} \frac{sqd}{I_x}}{s - \frac{d}{2v} \frac{sqd}{I_x} cl_p}$$

$$L\xi = cl_{\delta_a} sqd \quad \text{and} \quad Lp = cl_p sqd \frac{d}{2v} \quad [4]$$

$$\frac{L\xi}{I_x} = l\xi; \frac{Lp}{I_x} = lp, \quad \text{So}$$

$$\frac{\phi^*}{\delta_a}(s) = \frac{l\xi}{s - lp} = \frac{\frac{l\xi}{l_p}}{\frac{s}{l_p} - 1}$$

$$\frac{\phi}{\phi_D}(s) = (K K_{\phi}) / (s \{ T_a T_{\phi} s^2 + (T_a + T_{\phi}) s + (1 + GR K_{\phi}) \}) \quad (11)$$

So the char- equation became equal to

$$K_{\phi} = \frac{l\xi}{l_p}, \quad T_{\phi} = \frac{-1}{l_p}$$

$$\frac{\phi^*}{\delta_a}(s) = \frac{-K_{\phi}}{1 - T_{\phi} s} \quad (5)$$

The inner loop transfer function

$$Gi(s) = \frac{G_{A_1}(s)}{1 + GR(s)G_{A_1}(s)} \quad (6)$$

$$Gi(s) = \frac{-K_{\phi}}{T_{\phi}(s) + (1 - GRK_{\phi})}$$

While the overall transfer function

$$\frac{\phi}{\phi_D}(s) = \frac{-\frac{KGi(s)}{s}}{1 - \frac{KGi(s)}{s}} = \frac{KK_{\phi}}{s(T_{\phi}s + 1 - GRK_{\phi}) + KK_{\phi}} \quad (7)$$

If the actuator dynamic included the presentation can be seen in Figure-2 the inner loop transfer function

$$Gi(s) = \frac{K_{\phi}}{(1 + T_a s)(1 + T_{\phi} s) + GRK_{\phi}} \quad (8)$$

While the overall transfer function was [8]

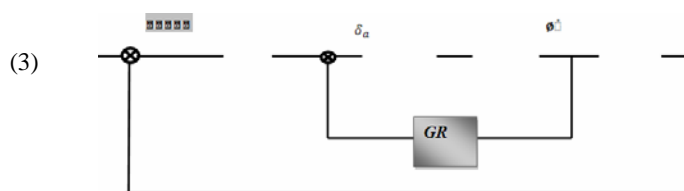


Figure-2. Roll attitude control system with actuator.

$$\frac{\phi}{\phi_D}(s) = \frac{-\frac{KGi(s)}{s}}{1 - \frac{KGi(s)}{s}} \quad (9)$$

$$\frac{\phi}{\phi_D}(s) = \frac{KK_{\phi}}{(T_a T_{\phi}) s^3 + (T_a T_{\phi}) s^2 + (1 - GRK_{\phi}) s + KK_{\phi}} \quad (10)$$

Open loop transfer function (K-Root Locus)

From open loop transfer function which was equal to [4]



$$(T_a T_\phi) s^3 + (T_a + T_\phi) s^2 + (1 + GR K_\phi) s = 0 \quad (12)$$

Dividing equation (12) by $(T_a T_\phi)$ we will get

$$s^3 + \frac{T_a + T_\phi}{T_a T_\phi} s^2 + \frac{1 + GR K_\phi}{T_a T_\phi} s = 0 \quad (13)$$

Solving equation above for different k and $[GR]$ and all the complex root found was studied to determine the effect of k .

Close loop transfer function (GR Root Locus)

$$\frac{\phi}{\phi_D}(s) = \frac{(K K_\phi / T_a T_\phi)}{(s^3 + (T_a + T_\phi) s^2 + (1 + GR K_\phi) s + K K_\phi)}$$

Also it can be written as

$$\frac{\phi}{\phi_D}(s) = \frac{D}{s(s + p_1)(s + p_2)(s + p_3)}$$

By partial fraction

$$\frac{1}{s} + \frac{\alpha_1 + \alpha_2 i}{s + g_1 - p_1 i} + \frac{\alpha_1 - \alpha_2 i}{s + g_2 - p_1 i} + \frac{-\alpha_3}{s + g_2}$$

The Inverse Laplace Transformation

$$\frac{\phi}{\phi_D}(t) = 1 - \alpha_3 e^{-g_2 t} - 2e^{-g_1 t} (\alpha_2 \cos p_1 t + \alpha_3 \sin p_1 t) \quad (16)$$

Solving equation above for different k and $[GR]$ and all the complex root found was studied to determine the effect of $[GR]$.

$$\frac{\phi}{\phi_D}(t) = +\alpha_3 g_2 e^{-g_2 t} + 2g_1 e^{-g_1 t} (\alpha_2 \cos p_1 t + \alpha_3 \sin p_1 t) + 2g_1 P_1 e^{-g_1 t} (-\alpha_2 \sin p_1 t + \alpha_3 \cos p_1 t)$$

$$+ 2P_1 g_1 e^{-g_1 t} (-\alpha_2 \sin p_1 t + \alpha_3 \cos p_1 t) + P_1^2 e^{-g_1 t} (\alpha_2 \cos p_1 t + \alpha_3 \sin p_1 t)$$

$$\frac{\phi}{\phi_D}(t) = -\alpha_3 g_2 e^{-g_2 t} - g_1 e^{-g_1 t} (\alpha_2 \cos p_1 t + \alpha_3 \sin p_1 t) + 2g_1 P_1 e^{-g_1 t} (-\alpha_2 \sin p_1 t + \alpha_3 \cos p_1 t) + P_1^2 e^{-g_1 t} (\alpha_2 \cos p_1 t + \alpha_3 \sin p_1 t)$$

$$\frac{\Delta S_a}{\phi_D}(t) = \left\{ \begin{aligned} &\alpha_3 g_2 e^{-g_2 t} + 2g_1 e^{-g_1 t} (\alpha_2 \cos p_1 t + \alpha_3 \sin p_1 t) - 2P_1 e^{-g_1 t} \\ &(-\alpha_2 \sin p_1 t + \alpha_3 \cos p_1 t) \phi (-\alpha_3 g_2 e^{-g_2 t} - 2g_1 e^{-g_1 t} (\alpha_2 \cos p_1 t + \alpha_3 \sin p_1 t) + \\ &2g_1 P_1 e^{-g_1 t} (-\alpha_2 \sin p_1 t + \alpha_3 \cos p_1 t) + 2P_1 g_1 e^{-g_1 t} (-\alpha_2 \sin p_1 t + \alpha_3 \cos p_1 t) \\ &+ 2P_1^2 e^{-g_1 t} (\alpha_2 \cos p_1 t + \alpha_3 \sin p_1 t) \end{aligned} \right\}$$

.....26

By varying time it can be solve for Aileron deflection.

Actuator slewing rate

It is the time required for actuator to deflect the aileron.

The close loop t.s which was equal to

$$\frac{\phi}{\phi_D}(s) = \frac{K K_\phi}{s^3 + \frac{(T_a + T_\phi)}{T_a T_\phi} s^2 + \frac{1 + GR K_\phi}{T_a T_\phi} s + \frac{K K_\phi}{T_a T_\phi}}$$

Divide equation (15) by $T_a T_\phi$

Dynamic response of aileron deflection
from equation

$$\frac{\phi^\circ}{\Delta \delta_a} = -\frac{K \phi}{1 + T_\phi S}$$

and it can be also written as

$$\Delta \delta_a(s)(-K_\phi) = (1 + T_\phi S)\phi(s) \quad \dots 16a$$

By taking the laplace inverse of equation (16a)

$$\Delta \delta_a(s)(-K_\phi) = \phi^\circ(t) + T_\phi \phi^{\circ\circ}(t) \quad \dots 16b$$

Divided equation (16b) by ϕ_0

$$\frac{\Delta S_a}{\phi_D}(t)(-k_\phi) = \frac{\phi(t)}{\phi_0} + T_\phi \frac{\phi^{\circ\circ}(t)}{\phi_0}$$

From equation



By differentiate equation (26).

$$\frac{\Delta S^{\circ} a}{\phi_D} = \frac{1}{k\phi} \left\{ \begin{aligned} & -\alpha_3 g_2^2 - 2g_1^2 e^{-g_1 t} (\alpha_2 \cos p_1 t + \alpha_3 \sin p_1 t) + 2g_1 p_1 e^{-g_1 t} (-\alpha_2 \sin p_1 t + \alpha_3 \cos p_1 t) + \\ & 2p_1 g_1 e^{-g_1 t} (-\alpha_2 \sin p_1 t + \alpha_3 \cos p_1 t) + 2p_1^2 e^{-g_1 t} (\alpha_2 \cos p_1 t + \alpha_3 \sin p_1 t) T\phi [\alpha_3 g_2^3 e^{-g_1 t} + \\ & 2g_1^3 e^{-g_1 t} (\alpha_2 \cos p_1 t + \alpha_3 \sin p_1 t) - 2g_1^2 p_1 e^{-g_1 t} (-\alpha_2 \sin p_1 t + \alpha_3 \cos p_1 t) - 2g_1 p_1^2 e^{-g_1 t} \\ & (\alpha_2 \cos p_1 t + \alpha_3 \sin p_1 t) - 2p_1 g_1^2 e^{-g_1 t} (-\alpha_2 \sin p_1 t + \alpha_3 \cos p_1 t) - 2p_1^2 g_1 e^{-g_1 t} (\alpha_2 \cos p_1 t \\ & + \alpha_3 \sin p_1 t) - 2p_1^2 g_1 e^{-g_1 t} (-\alpha_2 \sin p_1 t + \alpha_3 \cos p_1 t) \end{aligned} \right\} \quad (27)$$

So by solving the above equation at different GR, K and at $t = 0$, become max actuator slewing rate which occurs at $t = 0$, we will get different actuator slewing rate at different combinations of GR and K.

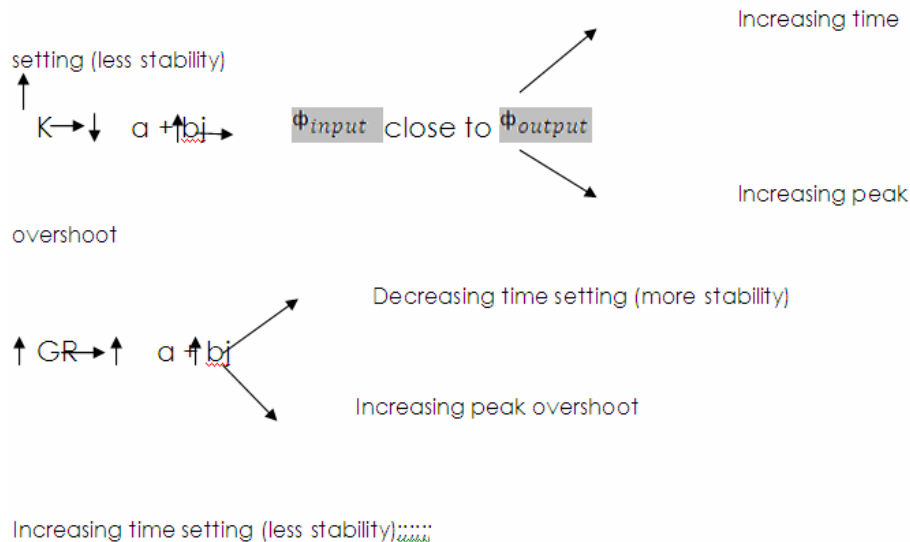
RESULTS AND DISCUSSIONS

DISCUSSIONS

The purpose of this design to obtain largest gain attainable required and the smallest peak overshoot and shortest time setting. From Table-1 and Figures 1 and 2,

GR = 0.1, maximum K is equal to 4 and for more than that the system is not stable.

It can be observe from Table-1 that by increasing K at constant value of GR, the real part of complex root will decrease and the imaginary part will increase, this mean that the overshoot will increase too and setting time will also increase, so it must have a choose between the largest gain attainable, shortest peak overshoot and shortest time setting. Also by increasing at constant K the real part of complex root will increase and imaginary part will increase too and that increase the peak overshoot and better stability.



See Figures (2-10)

It can be observed from Table-2 and aileron deflection curve (Figures 1 to 10) that by increasing K at constant GR, aileron deflection will also increase but it decrease by decreasing increasing GR at constant K.

↑ K → Aileron Deflection ↑ , GR → Aileron Deflection ↓

So it can't go beyond GR = 0.3 and K = 8 because it is limited by aileron deflection and case study design data which was equal to 5 rad.

After compromising all the values of K and GR searching for our design requirement which is given above, it is found that GR = 0.3, K = 8 is the best selection which must be checked with actuator slewing time (time of actuator to deflect). From Table-3 it can be notice that the maximum value of actuator slewing rate at GR = 0.3 and K = 10 was equal to 43.668 rad /sec and the design value of actuator slewing rate is equal to 90 rad/sec, that mean that the selection value was within the limit. Also it can be observed that as K increases the actuator slewing time will also increase and for increasing GR at constant K the actuator slewing time remains the same.

**CONCLUSIONS**

It can conclude that the response of a control system is examined by root locus method and it depend on real root of the characteristic equation of close loop transfer function and this root depend on K and GR so by increasing K the root shift to right of root locus which decrease the stability but by increasing GR the root shift to the left which increases the stability also increasing K and GR depend on max aileron deflection.

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Table-1. Roll angle response at different values of K and GR.

GR	K	Characteristics equations	Real root	Imaginary root	Steady time	Max peak over shoot	stability
0.1	1	$s^3 + 50s^2 + 695.38s + 6901.3$	-36.011	-6.994±12.053j	0.9999	1.2999	stable
0.1	2	$s^3 + 50s^2 + 695.38s + 13802.72$	-41.254	-4.372±17.61j	0.9925	1.4121	stable
0.1	3	$s^3 + 50s^2 + 695.38s + 20704.08$	-44.794	-2.602±21.34j	0.9591	1.6098	unstable
0.1	4	$s^3 + 50s^2 + 695.38s + 27605.44$	-47.579	-0.210±24.056j	0.1842	1.8670	unstable
0.1	5	$s^3 + 50s^2 + 695.38s + 34506.8$	-49.917	-0.041±26.292j	0.1495	1.8478	unstable
0.1	6	$s^3 + 50s^2 + 695.38s + 41408.16$	-51.955	-0.977±28.214j	0.3195	1.7854	unstable
0.1	7	$s^3 + 50s^2 + 695.38s + 48309.52$	-53.774	-1.887±29.913j	0.3195	1.7501	unstable
0.1	8	$s^3 + 50s^2 + 695.38s + 55210.88$	-55.425	-2.712±31.444j	0.4961	1.4779	unstable
0.1	9	$s^3 + 50s^2 + 695.38s + 62112.24$	-56.943	-3.471±32.843j	0.5106	1.4984	unstable
0.1	10	$s^3 + 50s^2 + 695.38s + 69013.6$	-58.351	-4.175±34.136j	0.5752	1.6190	unstable
0.2	1	$s^3 + 50s^2 + 1385.517s + 6901.36$	-6.197	-21.902±25.188j	0.1360	1.2999	unstable
0.2	2	$s^3 + 50s^2 + 1385.517s + 13802.72$	-16.612	-16.693±27.668j	0.6129	1.2999	unstable
0.2	3	$s^3 + 50s^2 + 1385.517s + 20704.08$	-27.071	-11.491±25.154j	0.9915	1.2999	stable
0.2	4	$s^3 + 50s^2 + 1385.517s + 27605.44$	-33.290	-8.354±27.557j	0.9670	1.2999	stable
0.2	5	$s^3 + 50s^2 + 1385.517s + 34506.8$	-37.568	-6.215±29.662j	0.8732	1.2999	stable
0.2	6	$s^3 + 50s^2 + 1385.517s + 41408.16$	-40.884	-4.557±31.496j	0.7698	1.48202	stable
0.2	7	$s^3 + 50s^2 + 1385.517s + 48309.52$	-43.624	-3.187±33.124j	0.5990	1.5690	unstable
0.2	8	$s^3 + 50s^2 + 1385.517s + 55210.88$	-45.981	-2.029±34.593j	0.4922	1.6550	unstable
0.2	9	$s^3 + 50s^2 + 1385.517s + 62112.24$	-48.061	-0.969±35.936j	0.3995	1.7212	unstable
0.2	10	$s^3 + 50s^2 + 1385.517s + 69013.61$	-49.932	-0.033±37.177j	0.2237	1.7956	unstable
0.3	1	$s^3 + 50s^2 + 2076.65s + 6901.3$	-3.617	-23.191±37.014j	0.1349	0.6701	unstable
0.3	2	$s^3 + 50s^2 + 2076.65s + 13802.72$	-7.922	-21.038±36.051j	0.8902	0.91354	unstable
0.3	3	$s^3 + 50s^2 + 2076.65s + 20704.08$	-12.979	-18.510±35.391j	0.9401	0.9837	unstable
0.3	4	$s^3 + 50s^2 + 2076.65s + 27605.44$	-18.489	-15.755±35.282j	0.9730	1.00048	unstable
0.3	5	$s^3 + 50s^2 + 2076.65s + 34506.8$	-23.761	-13.119±35.778j	0.9553	1.01173	Stable
0.3	6	$s^3 + 50s^2 + 2076.65s + 41408.16$	-28.328	-10.836±36.666j	0.9804	1.02265	stable
0.3	7	$s^3 + 50s^2 + 2076.65s + 48309.52$	-32.164	-8.918±37.715j	0.8704	1.0609	stable
0.3	8	$s^3 + 50s^2 + 2076.65s + 55210.88$	-35.412	-7.294±38.803j	0.8148	1.1045	stable
0.3	9	$s^3 + 50s^2 + 2076.65s + 62112.24$	-38.412	-7.294±38.803j	0.7619	1.1619	stable
0.3	10	$s^3 + 50s^2 + 2076.65s + 69013.61$	-38.215	-5.892±39.882j	0.6737	1.46103	stable
0.4	1	$s^3 + 50s^2 + 2765.79s + 6901.36$	-2.618	-23.693±45.613j	0.1075	1.2999	unstable
0.4	2	$s^3 + 50s^2 + 2765.79s + 13802.72$	-5.472	-22.263±45.016j	0.1274	1.2999	unstable
0.4	3	$s^3 + 50s^2 + 2765.79s + 20704.08$	-8.520	-20.705±44.513j	0.1046	1.2999	unstable
0.4	4	$s^3 + 50s^2 + 2765.79s + 27605.44$	-11.944	-19.028±44.149j	0.6192	1.2999	unstable
0.4	5	$s^3 + 50s^2 + 2765.79s + 34506.8$	-15.461	-17.269±43.972j	0.8044	1.2999	unstable
0.4	6	$s^3 + 50s^2 + 2765.79s + 41408.16$	-19.025	-15.487±44.007j	0.9151	1.2999	unstable
0.4	7	$s^3 + 50s^2 + 2765.79s + 48309.52$	-22.502	-13.570±44.251j	0.9983	1.2999	stable
0.4	8	$s^3 + 50s^2 + 2765.79s + 55210.88$	-25.782	-12.109±44.662j	0.9732	1.2999	stable
0.4	9	$s^3 + 50s^2 + 2765.79s + 62112.24$	-28.817	-10.591±45.201j	0.9568	1.2999	stable
0.4	10	$s^3 + 50s^2 + 2765.79s + 69013.61$	-31.595	-9.202±45.827j	0.9994	1.2999	stable
0.5	1	$S^3 + 50s^2 + 3455.92s + 6901.36$	-2.055	-23.972±52.251j	0.1144	1.2999	unstable
0.5	2	$S^3 + 50s^2 + 3455.92s + 13802.72$	-4.231	-22.88±52.331j	0.1191	1.2999	unstable
0.5	3	$S^3 + 50s^2 + 3455.92s + 20704.08$	-6.526	-21.736±51.958j	0.1121	1.2999	unstable
0.5	4	$S^3 + 50s^2 + 3455.92s + 27605.44$	-8.936	-20.531±51.640j	0.40454	1.2999	unstable
0.5	5	$S^3 + 50s^2 + 3455.92s + 34506.8$	-11.446	-19.276±51.401j	0.6363	1.2999	unstable
0.5	6	$S^3 + 50s^2 + 3455.92s + 41408.16$	-14.030	-17.984±51.262j	0.73411	1.2999	unstable
0.5	7	$S^3 + 50s^2 + 3455.92s + 48309.52$	-16.655	-16.672±53.302j	0.74412	1.2999	unstable
0.5	8	$S^3 + 50s^2 + 3455.92s + 55210.88$	-19.280	-15.360±54.084j	0.61467	1.2999	unstable
0.5	9	$S^3 + 50s^2 + 3455.92s + 62112.24$	-21.869	-14.060±54.872j	0.9728	1.2999	stable
0.5	10	$S^3 + 50s^2 + 3455.92s + 69013.61$	-24.375	-12.812±55.556j	0.5494	1.2999	stable

**Table-2.** Roll angle response at different values of K and GR.

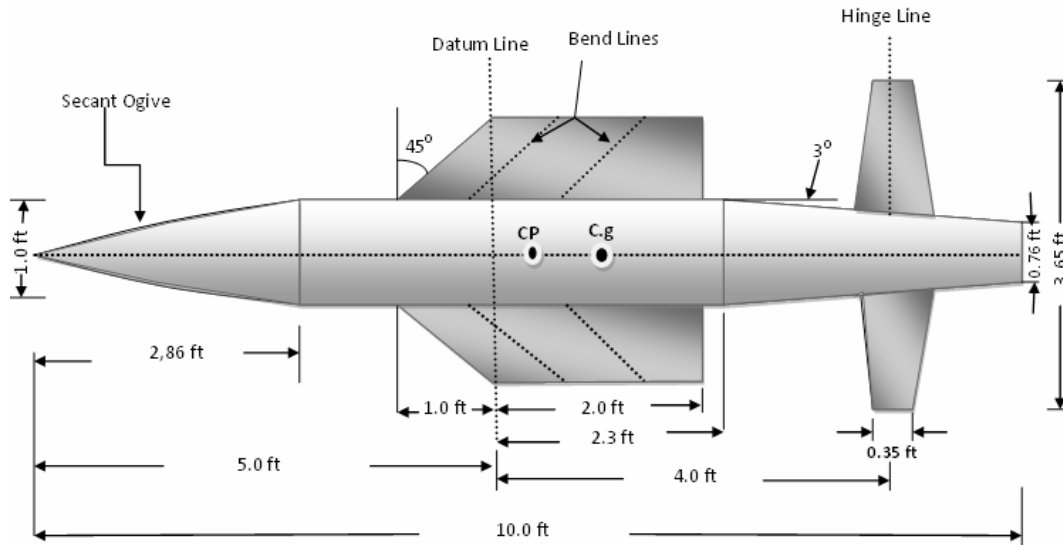
GR	Gain (K)	Overshoot (rad)	Time rise (sec)	Steady state time (sec)
0.1	1	0.15	0.205	0.55
0.1	2	0.42	0.102	0.60
0.1	3	0.6	0.1	1
0.1	4	0.75	0.1	More than 1
0.2	4	0.25	0.1	0.33
0.2	6	0.43	0.075	0.58
0.2	8	0.65	0.07	0.97
0.2	10	0.8	0.06	More than 1
0.3	4	Over damping	Over damping	0.25
0.3	6	0.16	0.09	0.4
0.3	8	0.32	0.75	0.46
0.3	10	0.45	0.65	0.8

Table-3. Result of maximum aileron deflection required for different values of K and CR.

GR	K	$\Delta \xi / \phi D)_{\max}$	$\Delta \xi)_{\max}$ (rad)
0.1	1	0.72	0.062
	2	1.38	0.120
	3	2.04	0.178
	4	3.19	0.278
0.2	4	2.42	0.2111
	6	3.56	0.311
	8	4.65	0.397
	10	5.18	0.452
0.3	4	2.17	0.189
	6	3.29	0.287
	8	4.29	0.374
	10	6.11	0.533

**Table-4.** Result of maximum actuator slewing rate required for different K and CR.

GR	K	$\Delta \xi / (\Phi D)_{\max}$ (1/Sec)	$\Delta \xi_{\max}$ (rad/sec)
0.1	1	49.688	4.336
	2	99.910	8.719
	3	149.870	13.078
	4	199.760	17.438
	4	199.760	17.437
0.2	6	299.649	26.149
	8	399.593	34.871
	10	499.501	43.589
0.3	4	199.816	17.437
	6	299.79	26.162
	8	399.796	34.864
	10	500.398	43.668



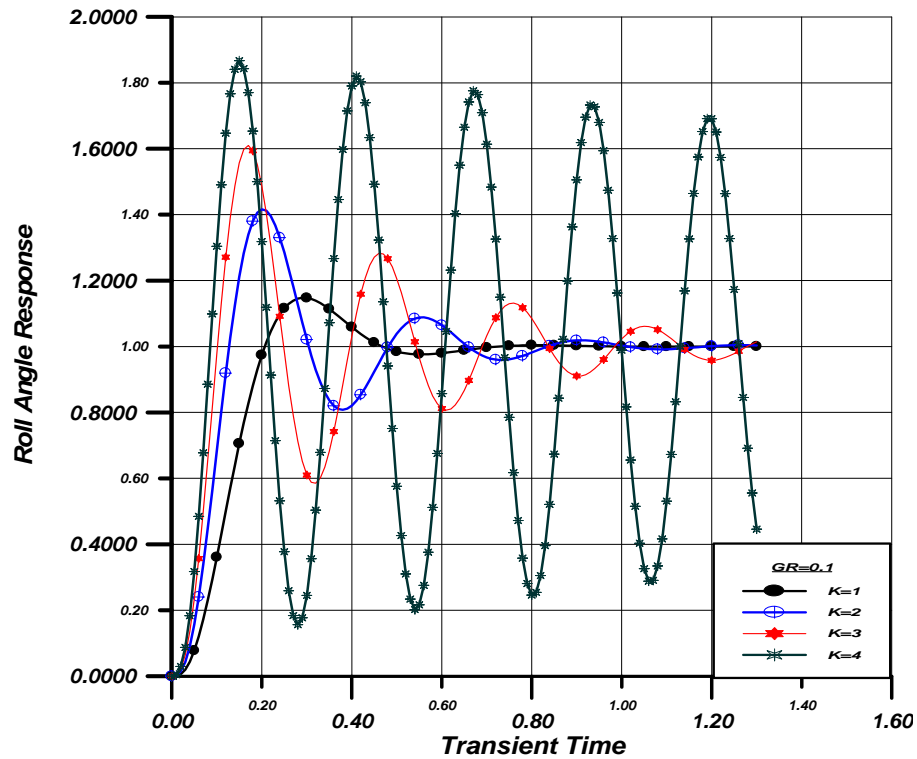


Figure-1. Rolling response versus time response for $GR = 0.1$ and different gain value.

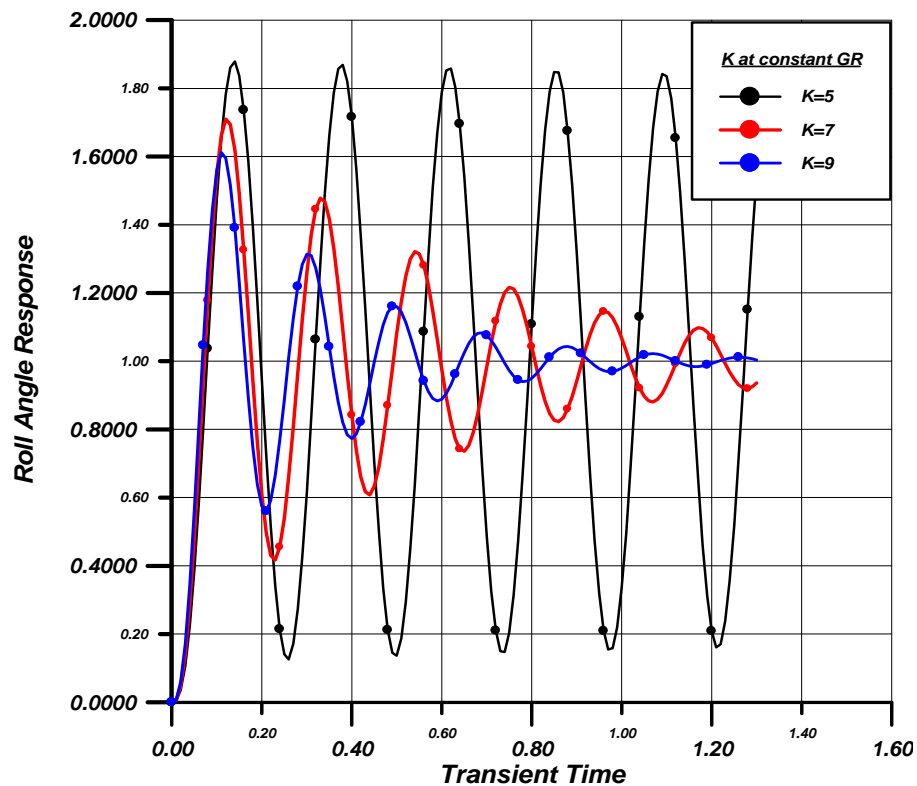


Figure-2. Rolling response versus time response for $GR = 0.1$ and different gain value.

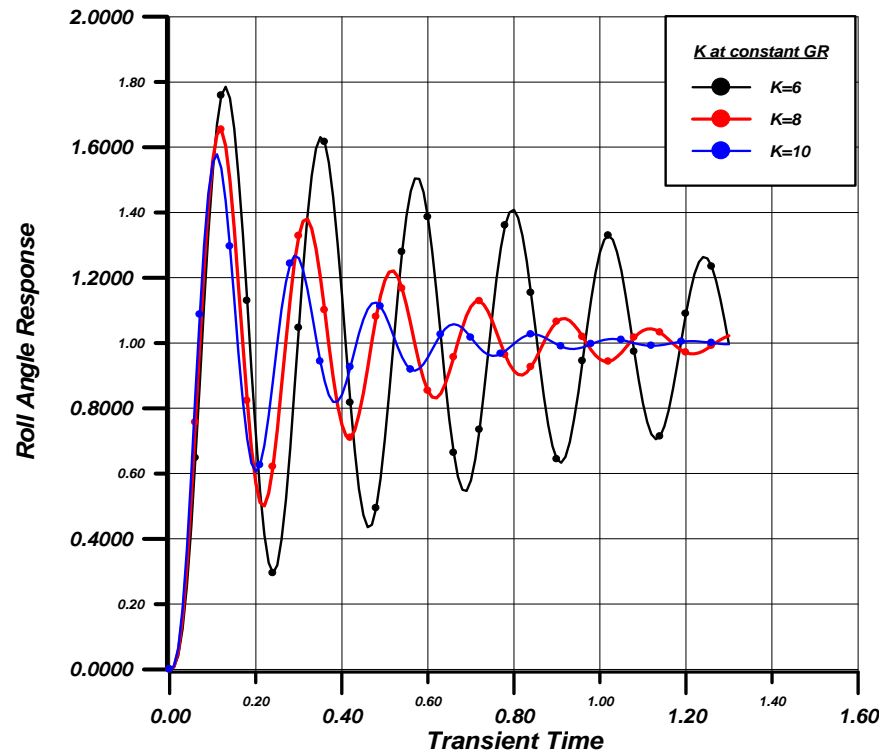


Figure-3. Rolling response verses time response for GR=0.1 and Different Gain value.

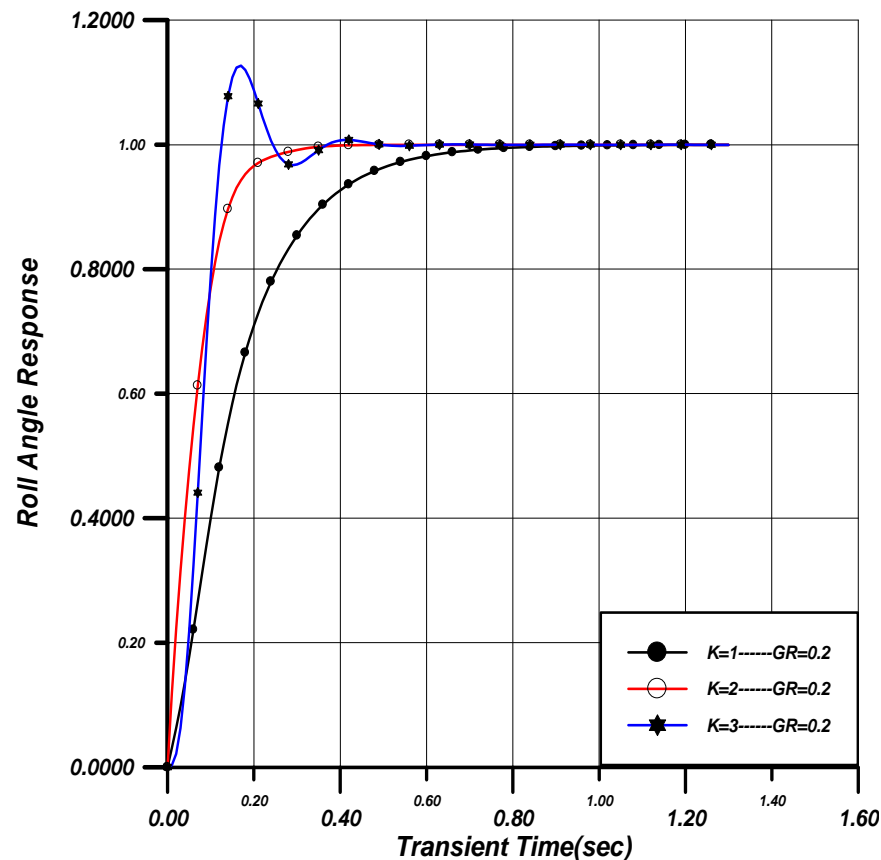


Figure-4. Rolling response verses time response for GR = 0.2 and different gain value.

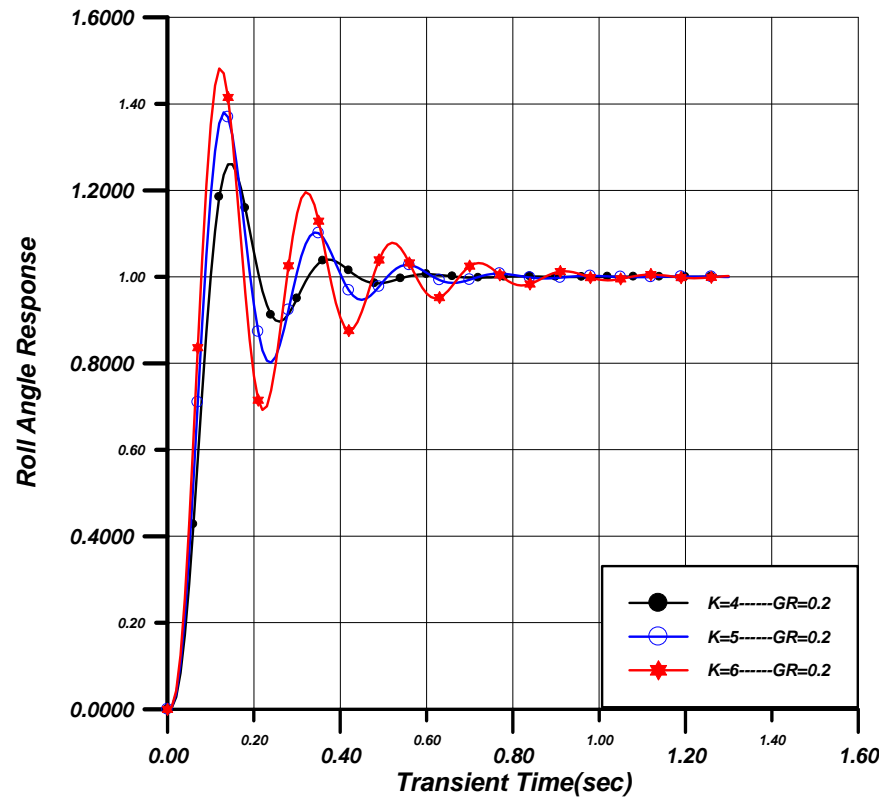


Figure-5. Rolling response verses time response for GR = 0.2 and different gain value.

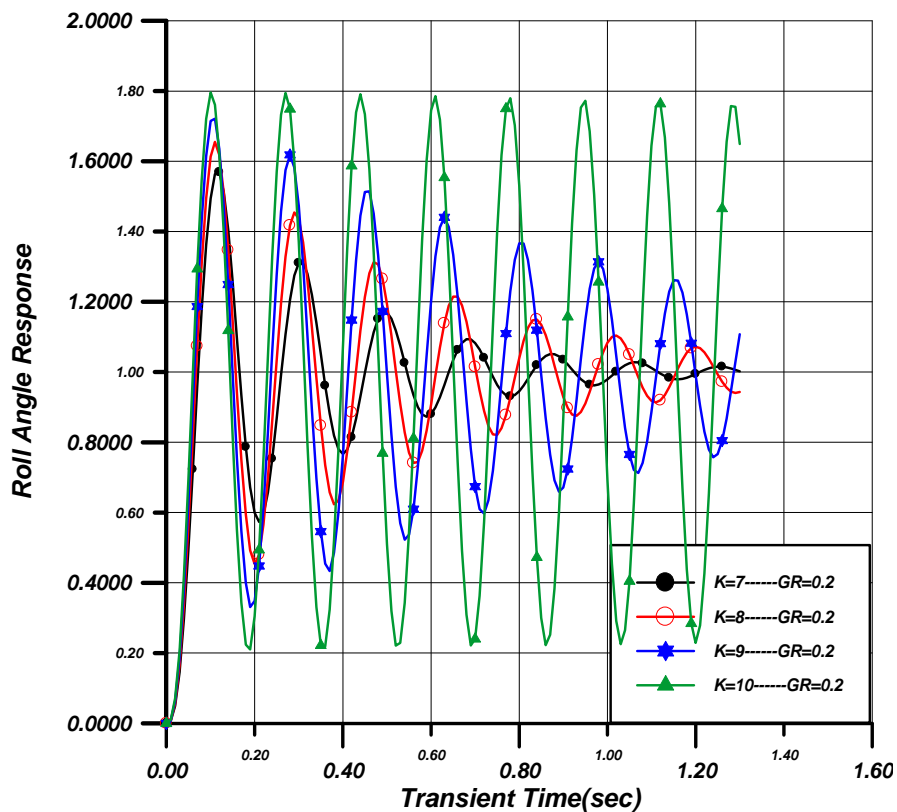


Figure-6. Rolling response verses time response for GR = 0.2 and different gain value.

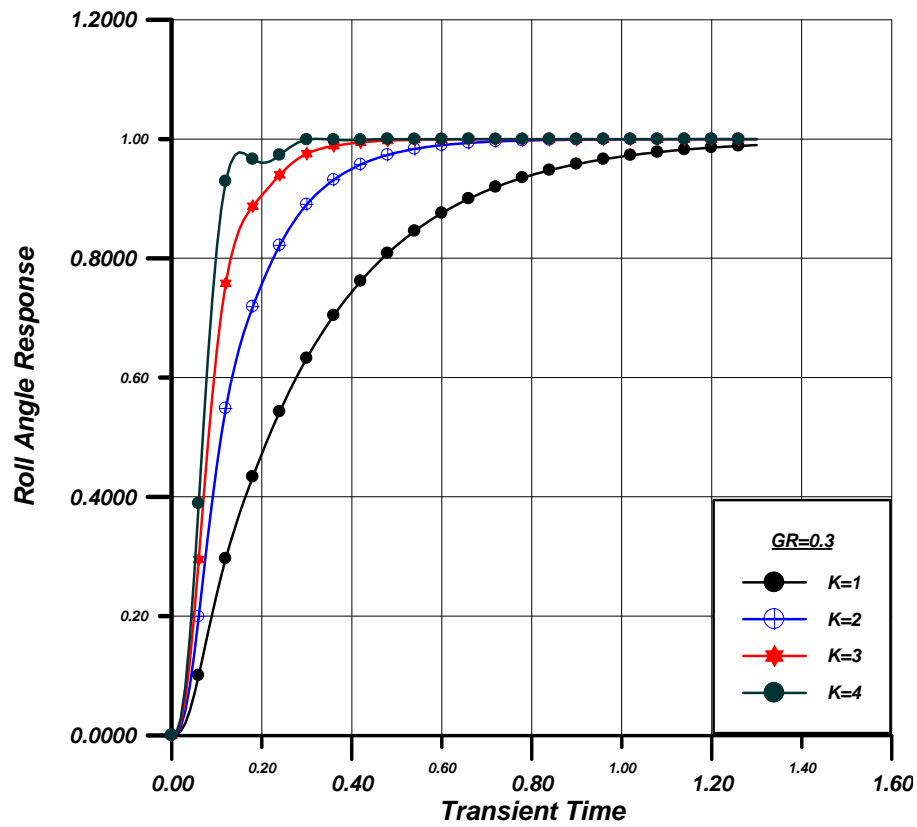


Figure-7. Rolling response verses time response for $GR = 0.3$ and different gain value.

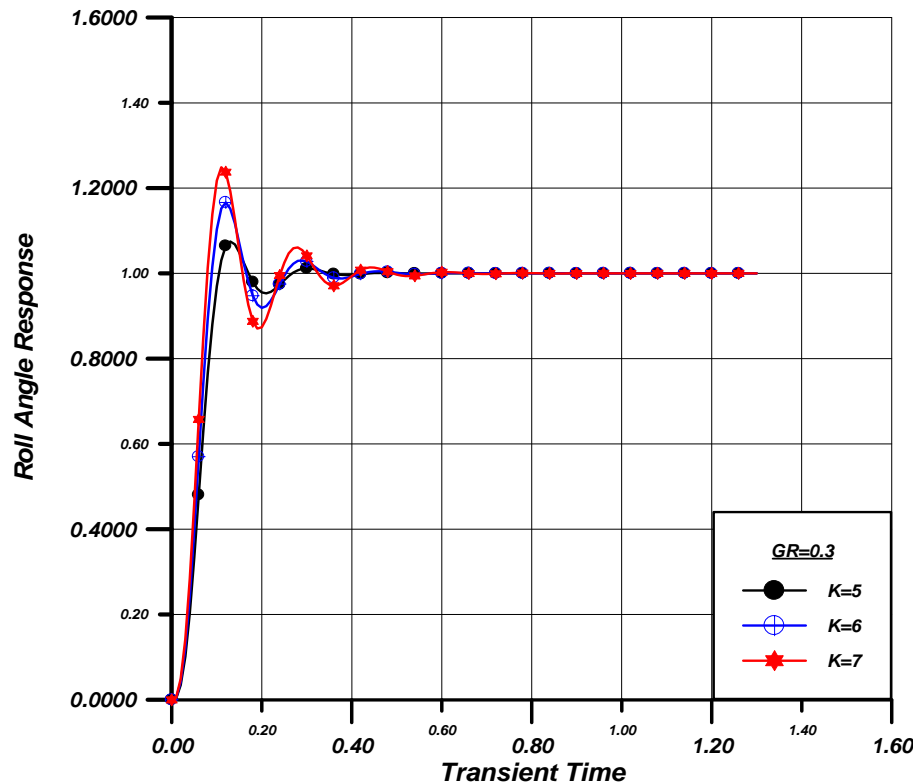


Figure-8. Rolling response verses time response for $GR = 0.3$ and different gain value.

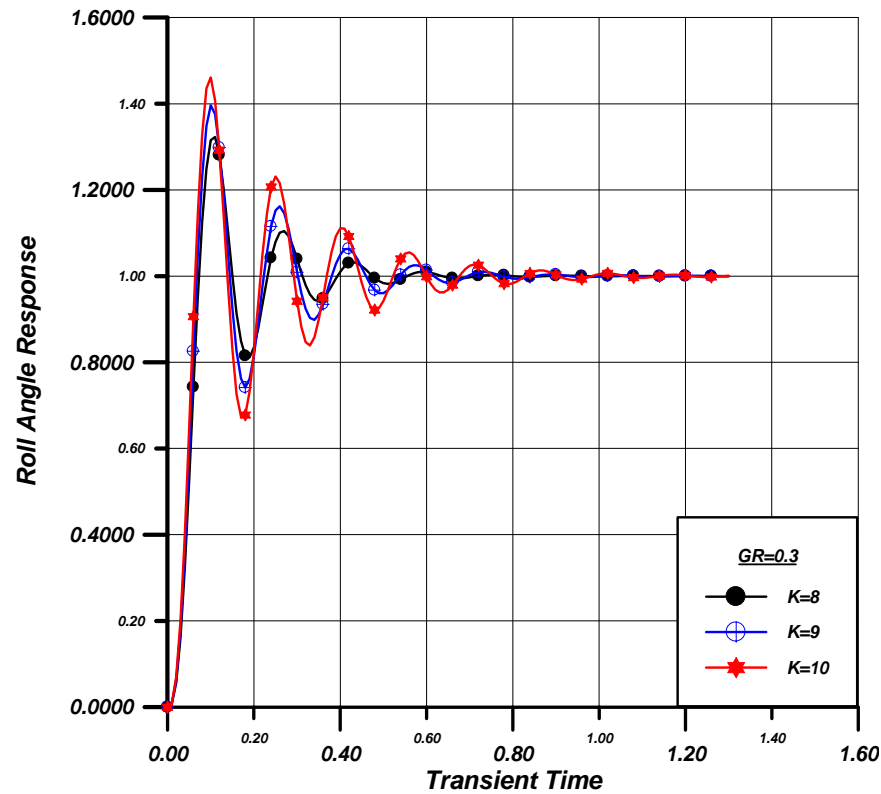


Figure-9. Rolling response verses time response for GR = 0.3 and different gain value.

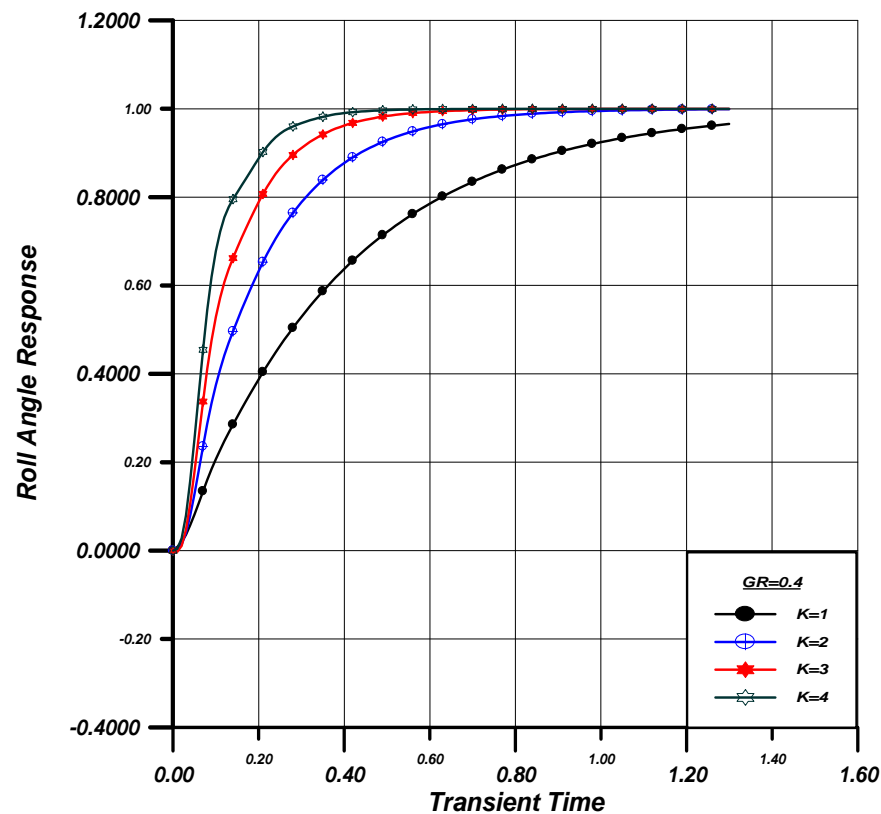


Figure-10. Rolling response verses time response for GR = 0.4 and different gain value.

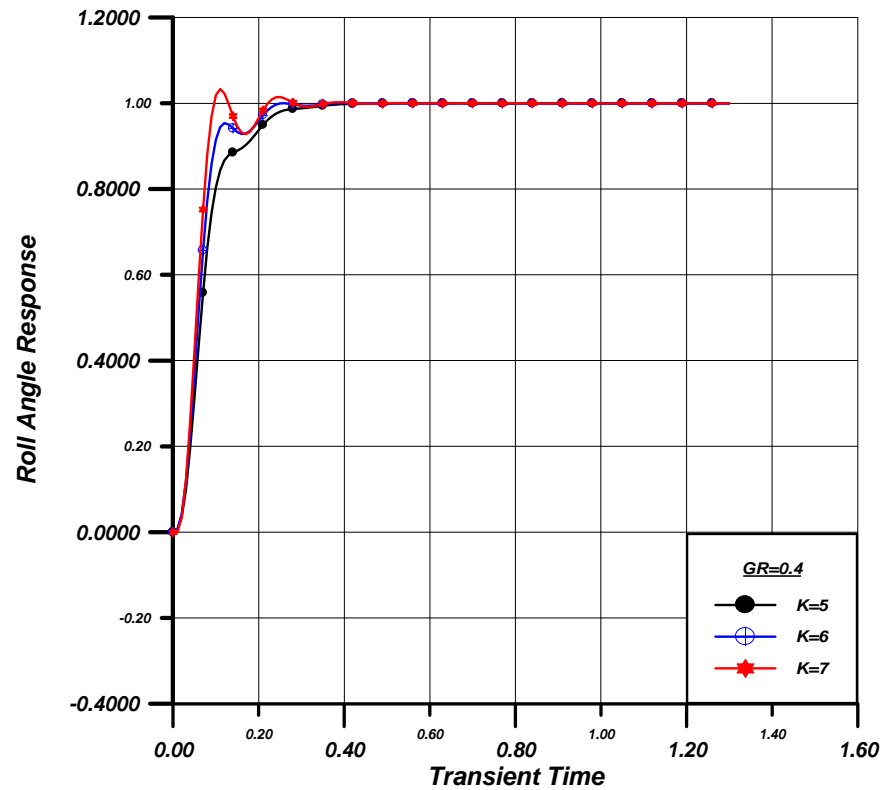


Figure-11. Rolling response verses time response for GR=0.4 and different gain value.

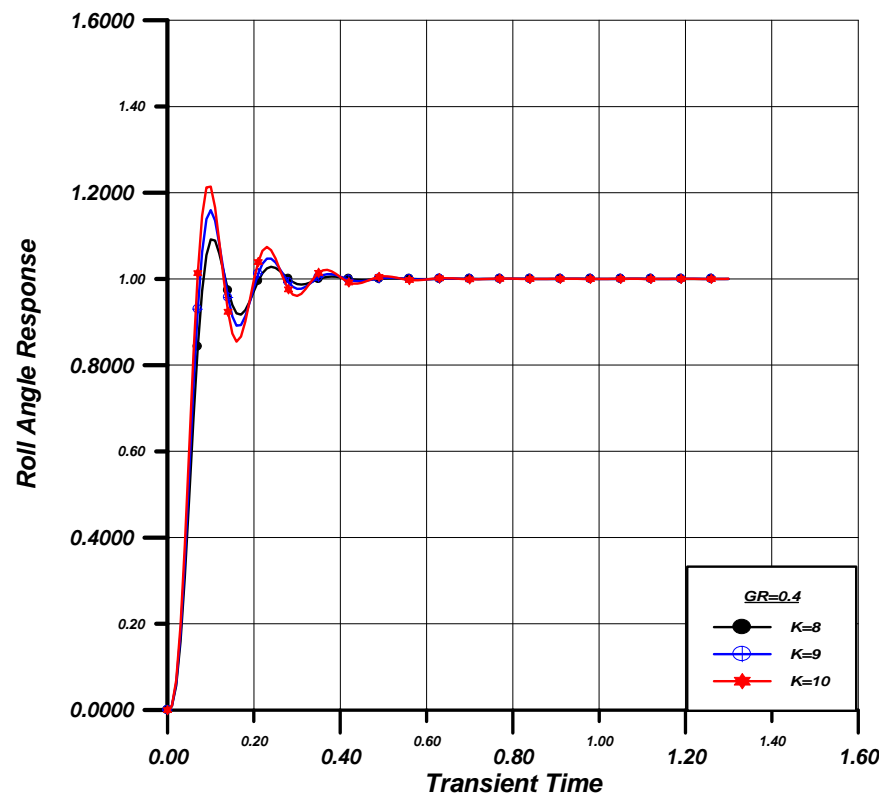


Figure-12. Rolling response verses time response for GR = 0.4 and different gain value.

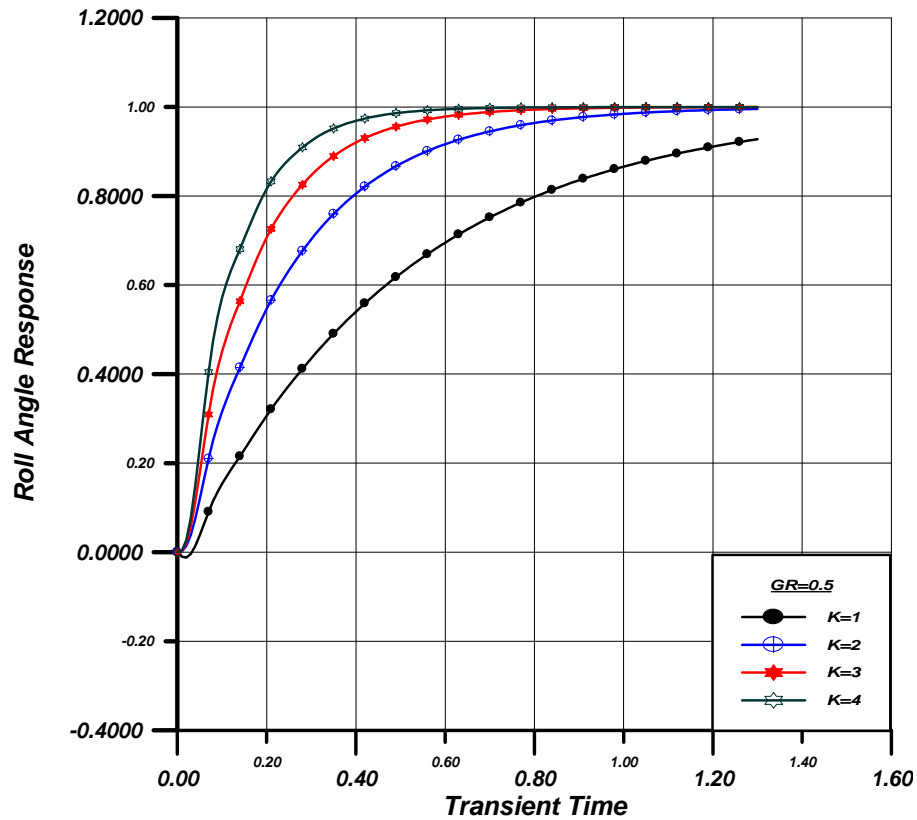


Figure-13. Rolling response versus time response for GR=0.5 and different gain value.

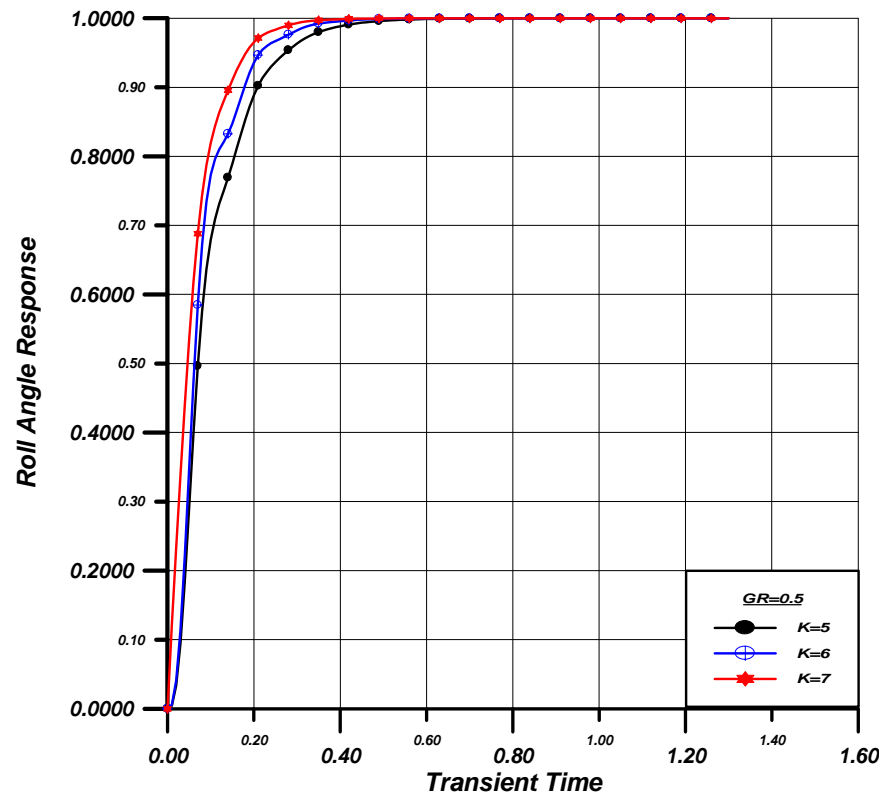


Figure-14. Rolling response versus time response for GR = 0.5 and different gain value.

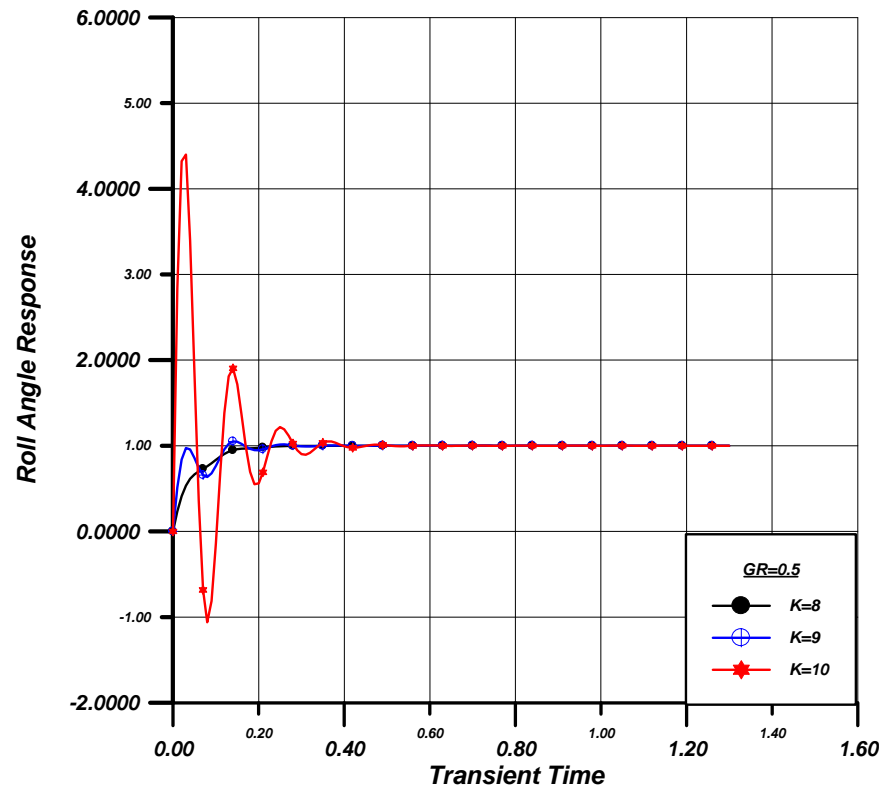


Figure-15. Rolling response versus time response for GR = 0.5 and different gain value.

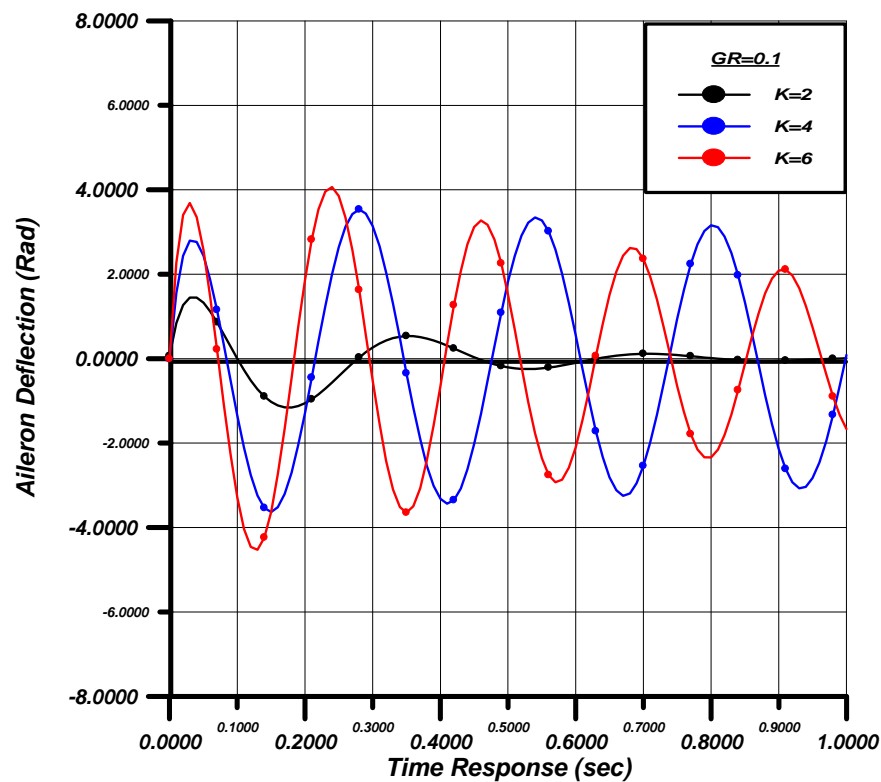


Figure-16. Aileron deflection versus time response for GR = 0.1 and different gain value.

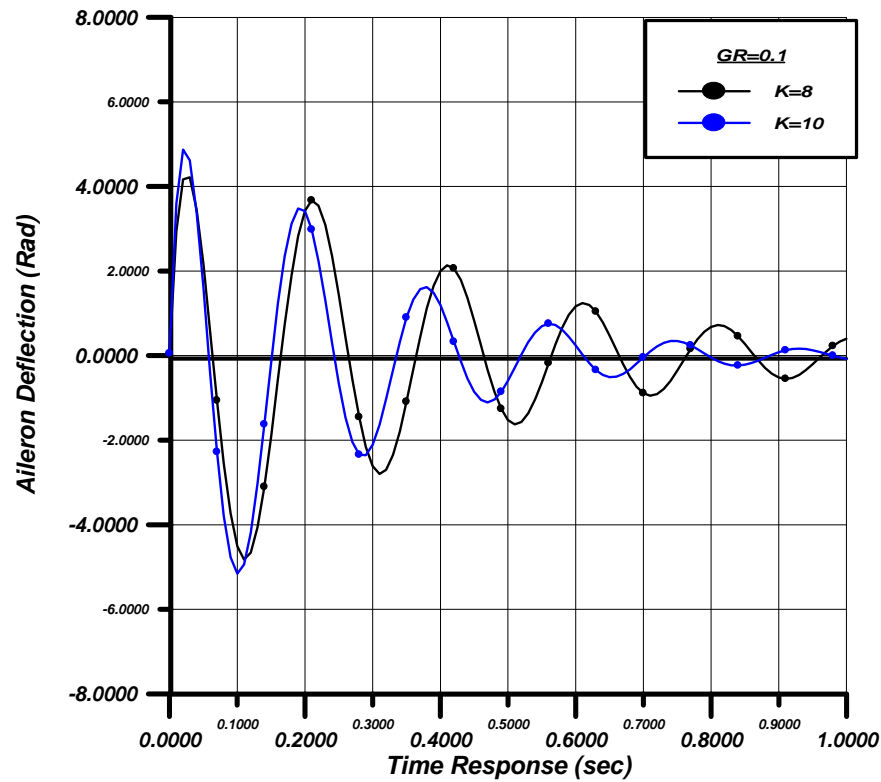


Figure-17. Aileron deflection verses time response for GR = 0.1 and different gain value.

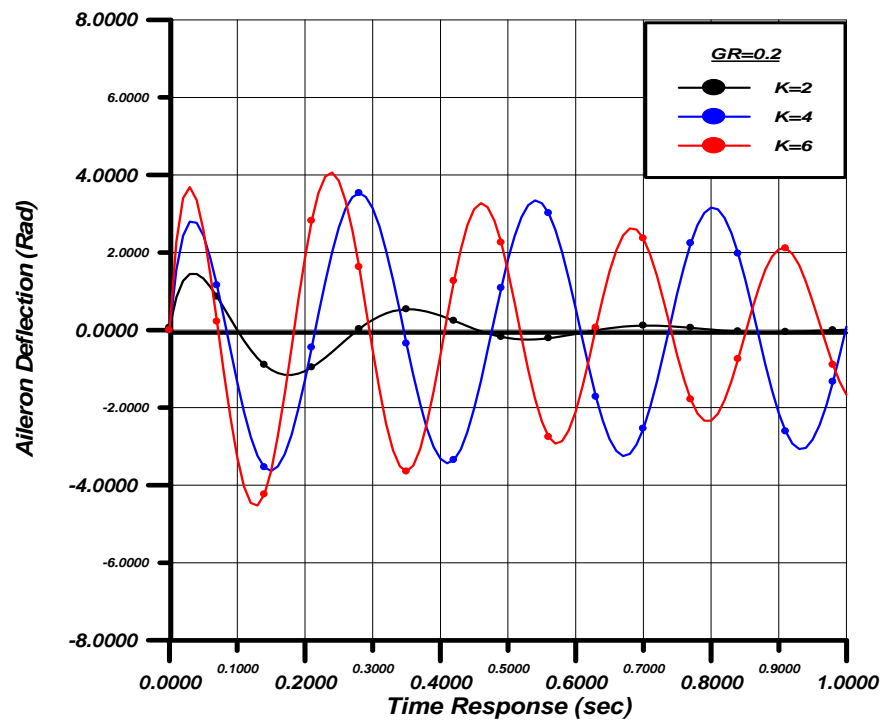
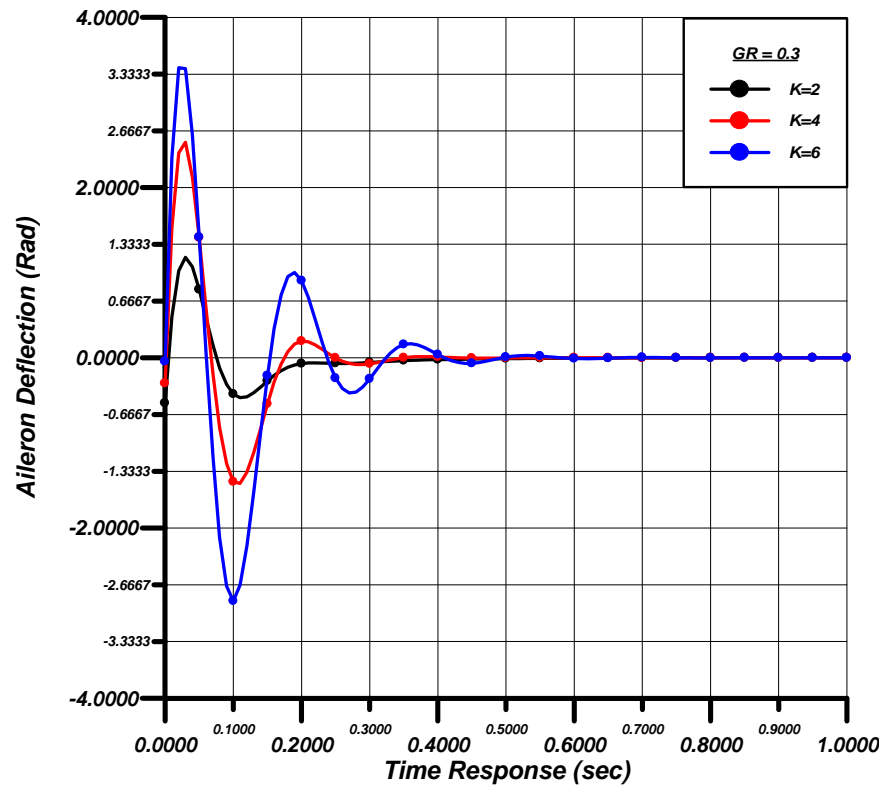
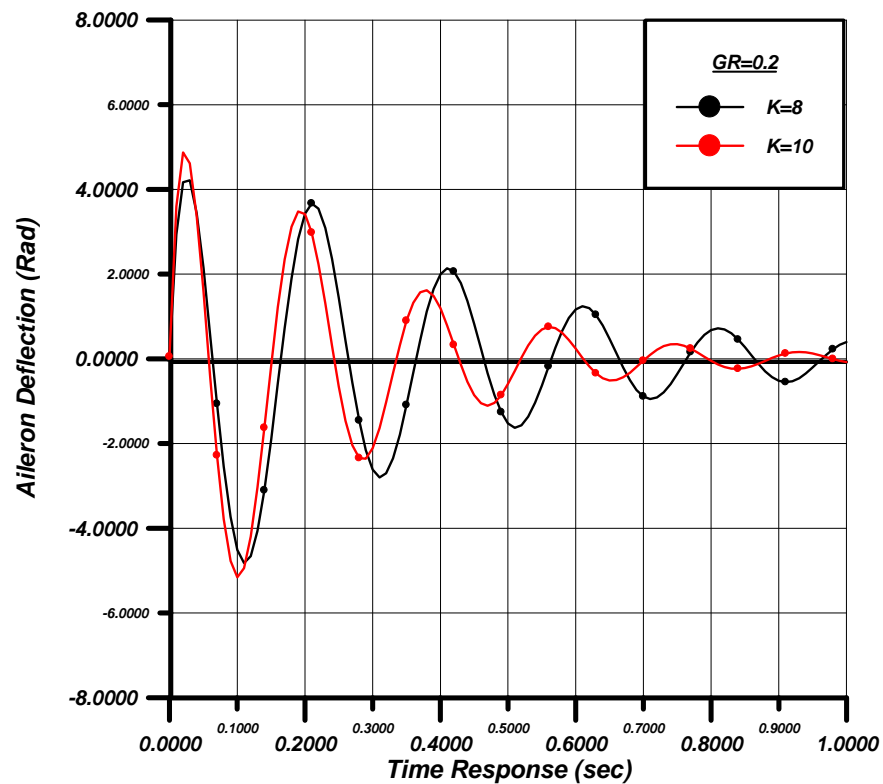


Figure-18. Aileron deflection verses time response for GR = 0.2 and different gain value.

Figure-19. Aileron deflection verses time response for $GR = 0.2$.Figure-20. Aileron deflection verses time response for $GR = 0.3$ and different gain value.

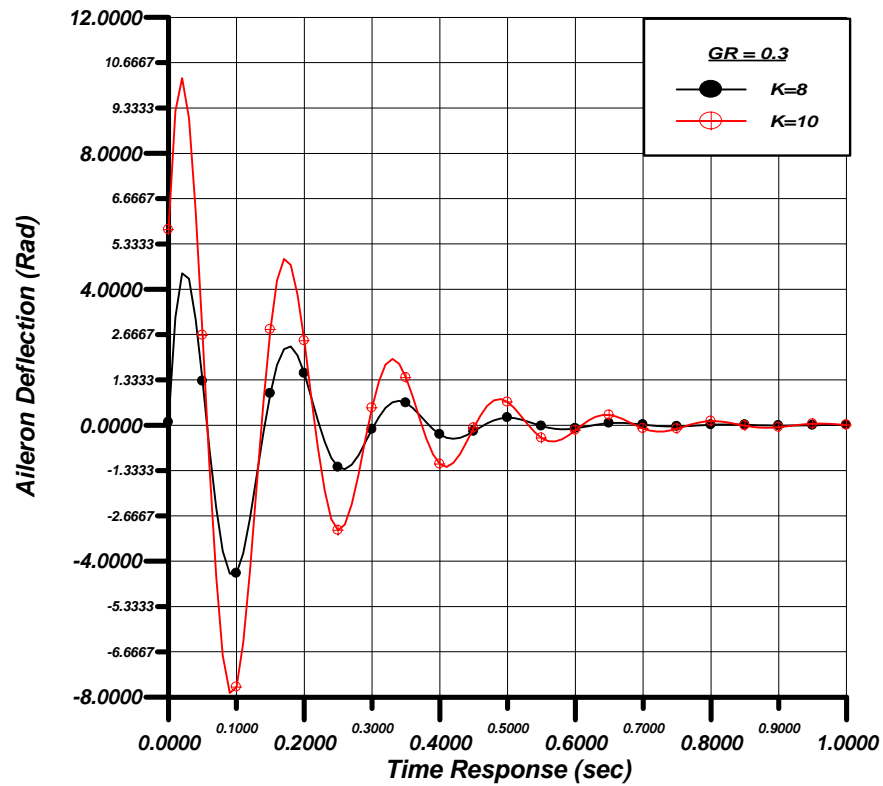


Figure-21. Aileron deflection verses time response for $GR = 0.3$ and different gain value.

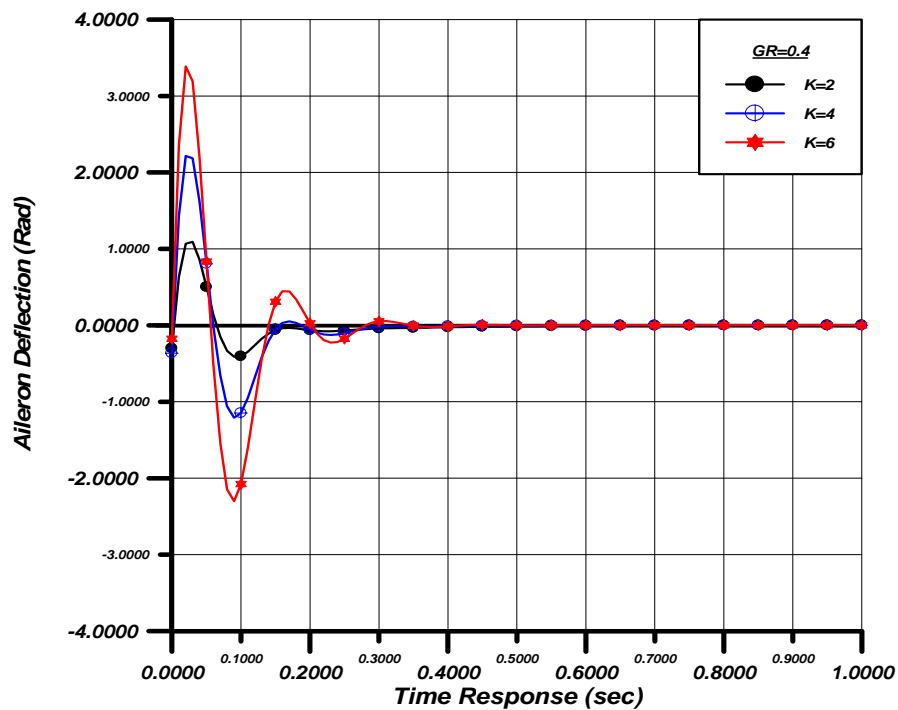


Figure-22. Aileron deflection verses time response for $GR = 0.4$ and different gain value.

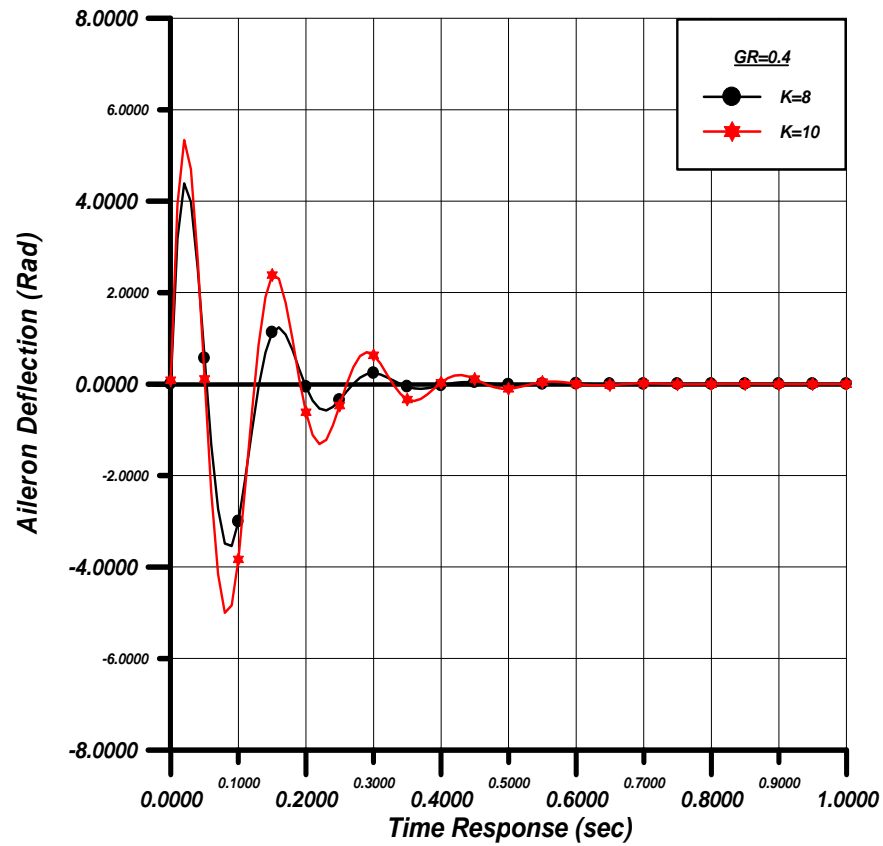


Figure-23. Aileron deflection verses time response for $GR = 0.4$ and different gain value.

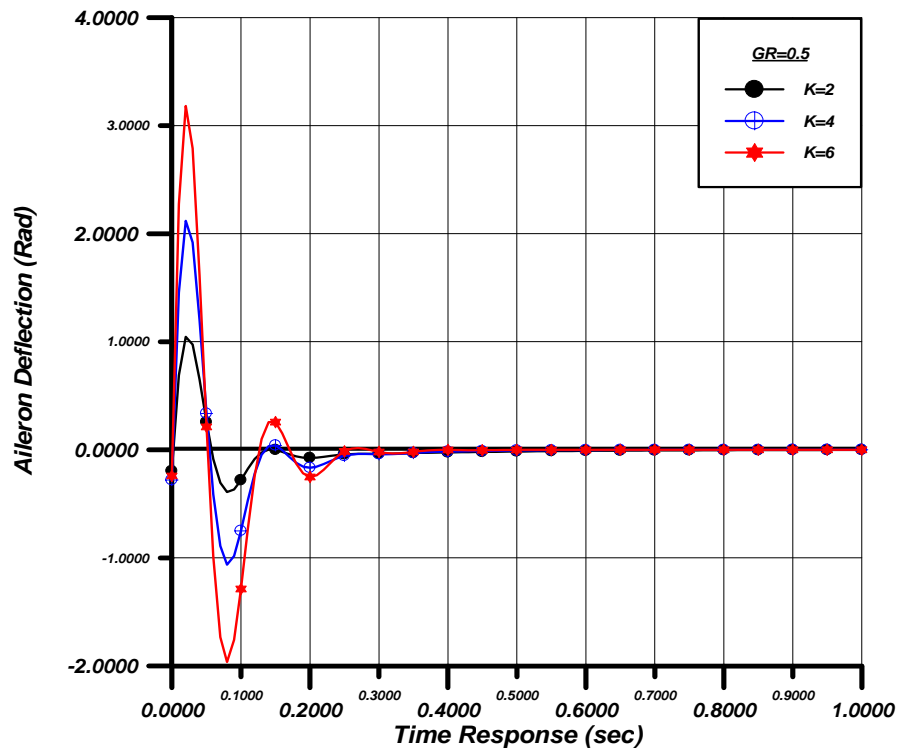


Figure-24. Aileron deflection verses time response for $GR = 0.5$ and different gain value.



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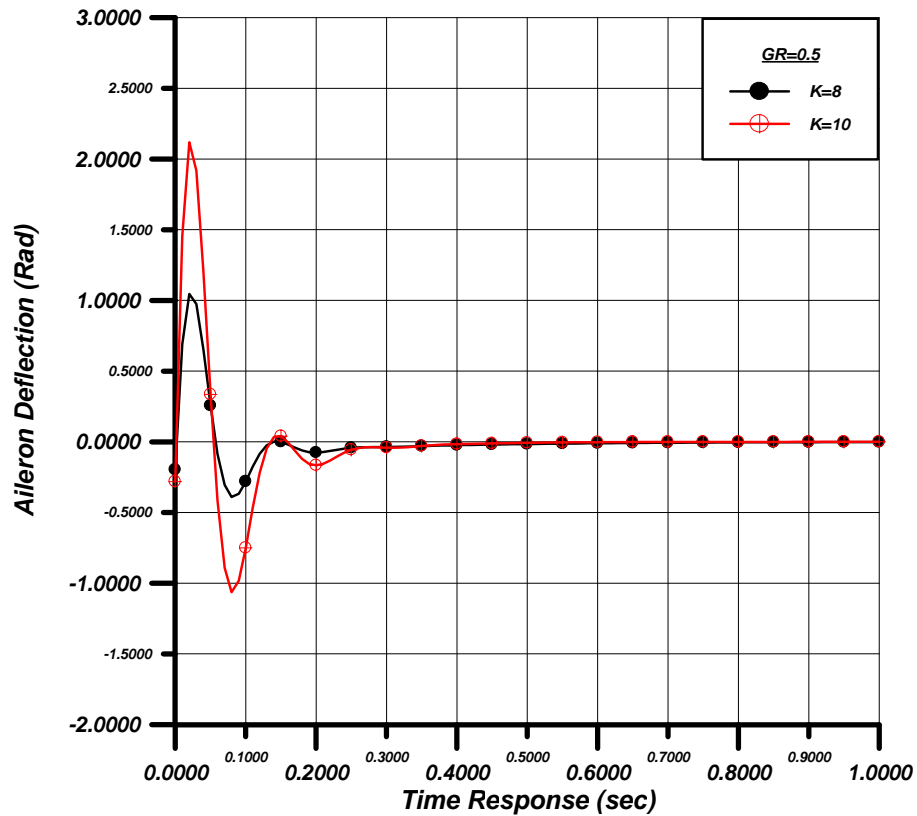


Figure-25. Aileron deflection versus time response for GR = 0.5 and different gain value.

Appendix A

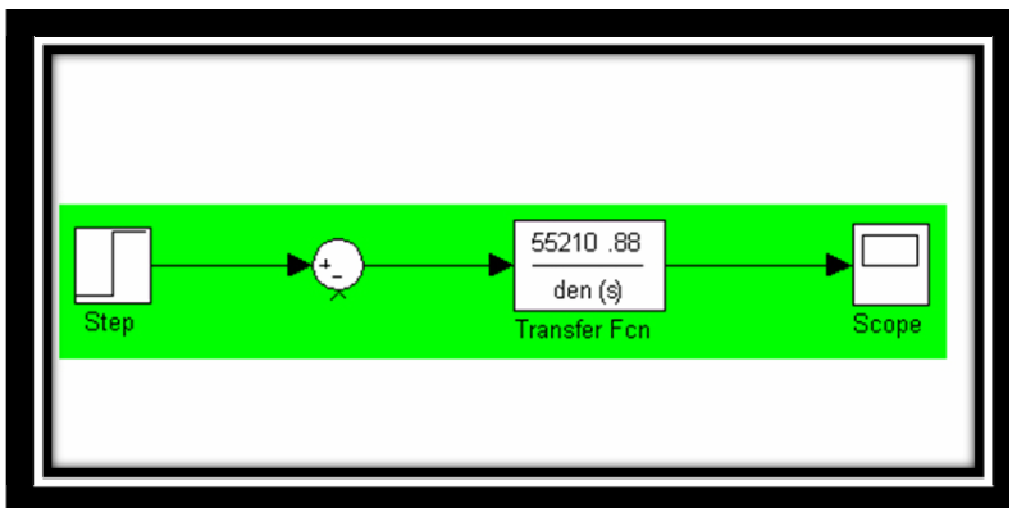


Figure-A1. Control system design simulation using Matlab.

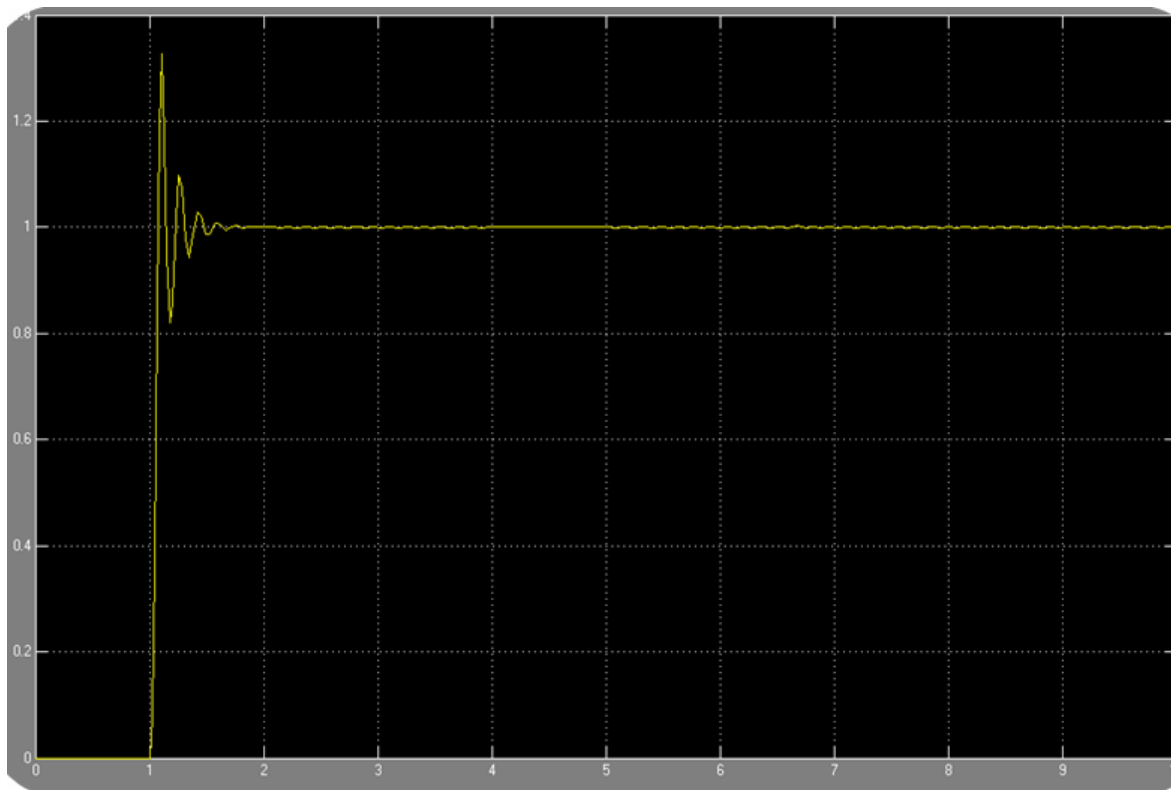


Figure-A2. Control system design result using Matlab, $G_r = 0.3$ $K = 8$.