VOL. 6, NO. 12, DECEMBER 2011 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

BENDING ANALYSIS OF COMPOSITE LAMINATED PLATES USING HIGHER-ORDER SHEAR DEFORMATION THEORY WITH ZIG-ZAG FUNCTION

T. Dharma Raju[1] and J. Suresh Kumar[2]
[1]Department of Mechanical Engineering, Godavari Institute of Engineering and Technology, Rajahmundry, India
[2]Department of Mechanical Engineering, JNTU College of Engineering, Hyderabad, India
E-Mail: raju_thummala@yahoo.com

ABSTRACT

In this paper an analytical procedure is developed to investigate the bending characteristics of laminated composite plates based on higher order shear displacement model with zig-zag function. This zig-zag function improves slope discontinuities at the interfaces of laminated composite plates. The equation of motion is obtained using the dynamic version of Hamilton's principle. The solutions are obtained using Navier's and numerical methods for anti-symmetric cross-ply and angle-ply laminates with a specific type of simply supported boundary conditions SS-1 and SS-2, respectively. In this paper the numerical results are presented for bending of anti-symmetric cross-ply and angle-ply laminated plates. All the solutions presented are close agreement with the theory of elasticity and closed form solutions available in the literature.

Keywords: laminated composite plates, bending analysis, higher order theory, zig-zag function, Hamilton's principle, Navier's method.

1. INTRODUCTION

The plates are straight and plane surface structures whose thickness is slight compared to other dimensions geometrically. Statically plates have simply supported and fixed boundary conditions, including elastic supports and elastic restraints or in some cases point supports. The static or dynamic loads are carried by plates are predominantly perpendicular to the plate surface. The accurate prediction of the response characteristics of laminated structures is a challenging task because of their intrinsic anisotropy, heterogeneity and low ratio of the transverse shear modulus to the in-plane Young's modulus. Hence, it is necessary to analyze the bending characteristics of laminated composite plates. Several plate theories have been developed by Reissner E; Stavsky Y [1] improved the CLPT by including the influence of bending-extensional coupling in un-symmetrical laminates. Pagano N. J. [2] studied the mechanical response of composite laminates by considering the problem of cylindrical bending of bi-directional laminates. Exact solutions within the frame work of the linear theory of elasticity are developed and compared with the respective solutions of CLPT. Pagano N. J. and Hatfield S. J. [3] assumed a uniform shear strain through the thickness of the plate and neglected local effects for the finite element formulation based on the non confirming rectangular plate bending elements. Kant T. *et al*., [4] presented a method for the numerical analysis of elastic plates with two opposite simply supported ends. Reddy J.N. [5] developed a higher order shear deformation theory of laminated composite plates. The theory contains the same dependent unknowns as in the first order shear deformation theory of Whitney and Pagano (1970), but accounts for parabolic distribution of the transverse shear strains through the thickness of the plate. Reddy J. N. [6] established a review of all third order, two dimensional technical thesis of plates that satisfy vanishing of transverse shear stresses on bounding planes of the plate is presented and their equivalence. Cho M. and Kim J. S. [7] developed a higher order zig-zag theory for laminated composite plates with multiple delaminations. M. Karama, K. S. Afaq, S. Mistou [8] presents a new multi-layer laminated composite structure model to predict the mechanical behaviour of multi-layered laminated composite structures. As a case study, the mechanical behaviour of laminated composite beam is examined from both a static and dynamic point of view. Sanjib Goswami [9] presented a simple C^0 finite element formulation with embedded higher-order shear deformation theory and a 3-dimensional state of stress and strain for thick and thin laminated composite plates. Peyman Khosravi, Rajamohan Ganesan and Ramin Sedaghati [10] developed an efficient facet shell element for the geometrically nonlinear analysis of laminated composite structures using the corotational approach. M. E. Faresand M. and Kh. Elmarghany [11] presented a refined nonlinear zigzag shear deformation theory of composite laminated plates using a modified mixed variational formulation. Diego Amadeu F. Torres *et al*., [12] implemented a formulation for the bending analysis of composite laminated plates with piezoelectric layers using the generalized finite element method. In this paper, a Higher -order shear deformation theory with Zig Zag function is proposed and develop the analytical procedure to analyze the bending characteristics of laminated composite plates which takes care of the sudden change of properties from lamina to lamina.

2. THE HIGHER-ORDER SHEAR DEFORMATION THEORY WITH ZIG-ZAG FUNCTION

A rectangular plate of $0 \le x \le a$; $0 \le y \le b$ and $-\frac{h}{2} \le z \le \frac{h}{2}$ is considered. The elasticity solution

VOL. 6, NO. 12, DECEMBER 2011 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

indicates that the transverse shear stress vary parabolically through the plate thickness. This requires the use of displacement field in which the in-plane displacements are expanded as cubic functions of the thickness coordinate in addition the transverse normal strain may vary in non linearly through the plate thickness. The higher-order displacement field with zig-zag function which satisfy the above criteria is assumed in the following form: ψ

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\psi_x(x,y) + z^2u_o^*(x,y) + z^3\psi_x^*(x,y) + \psi_k s_1(x,y) \\ v(x,y,z) &= v_o(x,y) + z\psi_y(x,y) + z^2v_o^*(x,y) + z^3\psi_y^*(x,y) + \psi_k s_2(x,y) \\ w(x,y,z) &= w_o(x,y) + z\psi_z(x,y) + z^2w_o^*(x,y) + z^3\psi_z^*(x,y) \end{aligned}\right\} \quad (1)$$

Where

u_0, v_0 , w_o, s_1 and s_2 denote the displacements of a point (x, y) on the mid-plane

ψ_x, ψ_y are rotations of the normal to the midplane about y-axis and x–axis

u_0^*, v_0^*, ψ_z^*, ψ_y^* are the higher–order deformation terms defined at the mid-plane

ψ_k is the Zig-Zag function, defined as:

$$\psi_k = 2(-1)^k \frac{Z_k}{h_k}$$

Z_k is the local transverse coordinate with its origin at the center of the k[th] layer

h_k is the corresponding layer thickness.

The strain components are:

$$\begin{aligned} \varepsilon_x &= \varepsilon_{xo} + zk_{sx} + z^2\varepsilon_{xo}^* + z^3k_x^* \\ \varepsilon_y &= \varepsilon_{yo} + zk_{sy} + z^2\varepsilon_{yo}^* + z^3k_y^* \\ \varepsilon_z &= \varepsilon_{z0} + zk_z + z^2\varepsilon_{z0}^* \\ \gamma_{xy} &= \varepsilon_{xyo} + zk_{sxy} + z^2\varepsilon_{xyo}^* + z^3k_{xy}^* \\ \gamma_{yz} &= \phi_{sy} + zK_{yz} + z^2\phi_y^* + z^3k_{yz}^* \\ \gamma_{xz} &= \phi_{sx} + zK_{xz} + z^2\phi_x^* + z^3k_{xz}^* \end{aligned} \quad (2)$$

The stress-strain relationships in the global x-y-z coordinate system can be written as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^L = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix}^L \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^L \quad (3)$$

The governing equations of displacement model will be derived using the principle of virtual work as

$$\int_0^T (\delta U + \delta V - \delta K)\, dt = 0 \quad (4)$$

The virtual work statement shown in Eq. (4), integrating through the thickness of laminate, the in-plane and transverse force and moment resultant relations in the form of matrix obtained as:

$$\begin{Bmatrix} N \\ N^* \\ -- \\ M \\ M^* \\ -- \\ Q \\ Q^* \end{Bmatrix} = \begin{bmatrix} A & | & B & | & 0 \\ \overline{B^t} & | & \overline{D_b} & | & \overline{0} \\ \overline{0} & | & 0 & | & \overline{D_s} \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \varepsilon_0^* \\ -- \\ K_s \\ K^* \\ -- \\ \phi \\ \phi^* \end{Bmatrix} \quad (5)$$

Equating the coefficients of each of virtual displacements δu_o, δv_0, δw_0, $\delta\psi_x$, $\delta\psi_y$, δu_0^*, δv_0^*, $\delta\psi_x^*$, $\delta\psi_y^*$, δs_1, δs_2 to zero, the equations of motion will be obtained and are expressed in terms of displacements u_o, v_0, w_0, ψ_x, ψ_y, u_0^*, v_0^*, ψ_x^*, ψ_y^*, s_1, s_2 by substituting for the force and moment resultants.

The Navier's solutions of simply supported anti symmetric cross ply laminated plates:

The SS-1 boundary conditions for the anti symmetric cross ply laminated plates are:

At edges x = 0 and x = a

$v_0 = 0$, $w_o = 0$, $\psi_y = 0$, $\psi_z = 0$, $M_x = 0$, $v_0^* = 0$, $w_0^* = 0$,

$\psi_y^* = 0$, $\psi_z^* = 0$, $M_x^* = 0$, $N_x = 0$, $N_x^* = 0$, $s_2 = 0$ 6 (a)

At edges y = 0 and y = b

$u_0 = 0$, $w_o = 0$, $\psi_x = 0$, $\psi_z = 0$, $M_y = 0$, $u_0^* = 0$, $w_0^* = 0$,

$\psi_x^* = 0$, $\psi_z^* = 0$, $M_y^* = 0$, $N_y = 0$, $N_y^* = 0$, $s_1 = 0$ 6(b)

The SS-2 boundary conditions for the anti symmetric angle ply laminated plates are:

VOL. 6, NO. 12, DECEMBER 2011 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

At edges x = 0 and x = a

$u_0 = 0$, $w_o = 0$, $\psi_y = 0$, $\psi_z = 0$, Nxy = 0, $M_x = 0$,

$w_0^* = 0$, $u_0^* = 0$, $\psi_y^* = 0$, $\psi_z^* = 0$, $M_x^* = 0$, $N_{xy}^* = 0$, s_1=0 7(a)

At edges y = 0 and y = b

$v_0 = 0$, $w_o = 0$, $\psi_x = 0$, $\psi_z = 0$, $N_{xy} = 0$, $M_y = 0$,

$v_0^* = 0$, $w_0^* = 0$, $\psi_x^* = 0$, $\psi_z^* = 0$, $M_y^* = 0$, $N_{xy}^* = 0$, s_2=0 7(b)

The displacements at the mid plane will be defined to satisfy the boundary conditions in Eq. (6) and (7). These displacements will be substituted in governing equations to obtain the equations in terms of A, B, D parameters. The obtained equations will be solved to find the behavior of the laminated composite plates.

3. RESULTS AND DISCUSSIONS

The simply supported boundary conditions (SS-1) shown in Eq. (6) are considered for solutions of anti-symmetric cross-ply laminates ,where as Eq. (7) for solutions of anti-symmetric angle-ply laminates using a higher order shear deformation theory with zig- zag function.

The material properties of graphite epoxy used for each lamina of the laminated composite plate are:

$$\frac{E_1}{E_2} = 40,\ E_2 = E_3 = 1X10^6\ N/cm^2,\ G_{13} = G_{12},\ G_{23} = 0.5\ X\ 10^6 N/cm^2,$$

$$G_{12} = 0.6\ X10^6 N/cm^2, \mu_{12} = \mu_{23} = \mu_{13} = 0.25$$

The deflection, internal stress-resultants and stresses are presented here in non-dimensional form using the following multipliers:

$$m_1 = \frac{10E_2h^3}{qa^4}, m_2 = \frac{10}{qa^2}, m_3 = \frac{10}{qa},\ m_4 = \frac{h^2}{qa^2}, m_5 = \frac{h}{qa}$$

Figure-1 shows the variation of non-dimensionalized maximum normal stresses as a function of no of layers and different theories for anti-symmetric cross-ply laminated plates under sinusoidal transverse load. The 2-layered plate experiences larger stresses than those of 4, 6 and 8 layered plates and the stress concentration is reduced in the latter. Thus the effect of bending-stretching coupling present in 2-layered plate on stresses is to increase the magnitude of stresses. Figure-2 shows the maximum transverse shear stresses for simply supported angle-ply laminates as a function of thickness, no of layers. From figure it is observed that the slope discontinuities in HSDT are taken care by the of zig-zag function in present theory. Figure-3 shows the variation of non-dimensionalized in plane and transverse stresses as a function of thickness coordinate (z/h). From the figure it is observed that the present models are in close agreement with 3D elasticity solutions.

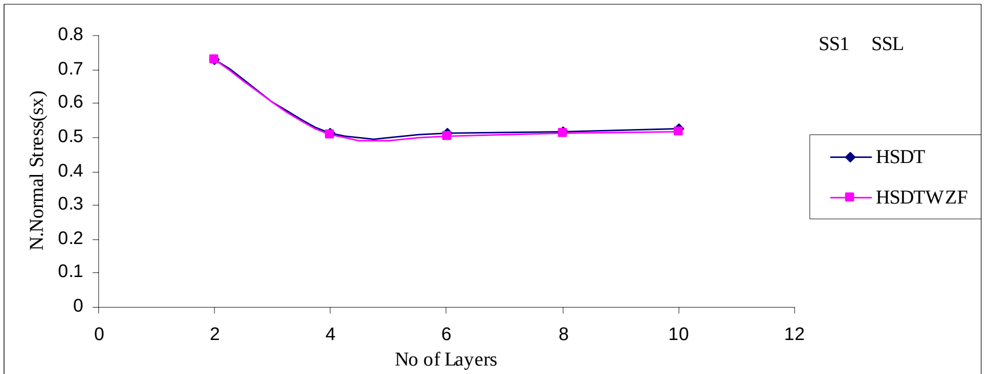

Figure 1. Non-dimensionalized max. transverse normal stress (sx) Vsno of layers for simply supported anti-symmetric cross-ply laminated square plate

VOL. 6, NO. 12, DECEMBER 2011 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

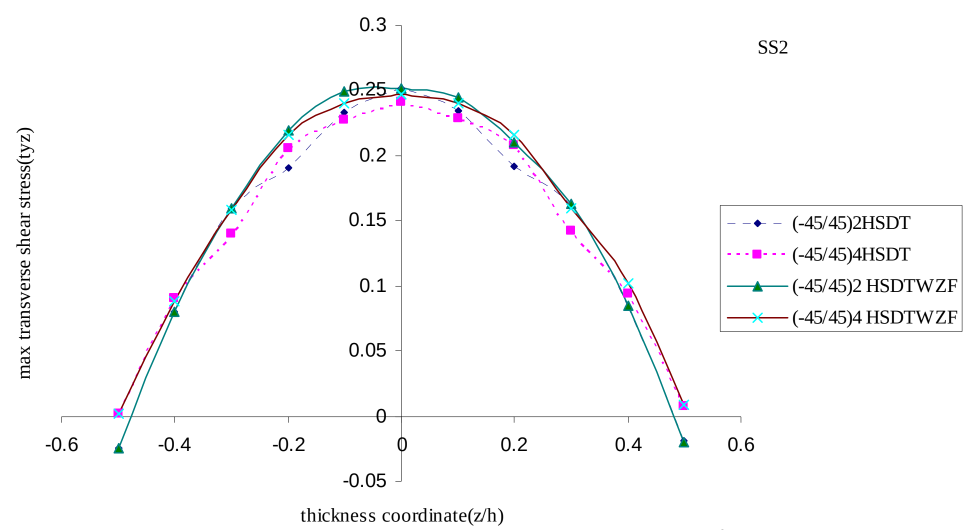

Figure 2. Non-dimensionalized max. transverse shear stress (tyz) Vs thickness coordinate for simply supported anti-symmetric angle-ply laminated square plate

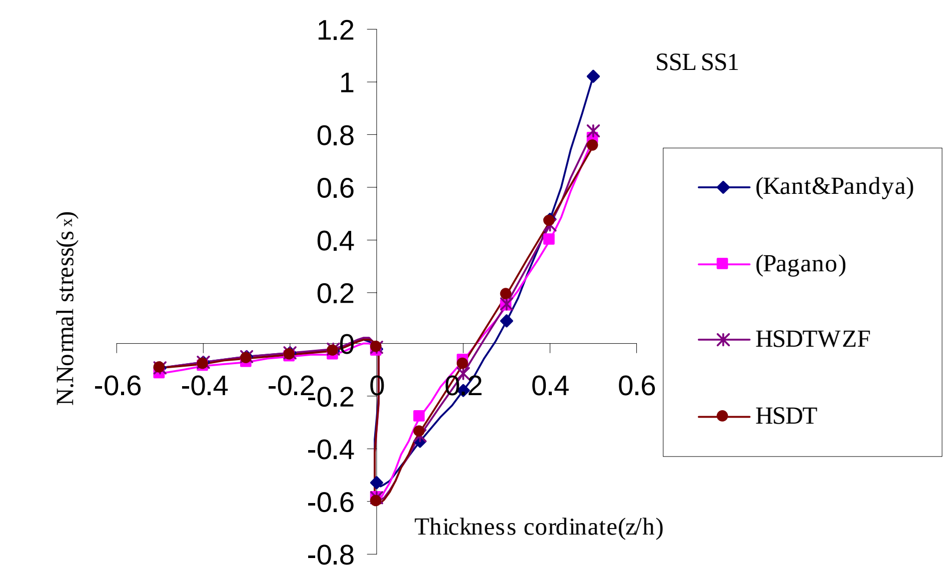

Figure 3. Variation of the normal stress(s_x)through the laminated thickness for a simply supported cross-ply square plate under sinusoidal transverse load

4. CONCLUSIONS

The variation with smooth slope-continuities at the interfaces is economically achieved by employing continuous linear and non-linear functions. The zig-zag function is a valuable tool to enhance the performance of both classical and advanced theories. The effect of bending-stretching-coupling is significant for all modulus ratios except for those close to unity on anti-symmetric angle-ply laminated composite plates of same thickness for any number of layers. The effect of bending stretching coupling present in two layered plates on stresses and deflections is to increase the magnitude than those of 4, 6, and 10 layered plates.

REFERENCES

[1] Reissner. E. 1947. On bending of elastic plates. Quarterly of applied Mathematics. 5: 55-68.

[2] Pagano N.J. 1969. Exact solutions for composite laminates in cylindrical bending. J. of composite materials. 3(3): 398-411.

VOL. 6, NO. 12, DECEMBER 2011 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

[3] Pagano. J.N. and J.H. Sharon. 1972. Elastic Behavior of Multilayered Bidirectional Composites. J. Structural Composite Materials. 10: 931-933.

[4] Kant. T. 1981. Numerical Analysis of Elastic Plates with Two opposite simply supported ends by segmentation method. J. of Computers and Structures. 14(3-4): 195-203.

[5] Reddy J.N. 1984. A simple Higher-Order Theory for Laminated Composite Plates. J. of Applied Mechanics. 51: 745-752.

[6] Reddy J.N. 1990. A General Non-linear Third-order theory of plates with Moderate Thickness. Int. J. of Non-linear Mechanics. 25(6): 677-686.

[7] Cho. M and J.S. Kim. 2001. Higher-Order Zig-Zag Theory for Laminated Composites with Multiple Delaminations. J. of Applied Mechanics. 68: 869-877.

[8] M. Karama *et al*., 2003. Mechanical behavior of laminated composite beam by the new multi-layered laminated composite structures model with transverse shear stress continuity. J. of Solids and Structures. 40:1525-1546.

[9] Sanjib Goswami. 2006. A C^0 plate bending element with refined shear deformation theory for composite structures. J. of Composite Structures. 72(3): 375-382.

[10] Peyman Khosravi *et al*., 2008. An efficient facet shell element for corotational nonlinear analysis of thin and moderately thick laminated composite structures. J of Computers and Structures. 86(9): 850-858.

[11] M.E. Faresand and M.Kh. Elmarghany. 2008. A refined zigzag nonlinear first-order shear deformation theory of composite laminated plates. J. of Composite Structures. pp. 71-83.

[12] Diego Amadeu F. Torres et al. 2011. Evaluation and verification of an HSDT-Layerwise generalized finite element formulation for adaptive piezoelectric laminated plates. J of Computer Methods in Applied Mechanics and Engineering. 200(5-8): 675-691.