AN APPROXIMATE THEORY OF STATIC FILTRATION OF DRILLING MUDS IN VERTICAL WELLS

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ABSTRACT

The adverse effects of filtrates during oilwell drilling may be reduced if the mechanism of filtration through mud cakes is well understood. Considerable efforts have therefore been made through empirical and theoretical studies to formulate a robust filtration theory for predicting the effects of mud parameters during drilling. Most of the existing models are either too simplistic or rather complex. This study developed a robust approximate analytical filtration theory that incorporated steady state and transient effects. The results show that in addition to viscosity, transient effects during filtration greatly affect cake build-up and filtrate volumes and that reduced filtrate volumes can be achieved when steady state is achieved rather than unsteady state filtration process.

Keywords: wells, drilling fluids, filtration theory, mud cakes, filtrate volume, approximate solution.

Nomenclature

A	Area of flow, ft^2
Ac	Cake compressibility, Psi^{-1}
ao	Local specific filtration resistance, ft/Lb
a1	Local specific filtration resistance at 1 Psi., ft/lb
X	Exponent of non-linearity (eqn 24)
B	Solid fraction in filter cake
C	Exponent defined by equation 10
β	Exponent defined by equation
B	Specific cake volume ft^3/lb
H	Cake thickness, ft
H	Formation thickness, ft
K	Permeability, ft^2
M	Exponent defined by equation 10
Ps	Solid Pressure, Psi
P	Fluid pressure, Psi
P_T	Total pressure, psi
Q	Flow rate, ft^3/sec
Q	Cumulative Volume, bbls
Rw	Well bore radius, ft
RI	Radius of invasion, ft.
X	Distance from well, ft.
T, t	Time, secs
μ	Time viscosity of mud filtrate, cp
ε	Stress function
ϕ_o	Initial cake porosity
N	Displacement efficiency
X	Exponent defined by equation 52
T	Shear rate
λ	Coefficient of cake thickness

INTRODUCTION

Filtration from drilling fluids arises from the pressure differential between the hydrostatic pressure of the mud column and the formation pressure. Since the hydrostatic pressure in the borehole is always greater than the formation pressure, water filters into the porous medium, depositing a porous, permeable and compressible cake of mud particles on the wall of the borehole.

Three classes of filtration: static, dynamic and beneath the bit filtration have been recognized and understanding the mechanism of each type of filtration and its practical implications could lead to significant reduction in mud filtration without compromising the beneficial functions of drilling fluids. While drilling muds help to hold formation pressure in place, the filtrates could cause shale swelling and sloughing in wellbores and formation of viscous emulsions with formation fluids leading to damage of reservoir permeability and reduction of oil well productivity. Filtrates lead to distortion of results obtained from drill stem tests, coring and electric log resistivity measurements.

REVIEW OF LITERATURE

The earliest research efforts on mud filtration were directed at formulating additives to provide properties compliant with API standards. In 1940, Williams carried out experimental study on filtration in typical Gulf coast muds and concluded that mud control through the use of appropriate additives minimizes filtration. Schremp and Johnson (1952) investigated filtration and mud cake formation at high temperatures and pressures. They established that bottom hole filter loss behaviours cannot be predicted from measurements at surface temperatures. Prokop (1952) investigated filtration under both static and dynamic conditions and established that cake accumulation depends upon whether or not the mud is circulated. He also observed that muds having high filtration rates deposited thick filter cakes in spite of very high eroding velocities; and that mud cakes deposited under either static or dynamic filtration possess different properties in terms of permeability, thickness and erodability. Ferguson and Klotz (1953) used a model oil well to simulate filtration under different bottom hole geometry and hydrodynamic conditions. They established

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three classes of filtration: static filtration, dynamic filtration, and filtration from beneath the bit during drilling.

Outman (1963) presented an analytic approach to the filtration theory. He described the mechanism of filtration by a theoretical-empirical non-linear equation which was linearized and solved explicitly under certain conditions. The study demonstrated the effects of mud properties like viscosity on filtration rate with a major conclusion that several quantities that affect dynamic filtration have no counterpart in static filtration and therefore static filtration cannot be relied on as a measure of dynamic filtration and vice versa.

Earshghi and Azari (1980) applied numerical simulation techniques to minimize the laboratory data requirement in filtration mathematical models. The study confirmed the effect of filtrate viscosity and cake permeability on filtrate volumes. Hassan (1980) noted that two distinct stages can be recognized during the dynamic filtration process: equilibrium and non-equilibrium stages. The differences in characteristics of static and dynamic filtration were highlighted by researchers such as Peden *et al*., (1984), Vaussard *et al*., (1986) and Fisk *et al*., (1991) with a conclusion that filtration loses could be much higher under dynamic than static conditions.

In recent times, there have been renewed efforts to obtain improved robust models for predicting filtration losses. Tien and Bai (2003) observed that the conventional filtration theory under-predicts parameters and suggested improvements based on better estimation of the average specific cake resistance and the wet cake to dry cake mass ratios. Xu *et al*., (2008) proposed the equivalent cake filtration modeling to describe filtration in Newtonian and non Newtonian fluids. Yim and Du Kwon (2010) suggested improvement in average cake resistance values using the concept of filtration permeation.

Most of the recent models however have not received widespread field applications due to their complexities. This study developed an approximate theory that incorporated steady state and unsteady state effects to describe mud filtration. The model proved accurate and simpler to use than many of the existing formulations.

THEORY AND MATHEMATICAL FORMULATION

The assumptions implicit in the derivations are: fluid flow through the compressible cake is governed by Darcy's law, the total pressure which is the sum of the fluid and solid pressure and is assumed constant for the system, while isothermal condition exists in the borehole. From stress theory, we observe that:

Total system Pressure = Solid pressure + Fluid pressure

Or,

$$P_t = P_s + P \tag{1}$$

The solid pressure, P_s, is responsible for all measurable effects of stress changes while the total pressure, P_t is constant. Therefore,

$$\frac{dps}{dx} = -\frac{dp}{dx}$$

Or,

$$\frac{dp}{dx} = -\frac{dps}{dx} \tag{2}$$

Cake compressibility under an external pressure is defined as:

$$ac = -\frac{1}{dv}\frac{d(dv)}{dPs}$$

Or, in terms of changes with time,

$$ac.dv.\frac{dPs}{dt} = -\frac{d}{dt}(dv) \tag{3}$$

Comparing equations (2) and (3) implies:

$$\frac{d}{dx}\left(\frac{k}{\mu}\frac{dPs}{dx}\right)dv = ac.dv.\frac{dPs}{dt} \tag{4}$$

Or,

$$\frac{1}{\mu}\frac{d}{dx}\left(k\frac{dPs}{dx}\right) = ac.\frac{dPs}{dt} \tag{5}$$

From Grace (1953) and Tiller (1975), we can establish equations (6) to (11).

$$\phi = \phi_o P_S^{-C} \tag{6}$$

$$k = \frac{1}{ax\rho_s(1-\phi)} \tag{7}$$

$$ax = a_o P_S^n \tag{8}$$

$$1-\phi = BP_S^m \tag{9}$$

$$\frac{d(1/1-\phi)}{dPs} = a_g \tag{10}$$

$$a_g = \frac{ac}{1-\phi} \tag{11}$$

From equations (6) to (10) we can establish equation (12).

$$k = \frac{1}{a_o\rho_s BP_s^{m+n}} \tag{12}$$

While,

$$ac = c\phi_o B^{-1}P_s^{-(m+c+1)} \tag{13}$$

We define a stress function:

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$$\varepsilon(x,t) = -\int_{o}^{P_s} a.dPs \tag{14}$$

Then,

$$P_s = \left(\frac{\varepsilon\ B(c+m)}{c\phi_o} \right)^{-\frac{1}{c+m}} \tag{15}$$

Substituting equation (15) into equation (5) and simplifying, we have

$$\frac{D}{\mu}\frac{d}{dx}\left(\varepsilon^{\frac{n-c-1}{c+m}} \frac{d\varepsilon}{dx} \right) = \frac{d\varepsilon}{dt} \tag{16}$$

or

$$\frac{d}{dx}\left(\varepsilon^{\alpha} \frac{d\varepsilon}{dx} \right) = \frac{\mu}{D}\frac{d\varepsilon}{dt} \tag{17}$$

Where,

$$\alpha = \frac{c+1-n}{-(c+m)}$$

And

$$D = \left(a_o \rho_s c \phi_o\right)^{-1} \left(\frac{B(c+m)}{c\phi_o} \right)^{\alpha}$$

Equation (17) is the diffusivity equation which defines filtration across a porous mud cake and is to be solved for various conditions of the filtration process.

MUD CAKE BUILD UP

The cake build up during filtration is proportional to the flow rate across the cake. Therefore,

$$\frac{dh}{dt} = bq \tag{18}$$

Where,

q = throughput of filtrate

b = constant, the specific cake volume.

Applying Darcy's law and simplifying, we have

$$\frac{dh}{dt} = -b\frac{k}{\mu}\frac{dPs}{dx} \tag{19}$$

Substituting equations (14) to (16) into equation (19), we obtain

$$\frac{dh}{dt} = \frac{bk}{\mu}\left[\left(B\frac{(c+m)}{c\phi_o} \right)^{-\frac{1}{c+m}} \cdot \frac{1}{c+m}\varepsilon^{-\frac{(c+m+1)}{c+m}} \frac{d\varepsilon}{dx} \right] \tag{20}$$

Equation (20) is non-linear and relates the cake stress function to the rate of build-up.

VOLUME OF FILTRATION

If the filtrate throughput is q, then the cumulative volume after time t is given as:

$$Q = \int_{O}^{t} qdt \tag{21}$$

From Darcy's law and using equation (2), we obtain

$$Q = \int_{O}^{t} \frac{k}{\mu}\frac{dPs}{dx}.dt \tag{22}$$

Substituting for $\frac{dps}{dx}$ and simplifying, we have

$$Q = \frac{K}{\mu}\int_{o}^{t} \varepsilon^{\alpha} \frac{d\varepsilon(0,t)}{dx} dt \tag{23}$$

Equation (23) describes the filtration volume and can be evaluated after the stress function of the cake $\varepsilon(x,t)$ is thoroughly defined.

SOLUTION OF THE FILTRATION THEORY EQUATIONS

The mechanism of filtration through any compressible cake medium described by the non-linear diffusivity equation given in equation (17) will apply to static, dynamic and filtration through multi-layered cakes depending on the boundary conditions.

BOUNDARY CONDITIONS FOR STATIC FILTRATION

The boundary conditions are given as:

i. $\varepsilon(h,t) = 0 \quad \forall\ t \geq 0$ (24)

ii. $\varepsilon(0,t) = \varepsilon_o \quad \forall\ t \geq 0$ (25)

Where,

$$\varepsilon_o = \frac{c\phi_o Pt^{-(c+m)}}{B(c+m)} \tag{26}$$

iii. $\frac{dh}{dt} = \frac{bD}{\mu} \cdot \varepsilon^{\alpha} \cdot \frac{d\varepsilon}{dx}, \quad x \to h(t)$ (27)

SOLUTION DEVELOPMENT

We integrate equation (17) and incorporate equation (20) to obtain:

$$\varepsilon^{\alpha}\frac{d\varepsilon}{dx} = \frac{\mu}{bD}\frac{dh}{dt} \tag{28}$$

Thus we can express the general equation which defines the filtration process as:

$$\frac{\mu}{D}\frac{d}{dt}\int_{0}^{x} \varepsilon\, dx = \frac{\mu}{bD}\frac{dh}{dt} - \varepsilon_o^{\alpha}\frac{d\varepsilon}{dx} \quad (0,t) \tag{29}$$

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From experimental observations, we assume a solution of the form:

$$\varepsilon(x,t) = \varepsilon s(x) + \varepsilon t(x,t) \tag{30}$$

Where, $\varepsilon s(x)$ represents the steady state effect or stress distribution after a long filtration time, while $\varepsilon t(x,t)$ is the transient effect which dies down with time.
We assume $\varepsilon s(x)$ is of the form:

$$\varepsilon s(x) = A_1 + A_2 x \tag{31}$$

While $\varepsilon t(x,t)$ is of the form:

$$\varepsilon t(x,t) = [c_1 \cos\gamma(x-h) + c_2 \sin\gamma(x-h)]e^{(\gamma x+a)^2} \tag{32}$$

After applying the boundary conditions, and simplifying, we have

$$\varepsilon(x,t) = \varepsilon_o\left(1-\frac{x}{h}\right) + \sum_{n=1}^{\infty} c_n \sin\frac{n\pi}{h}(h-x)e^{-\left(\frac{n\pi x}{h}+a\right)^2} \tag{33}$$

In this study, the analysis is further simplified by assuming that c_n is of the form:

$$c_n = \frac{2}{\pi n^2}\varepsilon_o e^{2\pi a n\frac{x}{h} - 2\pi\frac{x}{h} + a^2} \tag{34}$$

Hence,

$$\varepsilon(x,t) = \varepsilon_o\left[1-\frac{x}{h}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin n\pi\frac{(h-x)}{h}e^{-n^2\frac{\pi^2x^2}{h^2}-\frac{\pi x}{h}}\right] \tag{35}$$

Equation (35) is the approximate equation which defines the complete stress distribution in a static filtration process.

Differentiating equation (54) and simplifying, we have

$$\frac{d\varepsilon(0,t)}{dx} = \frac{-\varepsilon_o}{h} + \frac{2\varepsilon_o}{\pi}\sum_{n=1}^{\infty}\frac{-\pi(-1)^n}{nh} \tag{36}$$

Or,

$$\frac{d\varepsilon(0,t)}{dx} = -\frac{\varepsilon_o}{h} + \frac{2\ln 2\varepsilon_o}{h}$$

While,

$$\int_{0\ x\to h}^{x} \varepsilon dx = \varepsilon_o\frac{h}{2} + \frac{2\varepsilon_o h}{\pi^2}$$

$$= \varepsilon_o h\left(\frac{1}{2}+\frac{2}{\pi^2}\right) \tag{37}$$

Substituting equations (36) and (37) into equation (29) and simplifying, we have

$$\frac{\mu}{D}\left(\varepsilon_o\left(\frac{1}{2}+\frac{2}{\pi^2}\right)-\frac{1}{b}\right)\frac{dh}{dt} = 0.3863\frac{\varepsilon_o^{\alpha+1}}{h} \tag{38}$$

Therefore,

$$h = \left[\frac{1.545\pi^2 bD\varepsilon_o^{\alpha+1}}{\mu((\pi^2+4)\varepsilon_o b - 2\pi^2)}\right]^{\frac{1}{2}} t^{\frac{1}{2}} \tag{39}$$

This is of the well-known form:

$$h = \sqrt{\lambda t} \tag{40}$$

Where,

$$\lambda = \frac{1.545\pi^2 bD\varepsilon_o^{\alpha+1}}{\mu[(\pi^2+4)\varepsilon_o b - 2\pi^2]}$$

SOLID PRESSURE VARIATION IN THE CAKE

By substituting equation (35) into equation (15) and simplifying, the solid pressure within the cake can be related to the stress function distribution as:

$$P_S = P_t\left[1-\frac{x}{h}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin n\pi\left(1-\frac{x}{h}\right)e^{-\frac{n^2\pi^2x^2}{h^2}-\frac{2\pi x}{h}}\right]^{\frac{1}{c+m}} \tag{41}$$

CUMULATIVE VOLUME OF FILTRATION

The cumulative volume of filtrate after a time interval t is given by:

$$Q = -\int_0^t q\,dt \tag{42}$$

Applying Darcy's law,

$$Q = -\int_0^t \frac{k}{\mu}\frac{dps}{dx}dt \tag{43}$$

$$= -\frac{D}{\mu}\varepsilon_o^{\alpha}\int_0^t k\frac{d\varepsilon(0)t}{dx}dt$$

$$= \frac{0.3863D\varepsilon_o^{\alpha+1}}{\mu}\int_0^t\frac{dt}{\sqrt{\lambda t}}$$

$$= 0.7726\frac{D\varepsilon_0^{\alpha+1}}{\mu}\sqrt{t/\lambda} \tag{44}$$

By substituting for λ from equation (44) and simplifying, we have

$$Q = 1.66\left[\frac{(\varepsilon_o b-2)}{\mu b}D\varepsilon_o^{\alpha+1}\right]^{\frac{1}{2}} t^{\frac{1}{2}} \tag{45}$$

Equation (45) describes the cumulative filtrate volume per unit surface area. For a zone of thickness h and

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well bore radius rw, the total cumulative volume of filtrate can be expressed as:

$$Q = 3.32\mu\ r`w\ h\left[\frac{(\varepsilon_o b - 2)D\varepsilon_o^{\alpha+1}}{\mu b}\right]^{\frac{1}{2}} t^{\frac{1}{2}} \qquad (46)$$

DEPTH OF INVASION

For a homogeneous, isotropic and highly permeable formation where the filtrate is assumed to penetrate radially and uniformly into the formation, material balance yields:

$$Q = \frac{1}{4}\pi\phi hn\left(R1^2 - Rw^2\right) \qquad (47)$$

Or in field units,

$$R1 = \left(\frac{1.787Q}{\phi hn} + Rw^2\right)^{\frac{1}{2}} \qquad (48)$$

ANALYSIS AND DISCUSSION OF RESULTS

Equations (39) and (40) show the relationship between cake thickness and filtration time. They can be substituted into equation (29) to obtain a time-dependent stress function:

$$\varepsilon(x,t) = \varepsilon_o\left[1 - \frac{x}{\sqrt{\lambda t}} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\sin n\pi\frac{(1-x)}{\sqrt{\lambda t}}e^{-\frac{n^2\pi^2x^2}{\lambda t} - \frac{2\pi x}{\sqrt{\lambda t}}}\right)\right] \qquad (49)$$

As noted earlier, equation (49) combines both steady state and transient effects just as the solid pressure distribution given in equation (41). It can be readily observed that the stress function and solid pressure are not so dependent on the cake thickness, but on the ratio $\frac{x}{h}$. This observation agrees well with the results of Outman for the linearized situations. The solid pressure is maximum at the wall of borehole and reduces to zero at the free surface of the cake. It can be readily observed that at long times, the ratio $\frac{x}{h}$ becomes very small and hence the transient component vanishes, and the steady state approximation is predominant. Equation (45) shows that mud filtration is controlled by parameters involving the mud, cake and time. The term $\frac{(\varepsilon_o b - 2)}{\mu b}$ in equation (45) aggregates the influence of the mud properties, while the term $\left[D\varepsilon_o^{\alpha+1}\right]$ shows the influence of properties of the cake. When the mud viscosity or the specific cake volume is high, the filtrate volume is decreased. Thus, the volume of filtrate can be controlled by controlling properties of the mud like viscosity and the type of cake formed. The equation also shows that the cumulative volume of filtrate is very sensitive to α, the factor of non-linearity in the filtration equation and confirms the observation made by many investigators that the assumption of a linearized filtration equation introduces errors into the cake thickness and filtrate volume computations.

Furthermore, the cumulative volume obtained as expressed in equation (45) is higher than the volume obtained for steady-state filtration assumed by Outman and other researchers by the factor 1.66. This shows that steady state filtration represents an optimistic representation of volume of filtrate and is to be desired during filtration across compressible mud cakes.

EXAMPLE

The solutions developed above for static filtration was applied to a practical situation reported by Outman. Data are presented in Table-1. The results were compared to those of Outman and experimental results.

Table-1: Mud and cake parameters (from Outman (1963)

Symbol	Parameter	Value
m	Exponent	0.08
c+m	Exponent	-0.23
B	Specific cake volume	0.23 ft^3/lb
μ	Mud viscosity	0.80 cp
ρs	Mud density	12 ppg
a_o	Specific cake resistance	1 x 106 ft/lb
ϕ_o	Initial cake porosity	0.30
SG	Mud specific gravity	2.50
D	Depth of investigation	561 ft

The results are presented in Figures 1 to 3 and agree with all the observations made above.

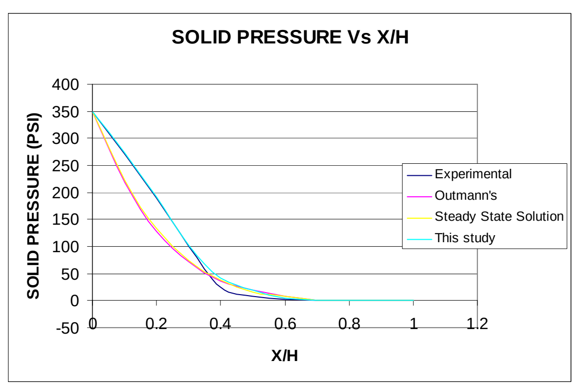

Figure-1. Solid pressure predictions for different models.

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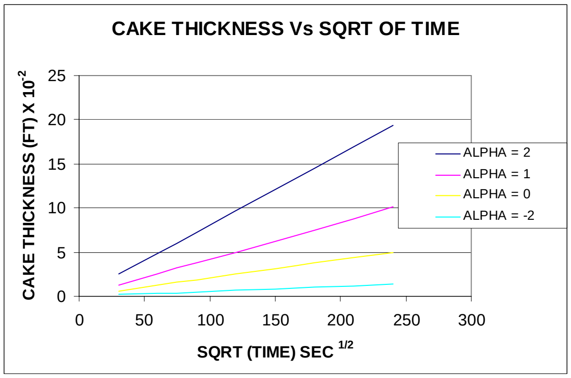

Figure-2. Cake thickness variation with square root of time.

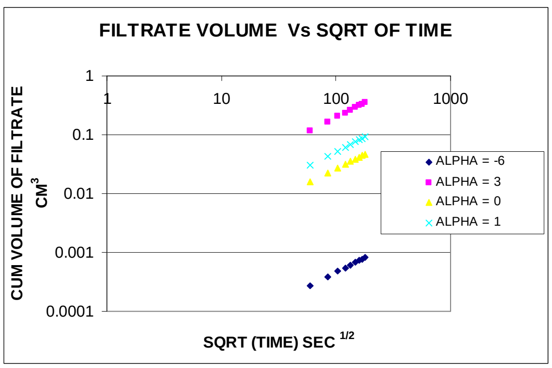

Figure-3. Filtrate volume variation with square root of time

CONCLUSIONS

An approximate analytical solution with a steady state component and transient effect has been applied to the filtration theory. The study shows that:

a) Mud properties, cake parameters and time are major factors that control drilling mud filtration.
b) Mud viscosity and specific cake volume can be controlled to achieve reduced volume of filtrate. The specific cake volume can be controlled by use of appropriate additives in a good filter-cake -forming gel.
c) A steady state solution to the filtration equation presents optimistic estimates of the volume of filtrate and radius of invasion.
d) Filtrate volume and radius of invasion are sensitive to the factor of non linearity in the filtration equation, and so the filtration equation has to be treated as strictly non linear.

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