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RADIATION EFFECTS ON MHD FLOW THROUGH POROUS MEDIA PAST AN IMPULSIVELY STARTED VERTICAL OSCILLATING PLATE WITH VARIABLE MASS DIFFUSION

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ABSTRACT

Radiation effects on MHD flow through porous media past an impulsively started vertical oscillating plate with variable mass diffusion is studied here. The fluid considered is gray, absorbing-emitting radiation but a non-scattering medium. The governing equations involved in the present analysis are solved by the Laplace-transform technique. The velocity, skin friction, Nusselt number and Sherwood number are studied for different parameters like radiation parameter, Schmidt number, Thermal Grashof number, mass Grashof number, phase angle and time.

Keywords: radiation, MHD, mass diffusion, oscillating plate, porous media.

INTRODUCTION

Study of MHD flow with heat and mass transfer plays an important role in chemical, mechanical and biological Sciences. Some important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aero dynamics. The response of laminar skin friction and heat transfer to fluctuations in the stream velocity was studied by Lighthill [1]. Free convection effects on the oscillating flow past an infinite vertical porous plate with constant suction - I, was studied by Soundalgekar [2] which was further improved by Vajravelu *et al*., [3]. Further researches in these areas were done by Gupta *et al*., [4], Jaiswal *et al*., [5] and Soundalgekar *et al*., [6] by taking different models. Some effects like radiation and mass transfer on MHD flow were studied by Muthucumaraswamy *et al*., [7, 8] and Prasad *et al*., [9]. Radiation effects on mixed convection along a vertical plate with uniform surface temperature were studied by Hossain and Takhar [10]. Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux was studied by Jha, Prasad and Rai [11].

On the other hand, radiation and free convection flow past a moving plate were considered by Raptis and Perdikis [12]. We are considering radiation effects on MHD flow through porous media past an impulsively started vertical oscillating plate with variable mass diffusion.

MATHEMATICAL ANALYSIS

In this paper we have considered the flow of unsteady viscous incompressible fluid. The x-axis is taken along the plate in the upward direction and y - axis is taken normal to the plate. Initially the fluid and plate are at the same temperature. A transverse magnetic field B_0, of uniform strength is applied normal to the plate. The viscous dissipation and induced magnetic field has been neglected due to its small effect. Initially, the fluid and plate are at the same temperature T_∞ and concentration C_∞ in the stationary condition. At time $t > 0$, the plate starts oscillating in its own plane with frequency ω and temperature of the plate is raised to T_w and the concentration level near the plate is raised linearly with respect to time. The flow modal is as under:

$$\frac{\partial u}{\partial t} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} - \upsilon\frac{u}{K}, \quad (1)$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2}, \quad (2)$$

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}. \quad (3)$$

Here, the symbols have the meaning as described below:

C_p = specific heat at constant pressure
T = temperature of the fluid near the plate
C = species concentration in the fluid
T_∞ = temperature of the fluid far away from the plate
C_∞ = concentration in the fluid far away from the plate
t = time
ρ = fluid density
g = acceleration due to gravity
β = volumetric coefficient of thermal expansion
β^* = volumetric coefficient of concentration expansion
υ = kinematics viscosity
B_0 = external magnetic field
q_r = radiative heat flux in the y – direction
σ = Stefan–Boltzmann constant
u = velocity of the fluid in the x - direction
k = thermal conductivity of the fluid

y = coordinate axis normal to the plate
D = mass diffusion coefficient
K = permeability of porous medium

The following boundary conditions have been assumed:

$$\left.\begin{array}{l} t\le 0: u=0,\ T=T_\infty,\ C=C_\infty \text{ for all the values of } y,\\ t>0: u=u_0 cos\omega t,\ T=T_w,\ C=C_\infty+\left(C_w-C_\infty\right)\frac{u_0^2 t}{\upsilon} \text{ at } y=0,\\ \qquad u\to 0,\ T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty, \end{array}\right\} \quad (4)$$

where ω is the angular velocity and T_w, is the temperature, C_w is the concentration of the fluid.

The local radiant for the case of an optically thin gray gas is expressed by:

$$\frac{\partial q_r}{\partial y}=-4a^*\sigma\left(T_\infty^4-T^4\right), \quad (5)$$

where a^* is absorption constant.

Considering the temperature difference within the flow sufficiently small, T^4 can be expressed as the linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms

Thus, $T^4 \cong 4T_\infty^3 T-3T_\infty^4$. (6)

Using equations (5) and (6), equation (3) becomes

$$\rho C_p\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial y^2}+16a^*\sigma T_\infty^3\left(T_\infty-T\right). \quad (7)$$

Introducing the following non- dimensional quantities:

$$\left.\begin{array}{l} \bar{y}=\frac{yu_0}{\upsilon},\ \bar{t}=\frac{tu_0^2}{\upsilon},\ \bar{u}=\frac{u}{u_0},\ \theta=\frac{(T-T_\infty)}{(T_w-T_\infty)},\ \bar{C}=\frac{(C-C_\infty)}{(C_w-C_\infty)},\\ G_r=\frac{g\beta\upsilon(T_w-T_\infty)}{u_0^3},M=\frac{\sigma B_0^2\upsilon}{\rho u_0^2},\ \bar{\omega}=\frac{\omega\upsilon}{u_0^2},\ \bar{K}=\frac{u_0^2}{\upsilon^2}K,\ P_r=\frac{\mu C_p}{k}\\ G_m=\frac{g\beta^*\upsilon(C_w-C_\infty)}{u_0^3},\ R=\frac{16a^*\upsilon^2\sigma T_\infty^3}{ku_0^2},\ S_c=\frac{\upsilon}{D},\ \mu=\rho\upsilon, \end{array}\right\} \quad (8)$$

with symbols: $\bar{u}$ - dimensionless velocity; P_r - Prandtl number; M - magnetic field parameter; $\bar{y}$ - dimensionless coordinate axis normal to the plate; θ - dimensionless temperature; G_r - thermal Grashof number; G_m - mass Grashof number; S_c - Schmidt number; R - radiation parameter; $\bar{C}$ - dimensionless concentration; $\overline{\omega t}$ - dimensionless Phase angle; μ - coefficient of viscosity and $\bar{K}$ - dimensionless permeability of porous medium.

Equations (1), (2) and (7) leads to

$$\frac{\partial\bar{u}}{\partial\bar{t}}=G_r\theta+G_m\bar{C}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}-\left(M+\frac{1}{\bar{K}}\right)\bar{u}, \quad (9)$$

$$\frac{\partial\bar{C}}{\partial\bar{t}}=\frac{1}{S_c}\frac{\partial^2\bar{C}}{\partial\bar{y}^2}, \text{ and} \quad (10)$$

$$\frac{\partial\theta}{\partial\bar{t}}=\frac{1}{P_r}\left(\frac{\partial^2\theta}{\partial\bar{y}^2}\right)-\frac{R}{P_r}\theta, \quad (11)$$

with the following boundary conditions:

$$\left.\begin{array}{l} \bar{t}\le 0: \bar{u}=0,\ \theta=0,\ \bar{C}=0 \text{ for all the values of } \bar{y},\\ \bar{t}>0: \bar{u}=\cos\overline{\omega t},\ \theta=1,\ \bar{C}=\bar{t} \text{ at } \bar{y}=0,\\ \qquad \bar{u}\to 0,\ \theta\to 0,\ \bar{C}\to 0 \text{ as } \bar{y}\to\infty. \end{array}\right\} \quad (12)$$

Dropping bars in the above equations, we have

$$\frac{\partial u}{\partial t}=G_r\theta+G_m C+\frac{\partial^2 u}{\partial y^2}-\left(M+\frac{1}{K}\right)u, \quad (13)$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}, \text{ and} \quad (14)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{P_r}\left(\frac{\partial^2\theta}{\partial y^2}\right)-\frac{R}{P_r}\theta, \quad (15)$$

with the following boundary conditions:

$$\left.\begin{array}{l} t\le 0: u=0,\ \theta=0,\ C=0 \text{ for all the values of } y,\\ t>0: u=\cos\omega t,\ \theta=1,\ C=t \text{ at } y=0,\\ \qquad u\to 0,\ \theta\to 0,\ C\to 0 \text{ as } y\to\infty. \end{array}\right\} \quad (16)$$

The dimensionless governing equations (13) to (15), subject to the boundary conditions (16), are solved by the usual Laplace- transform technique with some help from the article“ An algorithm for generating some inverse Laplace transforms of exponential form” by R.B. Hetnarski [13] . The solutions are derived as follows:

$$\bar{u}(y,t)=\frac{1}{4}\left[e^{i\omega t}\left\{\begin{array}{l} e^{-2\gamma\sqrt{\left(M^*+i\omega\right)t}}erfc\left(\gamma-\sqrt{\left(M^*+i\omega\right)t}\right)\\ +e^{2\gamma\sqrt{\left(M^*+i\omega\right)t}}erfc\left(\gamma+\sqrt{\left(M^*+i\omega\right)t}\right)\end{array}\right\}+e^{-i\omega t}\left\{\begin{array}{l} e^{-2\gamma\sqrt{\left(M^*-i\omega\right)t}}erfc\left(\gamma-\sqrt{\left(M^*-i\omega\right)t}\right)\\ +e^{2\gamma\sqrt{\left(M^*-i\omega\right)t}}erfc\left(\gamma+\sqrt{\left(M^*-i\omega\right)t}\right)\end{array}\right\}\right]$$

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$$
\begin{aligned}
&+\frac{1}{2}\left(G_3-\frac{tG_2}{d}\right)\left[e^{-2\gamma\sqrt{M^*t}}erfc\left(\gamma-\sqrt{M^*t}\right)+e^{2\gamma\sqrt{M^*t}}erfc\left(\gamma+\sqrt{M^*t}\right)\right]\\
&+\frac{yG_2}{4d\sqrt{M^*}}\left[e^{-2\gamma\sqrt{M^*t}}erfc\left(\gamma-\sqrt{M^*t}\right)-e^{2\gamma\sqrt{M^*t}}erfc\left(\gamma+\sqrt{M^*t}\right)\right]\\
&-\frac{G_1e^{-bt}}{2b}\left[\begin{array}{l}\left\{e^{-2\gamma\sqrt{ct}}erfc\left(\gamma-\sqrt{ct}\right)+e^{2\gamma\sqrt{ct}}erfc\left(\gamma+\sqrt{ct}\right)\right\}-\\ \left\{e^{-2\gamma\sqrt{P_r ft}}erfc\left(\gamma\sqrt{P_r}-\sqrt{ft}\right)+e^{2\gamma\sqrt{P_r ft}}erfc\left(\gamma\sqrt{P_r}+\sqrt{ft}\right)\right\}\end{array}\right]\\
&-\frac{G_1}{2b}\left[e^{-2\gamma\sqrt{P_r at}}erfc\left(\gamma\sqrt{P_r}-\sqrt{at}\right)+e^{2\gamma\sqrt{P_r at}}erfc\left(\gamma\sqrt{P_r}+\sqrt{at}\right)\right]\\
&+\frac{G_2e^{dt}}{d^2}\left[\begin{array}{l}\left\{e^{-2\gamma\sqrt{ht}}erfc\left(\gamma-\sqrt{ht}\right)+e^{2\gamma\sqrt{ht}}erfc\left(\gamma+\sqrt{ht}\right)\right\}-\\ \left\{e^{-2\gamma\sqrt{dS_c t}}erfc\left(\gamma\sqrt{S_c}-\sqrt{dt}\right)+e^{2\gamma\sqrt{dS_c t}}erfc\left(\gamma\sqrt{S_c}+\sqrt{dt}\right)\right\}\\ +\left(1+dt+2t\gamma^2dS_c\right)erfc\left(\gamma\sqrt{S_c}\right)-\frac{2\gamma t\sqrt{S_c}}{\sqrt{\pi}}e^{-\gamma^2S_c}\end{array}\right],
\end{aligned}
\tag{17}
$$

$$C(y,t)=t\left[\left(1+2\gamma^2S_c\right)erfc\left(\gamma\sqrt{S_c}\right)-\frac{2\gamma\sqrt{S_c}}{\sqrt{\pi}}e^{-\gamma^2S_c}\right],\text{ and}\tag{18}$$

$$\theta(y,t)=\frac{1}{2}\left[e^{-2\gamma\sqrt{aP_rt}}erfc\left(\gamma\sqrt{P_r}-\sqrt{at}\right)+e^{2\gamma\sqrt{aP_rt}}erfc\left(\gamma\sqrt{P_r}+\sqrt{at}\right)\right].\tag{19}$$

For making the solution concise, the following symbols have been used above:

$$M^*=M+\frac{1}{K},a=\frac{R}{P_r},b=\frac{R-M^*}{P_r-1},c=M^*-b,d=\frac{M^*}{S_c-1},f=a-b,$$

$$h=M^*+d,G_1=\frac{G_r}{P_r-1},G_2=\frac{G_m}{S_c-1},G_3=\frac{G_1}{b}-\frac{G_2}{d^2},\gamma=\frac{y}{2\sqrt{t}}.$$

Skin friction (τ)

The skin friction is given as:

$$\tau=-\left(\frac{\partial u}{\partial y}\right)_{y=0}=-\frac{1}{2\sqrt{t}}\left(\frac{\partial u}{\partial \gamma}\right)_{\gamma=0},\tag{20}$$

Therefore, using equation (17), we obtain τ :

$$
\begin{aligned}
\tau=&\frac{1}{2}\left[e^{i\omega t}\left(\sqrt{M^*+i\omega}\right)erf\left(\sqrt{\left(M^*+i\omega\right)t}\right)+e^{-i\omega t}\left(\sqrt{M^*-i\omega}\right)erf\left(\sqrt{\left(M^*-i\omega\right)t}\right)\right]\\
&+G_3\sqrt{M^*}erf\left(\sqrt{M^*t}\right)+\frac{e^{-M^*t}}{\sqrt{\pi t}}+\frac{G_2}{d^2}\left[e^{-dt}\left\{\sqrt{h}erf\left(\sqrt{ht}\right)-\sqrt{dS_c}erf\left(\sqrt{dt}\right)\right\}\right]\\
&-\frac{G_1}{b}\left[e^{-bt}\left\{\sqrt{c}erf\left(\sqrt{ct}\right)-\sqrt{fP_r}erf\left(\sqrt{ft}\right)\right\}+\sqrt{aP_r}erf\left(\sqrt{at}\right)\right]\\
&-\frac{G_2}{d}\left[\left(t\sqrt{M^*}+\frac{1}{2\sqrt{M^*}}\right)erf\left(\sqrt{M^*t}\right)+\sqrt{\frac{t}{\pi}}\left(e^{-M^*t}-2\sqrt{S_c}\right)\right],
\end{aligned}
\tag{21}
$$

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Sherwood number $\left(S_h\right)$

The Sherwood Number is given as:

$$S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0}, \tag{22}$$

Then from equation (18), we have

$$S_h = 2\sqrt{\frac{tS_c}{\pi}}. \tag{23}$$

Nusselt number $\left(N_u\right)$

The Nusselt Number is given as:

$$N_u = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{\partial \theta}{\partial \gamma}\right)_{\gamma=0}, \tag{24}$$

Therefore from equation (19), we get

$$N_u = \frac{\sqrt{P_r}}{\sqrt{\pi t}} e^{-at} - \sqrt{aP_r}\, erf\left(\sqrt{at}\right). \tag{25}$$

RESULTS AND DISCUSSIONS

The velocity profiles for different parameters are shown in Figures 1 to 6. In Figure-1 when value of K is increased, the velocity decreases near the plate up to certain point and then it increases slightly. In Figure-2 if phase angle ωt is increased the velocity profile shifts towards the origin. i.e., the velocity decreases near the plate. When the value of R is increased the velocity decreases which is shown in Figure-3. It is observed in Figure-4, that when the mass Grashof number is increased velocity decreases near the plate and then starts increasing slightly away from the plate before becoming steady. The effect of the thermal Grashof number for cooling and heating of the plate is shown by the Figures 5 and 6. Figure-7 shows that when the Schmidt number is increased, the concentration profile shifts towards the origin. Figure-8 shows that when t is increased, there is sufficient increases in the concentration near the plate while it gets steady away from the plate. It is observed in Figure-9 that when Schmidt number increases, Sherwood number also increases. Figure-10 shows the pattern of Nusselt number for different values of radiation parameter; it decreases with the increase in R. The values of P_r and M are taken as 0.71 and 0.5 respectively in all the cases. Wall shear stress is tabulated in Table-1. Here the values of different parameters are changed diagonally to make the comparison possible with first row.

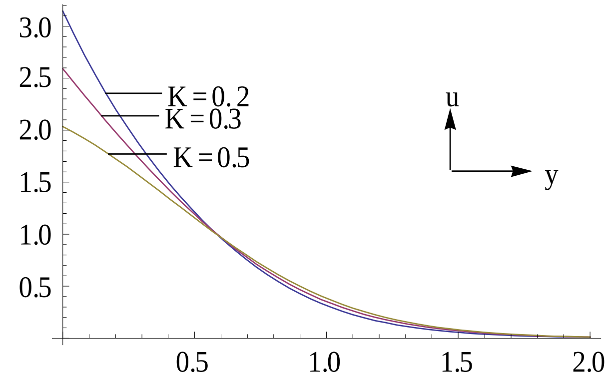

Figure-1. Velocity profile. ($G_r = 10$, $\omega t = \frac{\pi}{4}, t = 0.2, R = 2, G_m = 5, S_c = 2.01$).

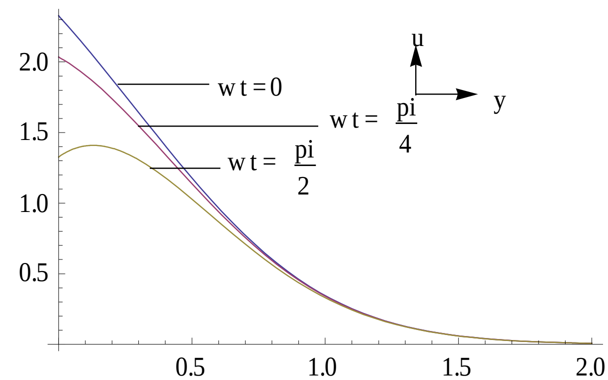

Figure-2. Velocity profile. ($G_r = 10, G_m = 5,\ t = 0.2, R = 2, S_c = 2.01$).

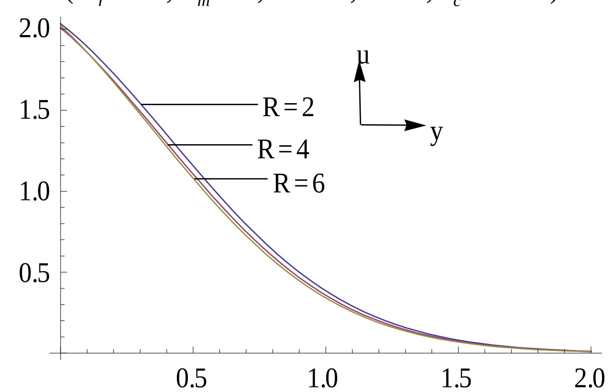

Figure-3. Velocity profile. ($\omega t = \frac{\pi}{4}$, $G_r = 10, G_m = 5,\ t = 0.2, S_c = 2.01$).

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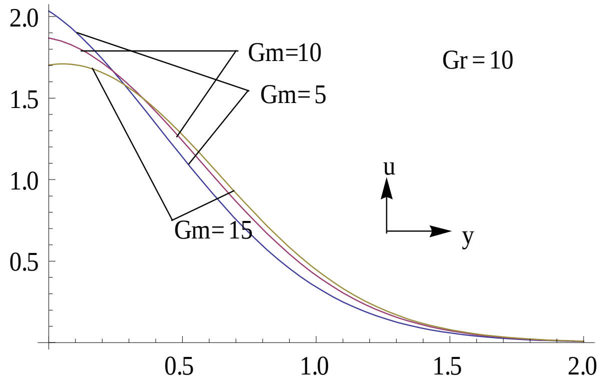

Figure-4. Velocity profile.

$(\omega t = \frac{\pi}{4}, t = 0.2, R = 2, S_c = 2.01)$.

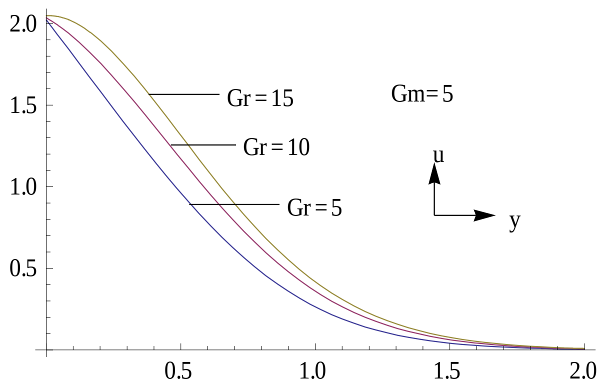

Figure-5. Velocity profile-cooling of the plate.

$(\omega t = \frac{\pi}{4}, t = 0.2, R = 2, S_c = 2.01)$.

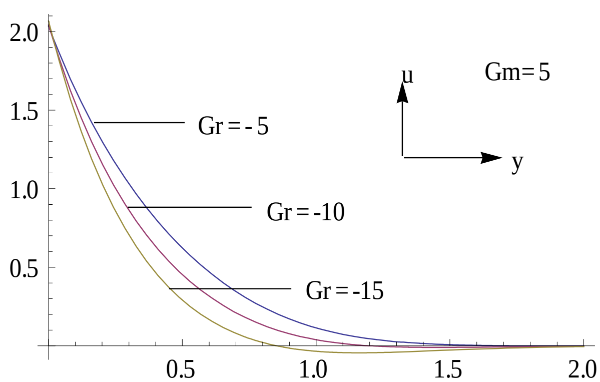

Figure-6. Velocity profile- heating of the plate.

$(\omega t = \frac{\pi}{4}, t = 0.2, R = 2, S_c = 2.01)$.

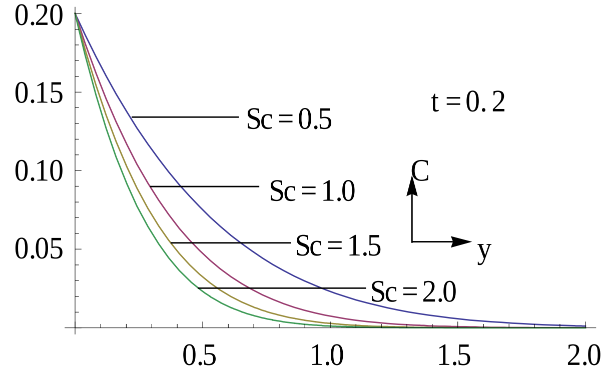

Figure-7. Concentration profile.

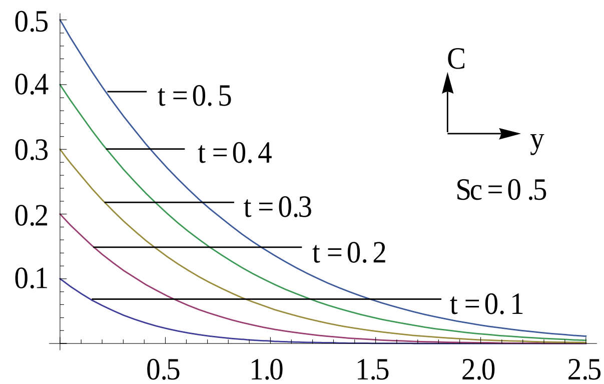

Figure-8. Concentration profile.

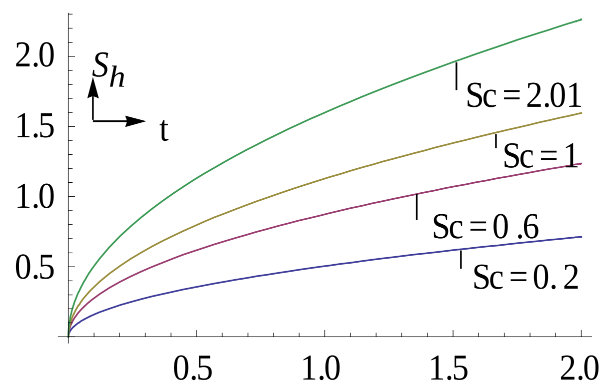

Figure-9. Sherwood number.

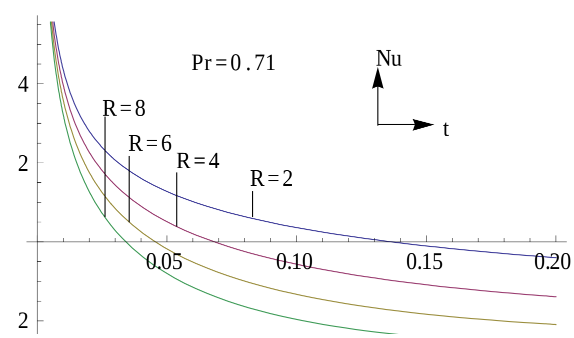

Figure-10. Nusselt number.

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Table-1. Skin friction for different parameters.

K	P_r	G_r	G_m	S_c	M	R	t	τ
0.1	0.71	10	5	2.01	0.5	2	0.2	-2.55681
0.1	0.71	10	5	2.01	0.5	2	0.4	-4.31938
0.1	0.71	10	5	2.01	0.5	4	0.2	-2.04277
0.1	0.71	10	5	2.01	1	2	0.2	-2.60225
0.1	0.71	10	5	3	0.5	2	0.2	-1.15201
0.1	0.71	10	10	2.01	0.5	2	0.2	-1.65284
0.1	0.71	15	5	2.01	0.5	2	0.2	-3.47113
0.1	0.1	10	5	2.01	0.5	2	0.2	-2.9577
0.3	0.71	10	5	2.01	0.5	2	0.2	-6.35318

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REFERENCES

[1] M.J. Lighthill. 1994. The response of laminar skin friction and heat transfer to fluctuations in the stream velocity. Proc. R. Soc., A, 224: 1-23.

[2] V. M. Soundalgekar. 1973. Free convection effect on the oscillatory flow an infinite vertical porous, plate with constant suction. I, Proc. R. Soc., A. 333: 25-36.

[3] K. Vajravelu and K. S. Sastri. 1977. Correction to free convection effects on the oscillatory flow an infinite, vertical porous, plate with constant suction. I, Proc. R. Soc., A. 353: 221-223.

[4] A.S. Gupta, I. Pop and V.M. Soundalgekar. 1979. Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid. Rev. Roum. Sci. Techn. -Mec. Apl. 24: 561-568.

[5] B.S. Jaiswal and V.M. Soundalgekar. 2001. Oscillating plate temperature effects on a flow past an infinite porous plate with constant suction and embedded in a porous medium. Heat and Mass Transfer. 37: 125-131.

[6] V. M. Soundalgekar and H. S. Takhar. 1993. Radiation effects on free convection flow past a semi-infinite vertical plate. Modeling, Measurement and Control. B51: 31- 40.

[7] R. Muthucumaraswamy and Janakiraman. 2006. MHD and Radiation effects on moving isothermal vertical plate with variable mass diffusion. Theoret. Appl. Mech. 33(1): 17-29.

[8] R. Muthucumaraswamy, K.E. Sathappan and R. Natarajan. 2008. Mass transfer effects on exponentially accelerated isothermal vertical plate. Int. J. of Appl. Math. and Mech. 4(6): 19-25.

[9] V.R. Prasad, N.B. Reddy and R. Muthucumaraswamy. 2007. Radiation and mass transfer effects on two-dimensional flow past an impulsively started infinite vertical plate. Int. J. Thermal Sci. 46(12): 1251-1258.

[10] M.A. Hossain and H.S. Takhar. 1996. Radiation effect on mixed convection along a vertical plate with uniform surface temperature. Heat and Mass Transfer. 31: 243-248.

[11] B.K Jha, R. Prasad and S. Rai. 1991. Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux. Astrophysics and Space Science. 181: 125-134.

[12] A. Raptis and C. Perdikis. 1999. Radiation and free convection flow past a moving plate. Int. J. of App. Mech. and Engg. 4: 817-821.

[13] R.B. Hetnarski. 1975. An algorithm for generating some inverse Laplace transforms of exponential form. ZAMP. 26: 249-253.