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ON THE STABILITY OF A FOUR SPECIES SYN ECO-SYSTEM WITH COMMENSAL PREY-PREDATOR PAIR WITH PREY-PREDATOR PAIR OF HOSTS-VIII

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ABSTRACT

The present paper is devoted to an investigation on a four species (S_1, S_2, S_3, S_4) Syn Eco-System with Commensal Prey-Predator pair with Prey-Predator pair of Hosts (host of S_1 washed out states). The system comprises of a Prey (S_1), a Predator (S_2) that survives upon S_1, two Hosts S_3 and S_4 for which S_1, S_2 are Commensal, respectively i.e., S_3 and S_4 benefit S_1 and S_2, respectively without getting effected either positively or adversely. Further S_3 is Prey for S_4 and S_4 is Predator for S_3. The pair (S_1, S_2) may be referred as 1st level Prey-Predator and the pair (S_3, S_4), the 2nd level Prey-Predator. The model equations of the system constitute a set of four first order non-linear ordinary differential coupled equations. In all, there are sixteen equilibrium points. Criteria for the asymptotic stability of four of these sixteen equilibrium points: Host of S_1 washed out states is established. The system would be stable if all the characteristic roots are negative, in case they are real, and have negative real parts, in case they are complex. The linearized equations for the perturbations over the equilibrium points are analyzed to establish the criteria for stability and the trajectories are illustrated.

Keywords: commensal, eco-system, prey, predator, equillibrium point, host, neutrally stable, quasi-linearization, trajectories.

1. INTRODUCTION

Research in the area of theoretical Ecology was initiated in 1925 by Lotka [1] and in 1931 by Volterra [2]. Since then many Mathematicians and Ecologists contributed to the growth of this area of knowledge reported in the treatises of May [3], Smith [4], Kushing [5], Kapur [6] etc. The ecological interactions can be broadly classified as Prey-Predator, Commensalism, Competition, Neutralism, Mutualism and so on. Srinivas [7] studied competitive eco-systems of two species and three species with limited and unlimited resources. Later Lakshminarayan and Pattabhi Ramacharyulu [8] studied Prey-Predator ecological models with partial cover for the Prey and alternate food for the Predator. Recently, Archana Reddy [9] and Bhaskara Rama Sharma [10] investigated diverse problems related to two species competitive systems with time delay, employing analytical and numerical techniques. Further Phani Kumar and Pattabhi Ramacharyulu [11] studied Three Species Ecosystem Consisting of a Prey, Predator and a Host Commensal to the Prey. The present authors Hari Prasad and Pattabhi Ramacharyulu [12, 13, 14] discussed on the stability of a four species: A Prey-Predator-Host-Commensal Syn Eco-System.

The paper is organized as follows: Section 2 discusses the basic equations and notations. Section 3 shows investigation of equilibrium states. Sections 4 and 5 discuss stability of the equilibrium states. Section 6 gives the trajectories of perturbations. Section 7 shows perturbation graphs. Section 8 gives conclusion and section 9 presents future work.

2. BASIC EQUATIONS

The model equations for a four species syn eco-system are given by the following system of first order non-linear ordinary differential equations employing the following notation:

Notation

S_1: Prey for S_2 and commensal for S_3.
S_2: Predator surviving upon S_1 and commensal for S_4.
S_3: Host for the commensal (S_1) and Prey for S_4.
S_4: Host of the commensal (S_2) and Predator surviving upon S_4.
N_i (t): The population strength of S_i at time t, i = 1, 2, 3, 4
t: Time instant
a_i: Natural growth rate of S_i, i = 1, 2, 3, 4
a_{ii}: Self inhibition coefficient of S_i, i = 1, 2, 3, 4
a_{12}, a_{21}: Interaction (Prey-Predator) coefficients of S_1 due to S_2 and S_2 due to S_1
a_{34}, a_{43}: Interaction (Prey-Predator) coefficients of S_3 due to S_4 and S_4 due to S_3
a_{13}, a_{24}: Coefficients for commensal for S_1 due to the Host S_3 and S_2 due to the Host S_4
$K_i = \dfrac{a_i}{a_{ii}}$: Carrying capacities of S_i, i = 1, 2, 3, 4

Further the variables N_1, N_2, N_3, N_4 are non-negative and the model parameters a_1, a_2, a_3, a_4; a_{11}, a_{22}, a_{33}, a_{44}; a_{12}, a_{21}, a_{13}, a_{24}, a_{34}, a_{43} are assumed to be non-negative constants.

The model equations for the growth rates of S_1, S_2, S_3, S_4 are:

$$\frac{dN_1}{dt} = a_1N_1 - a_{11}N_1^2 - a_{12}N_1N_2 + a_{13}N_1N_3 \quad (1)$$

$$\frac{dN_2}{dt} = a_2N_2 - a_{22}N_2^2 + a_{21}N_1N_2 + a_{24}N_2N_4 \quad (2)$$

$$\frac{dN_3}{dt} = a_3N_3 - a_{33}N_3^2 - a_{34}N_3N_4 \quad (3)$$

$$\frac{dN_4}{dt} = a_4N_4 - a_{44}N_4^2 + a_{43}N_3N_4 \quad (4)$$

A Schematic Sketch of the system under investigation is shown in Figure-1.

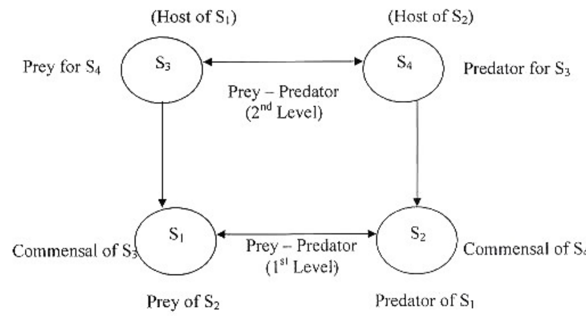

Figure-1. Schematic sketch of the Syn eco-system.

3. EQUILIBRIUM STATES

The system under investigation has sixteen equilibrium states defined by:

$$\frac{dN_i}{dt} = 0,\ i = 1,2,3,4 \quad (5)$$

As given in the following Table-1.

Table-1.

S. No.	Equilibrium state	Equilibrium point
1	Fully Washed out state	$\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$
2*	Only the host (S_4) of S_2 survives	$\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$
3	Only the host (S_3)of S_1 survives	$\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$
4	Only the predator (S_2) survives	$\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = 0, \overline{N_4} = 0$
5	Only the prey (S_1) survives	$\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$
6	Prey (S_1) and predator (S_2) washed out	$\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \frac{\alpha}{\beta}, \overline{N_4} = \frac{\gamma}{\beta}$ where $\alpha = a_3a_{44} - a_4a_{34},\ \beta = a_{33}a_{44} + a_{34}a_{43} > 0$ $\gamma = a_3a_{43} + a_4a_{33} > 0$
7*	Prey (S_1) and host (S_3) of S_1 washed out	$\overline{N_1} = 0, \overline{N_2} = \frac{\delta_1}{a_{22}a_{44}}, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$ where $\delta_1 = a_2a_{44} + a_4a_{24} > 0$
8	Prey (S_1) and host (S_4) of S_2 washed out	$\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$
9*	Predator (S_2) and host (S_3) of S_1 washed out	$\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$
10	Predator (S_2) and host (S_4) of S_2 washed out	$\overline{N_1} = \frac{\delta_2}{a_{11}a_{33}}, \overline{N_2} = 0, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$ where $\delta_2 = a_1a_{33} + a_3a_{13} > 0$
11	Prey (S_1) and predator (S_2) survives	$\overline{N_1} = \frac{\alpha_1}{\beta_1}, \overline{N_2} = \frac{\gamma_1}{\beta_1}, \overline{N_3} = 0, \overline{N_4} = 0$ where $\alpha_1 = a_1a_{22} - a_2a_{12},\ \beta_1 = a_{11}a_{22} + a_{12}a_{21} > 0$

		$\gamma_1 = a_1 a_{21} + a_2 a_{11} > 0$
12	Only the prey (S_1) washed out	$\overline{N_1} = 0, \overline{N_2} = \frac{a_2\beta + a_{24}\gamma}{a_{22}\beta}, \overline{N_3} = \frac{\alpha}{\beta}, \overline{N_4} = \frac{\gamma}{\beta}$
13	Only the predator (S_2) washed out	$\overline{N_1} = \frac{a_1\beta + a_{13}\alpha}{a_{11}\beta}, \overline{N_2} = 0, \overline{N_3} = \frac{\alpha}{\beta}, \overline{N_4} = \frac{\gamma}{\beta}$
14*	Only the host (S_3) of S_1 washed out	$\overline{N_1} = \frac{a_1 a_{22} a_{44} - a_{12}\delta_1}{a_{44}\beta_1}, \overline{N_2} = \frac{a_1 a_{21} a_{44} + a_{11}\delta_1}{a_{44}\beta_1},$ $\overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$
15	Only the Host (S_4) of S_2 washed out	$\overline{N_1} = \frac{a_{22}\delta_2 - a_2 a_{12} a_{33}}{a_{33}\beta_1}, \overline{N_2} = \frac{a_{21}\delta_2 + a_2 a_{11} a_{33}}{a_{33}\beta_1},$ $\overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$
16	The co-existent state (or) Normal steady state	$\overline{N_1} = \frac{a_{22}\alpha_2 - a_{12}\gamma_2}{\beta_1}, \overline{N_2} = \frac{a_{11}\gamma_2 + a_{21}\alpha_2}{\beta_1},$ $\overline{N_3} = \frac{\alpha}{\beta}, \overline{N_4} = \frac{\gamma}{\beta}$ where $\alpha_2 = a_1 + a_{13}\frac{\alpha}{\beta}, \gamma_2 = a_2 + a_{24}\frac{\gamma}{\beta} > 0$

The present paper deals with the Host of S_1 washed out states only (Sl. Nos. 2, 7, 9, 14 marked * in the above Table-1). The stability of the other equilibrium states was already discussed and communicated to several International Journals.

4. STABILITY OF THE EQUILIBRIUM STATES

Let $N = (N_1, N_2, N_3, N_4) = \bar{N} + U$ (6)

where $U = (u_1, u_2, u_3, u_4)$ is a perturbation over the equilibrium state $\bar{N} = (\bar{N}_1, \bar{N}_2, \bar{N}_3, \bar{N}_4)$.

The basic equations (1), (2), (3), (4) are quasi-linearized to obtain the equations for the perturbed state.

$$\frac{dU}{dt} = AU \tag{7}$$

Where

$$A = \begin{bmatrix} a_1 - 2a_{11}\bar{N}_1 - a_{12}\bar{N}_2 + a_{13}\bar{N}_3 & -a_{12}\bar{N}_1 & a_{13}\bar{N}_1 & 0 \\ a_{21}\bar{N}_1 & a_2 - 2a_{22}\bar{N}_2 + a_{21}\bar{N}_1 + a_{24}\bar{N}_4 & 0 & a_{24}\bar{N}_2 \\ 0 & 0 & a_3 - 2a_{33}\bar{N}_3 - a_{34}\bar{N}_3 & -a_{34}\bar{N}_3 \\ 0 & 0 & a_{34}\bar{N}_4 & a_4 - 2a_{44}\bar{N}_4 + a_{43}\bar{N}_3 \end{bmatrix} \tag{8}$$

The characteristic equation for the system is $\det[A - \lambda I] = 0$ (9)

The equilibrium state is stable, if both the roots of the equation (9) are negative in case they are real or have negative real parts in case they are complex.

5. STABILITY OF THE HOST (S_3) OF S_1 WASHED OUT EQUILIBRIUM SATES: (Sl. No's 2, 7, 9, 14 marked * in table-1)

The equilibrium states (Sl. No's 2, 7, 9) were already discussed in the papers "On the stability of a four species syn Eco-system with commensal prey-predator pair with prey-predator pair of hosts - II, IV, V" communicated to JJMS, IJPAMS, IJAMM, respectively. Now discuss about the Equilibrium point.

$\bar{N}_1 = \frac{a_{12}a_{22}a_{44} - a_{12}\delta_1}{a_{44}\beta_1}$, $\bar{N}_2 = \frac{a_{12}a_{21}a_{44} + a_{11}\delta_1}{a_{44}\beta_1}$, $\bar{N}_3 = 0, \bar{N}_4 = k_4$:

This would exists only when $a_1 a_{22} a_{44} > a_{12}\delta_1$ (10)

The corresponding linearized equations for the perturbations u_1, u_2, u_3, u_4 are:

$$\frac{du_1}{dt} = \mu_1 u_1 - a_{12}\,\overline{N}_1 u_2 + a_{13}\,\overline{N}_1 u_3 \tag{11}$$

$$\frac{du_2}{dt} = a_{21}\overline{N}_2 u_1 + \mu_2 u_2 + a_{24}\overline{N}_2 u_4 \tag{12}$$

$$\frac{du_3}{dt} = \mu_3 u_3, \frac{du_4}{dt} = a_{43}k_4 u_3 - a_4 u_4 \quad (13)$$

Where

$$\mu_1 = a_1 - \left(2a_{11}\overline{N}_1 + a_{12}\overline{N}_2\right) \quad (14)$$

$$\mu_2 = a_2 + a_{21}\overline{N}_1 + a_{24}k_4 - 2a_{22}\overline{N}_2 \quad (15)$$

$$\mu_3 = a_3 - a_{34}k_4 \quad (16)$$

The characteristic equation for which is:

$$\left[\lambda^2 - (\mu_1 + \mu_2)\lambda + \left(\mu_1\mu_2 - a_{12}a_{21}\overline{N}_1\overline{N}_2\right)\right] (\lambda - \mu_3)(\lambda + a_4) \quad (17)$$

One of the four roots $-a_4$ is negative. Let λ_1, λ_2 be the zeros of the quadratic polynomial on the L.H.S. of the characteristic equation (17).

Case (A): When $a_3 < a_{34}k_4$, i.e., the root μ_3 is negative

Case (a): If the roots λ_1, λ_2 noted to be negative. Hence the state is stable and the equations (11), (12), (13) yield the solutions.

$$u_1 = \frac{(\mu_1 - \lambda_2)(\overline{\gamma} + \overline{\mu} - u_{10}) - a_{12}\overline{N}_1(\Psi_1 + \psi_2 - u_{20})}{\lambda_2 - \lambda_1} e^{\lambda_1 t} + \frac{(\mu_1 - \lambda_1)(\overline{\gamma} + \overline{\mu} - u_{10}) - a_{12}\overline{N}_1(\Psi_1 + \psi_2 - u_{20})}{\lambda_1 - \lambda_2} e^{\lambda_2 t} + \overline{\gamma} e^{\mu_3 t} + \overline{\mu}\ \overline{e}^{a_4 t} \quad (18)$$

$$u_2 = \frac{(\mu_1 - \lambda_2)(\overline{\gamma} + \overline{\mu} - u_{10}) - a_{12}\overline{N}_1(\Psi_1 + \psi_2 - u_{20})}{(\lambda_2 - \lambda_1) a_{12}\overline{N}_1} (\mu_1 - \lambda_1) e^{\lambda_1 t} + \frac{(\mu_1 - \lambda_1)(\overline{\gamma} + \overline{\mu} - u_{10}) - a_{12}\overline{N}_1(\Psi_1 + \psi_2 - u_{20})}{(\lambda_1 - \lambda_2) a_{12}\overline{N}_1} (\mu_1 - \lambda_2) e^{\lambda_2 t} + \psi_1 e^{\mu_3 t} + \psi_2 \overline{e}^{a_4 t} \quad (19)$$

$$u_3 = u_{30}\, e^{\mu_3 t}, u_4 = \overline{\mu}_3 e^{\mu_3 t} + (u_{40} - \overline{\mu}_3) e^{-a_4 t} \quad (20)$$

Where

$$\overline{\mu}_3 = \frac{a_{43}k_4 u_{30}}{\mu_3 + a_4}, \overline{\beta} = a_{12}a_{24}\overline{N}_2(u_{40} - \overline{\mu}_3) \quad (21)$$

$$\overline{\alpha} = \left(\mu_3 a_{13}u_{30} - \mu_2 a_{13}u_{30} + a_{12}a_{24}\overline{N}_2\overline{\mu}_3\right)\overline{N}_1 \quad (22)$$

$$\overline{\gamma} = \frac{\overline{\alpha}}{\mu_3^2 - (\mu_1 + \mu_2)\mu_3 + \mu_1\mu_2 + a_{12}a_{21}\overline{N}_1\overline{N}_2} \quad (23)$$

$$\overline{\mu} = \frac{\overline{\beta}}{a_4^2 + (\mu_1 + \mu_2)a_4 + \mu_1\mu_2 + a_{12}a_{21}\overline{N}_1\overline{N}_2} \quad (24)$$

$$\psi_1 = \frac{a_{12}\overline{N}_1 u_{30} + \mu_1\overline{\gamma} - \mu_3\overline{\gamma}}{a_{12}\overline{N}_1}, \psi_2 = \frac{\mu_1\overline{\mu} + a_4\overline{\mu}}{a_{12}\overline{N}_1} \quad (25)$$

and $u_{10}, u_{20}, u_{30}, u_{40}$ are the initial values of u_1, u_2, u_3, u_4, respectively

There would arise in all 576 cases depending upon the ordering of the magnitudes of the growth rates a_1, a_2, a_3, a_4 and the initial values of the perturbations $u_{10}(t), u_{20}(t), u_{30}(t), u_{40}(t)$ of the species S_1, S_2, S_3, S_4. Of these 576 situations some typical variations are illustrated through respective solution curves that would facilitate to make some reasonable observations and the solution curves are illustrated in Figures-2 and 3.

Case (i): If $u_{10} < u_{20} < u_{30} < u_{40}$ and $a_1 < \mu_3 < a_2 < a_4$

In this case the natural birth rates of the prey (S_1), host (S_3) of S_1, predator (S_2) and the host (S_4) of S_2 are in ascending order. Initially the host (S_3) of S_1 dominates over the predator (S_2) till the time instant t_{23}^* and thereafter the dominance is reversed. The time t_{23}^* may be called the dominance time of S_3 over S_2.

Case (ii): If $u_{20} < u_{40} < u_{10} < u_{30}$ and $\mu_3 < a_2 < a_4 < a_1$

In this case the host (S_3) of S_1 has the least natural birth rate. Initially it is dominated over by the prey (S_1), host (S_4) of S_2, predator (S_2) till the time instant $t_{13}^*, t_{43}^*, t_{23}^*$, respectively and thereafter the dominance is reversed.

Case (b): If one root (λ_1) is negative while the other root (λ_2) is positive. Hence the state is unstable and the solution curves are illustrated in Figures-4 and 5.

Case (i): If $u_{30} < u_{20} < u_{10} < u_{40}$ and $a_2 < \mu_3 < a_1 < a_4$

In this case the natural birth rates of the host (S_3) of S_1, host (S_4) of S_2, predator (S_2) and the prey (S_1) are in ascending order. Initially the host (S_4) of S_2 dominates over the prey (S_1), predator (S_2) till the time instant t_{14}^*, t_{24}^*, respectively and thereafter the dominance is reversed.

Case (ii): If $u_{40} < u_{20} < u_{30} < u_{10}$ and $\mu_3 < a_1 < a_4 < a_2$

In this case the host (S_3) of S_1 has the least natural birth rate. Initially it is dominated over by the predator (S_2), host (S_4) of S_2 till the time instant t_{23}^*, t_{43}^*, respectively and thereafter the dominance is reversed. Also the prey (S_1) dominates over the predator (S_2) till the time instant t_{21}^* and the dominance gets reversed thereafter.

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Case (B): When $a_3 > a_{34}k_4$, i.e., the root μ_3 is positive. Hence the state is unstable and the solutions in this case are same as in case (A). The solution curves are illustrated in Figures-6 to 9.

Case (a): If the roots λ_1 and λ_2 noted to be negative.

Case (i): If $u_{10} < u_{40} < u_{30} < u_{20}$ and $a_4 < \mu_3 < a_2 < a_1$

In this case the natural birth rates of host (S_4) of S_1, host (S_3) of S_1, predator (S_2) and the prey (S_1) are in ascending order. Initially the predator (S_2), host (S_3) of S_1, host (S_4) of S_2 dominates over the prey (S_1) till the time instant $t^*_{12}, t^*_{13}, t^*_{14}$, respectively and thereafter the dominance is reversed.

Case (ii): If $u_{20} < u_{30} < u_{10} < u_{40}$ and $a_1 < a_4 < a_2 < \mu_3$

In this case the prey (S_1) has the least natural birth rate. Initially it is dominated over by the host (S_3) of S_1, predator (S_2) till the time instant t^*_{31}, t^*_{21} respectively and thereafter the dominance is reversed. Also the host (S_4) of S_2 dominates over the host (S_3) of S_1, predator (S_2) till the time instant t^*_{34}, t^*_{24} respectively and the dominance gets reversed thereafter.

Case (b): If one root (λ_1) is negative while the other root (λ_2) is positive. Hence the state is unstable.

Case (i): If $u_{30} < u_{10} < u_{20} < u_{40}$ and $\mu_3 < a_2 < a_1 < a_4$

In this case the natural birth rates of the host (S_3) of S_1, predator (S_2), prey (S_1) and the host (S_4) of S_2 are in ascending order. Initially the predator (S_2) dominates over the prey (S_1) till the time instant t^*_{12} and thereafter the dominance is reversed.

Case (ii): If $u_{40} < u_{20} < u_{30} < u_{10}$ and $a_4 < a_1 < a_2 < \mu_3$.

In this case host (S_4) of S_2 has the least natural birth rate. Initially the prey (S_1) dominates its host (S_3) and predator (S_2) till the time instant t^*_{31} and t^*_{21}, respectively and thereafter the dominance is reversed.

Case(C): when $a_3 = a_{34}k_4$ $(ie, \mu_3 = 0)$

In the case the equations (18), (19), (20) becomes

$$u_1 = \frac{(\mu_1-\lambda_2)(\bar{\gamma}_1+\bar{\mu}_1-u_{10})-a_{12}\bar{N}_1(\bar{\psi}_1+\bar{\psi}_2-u_{20})}{\lambda_2-\lambda_1}e^{\lambda_1 t}$$
$$+\frac{(\mu_1-\lambda_1)(\bar{\gamma}+\bar{\mu}-u_{10})-a_{12}\bar{N}_1(\bar{\psi}_1+\bar{\psi}_2-u_{20})}{\lambda_1-\lambda_2}e^{\lambda_2 t}$$
$$+\bar{\gamma}_1+\bar{\mu}_1\,\bar{e}^{a_4 t} \quad (26)$$

$$u_2 = \frac{(\mu_1-\lambda_2)(\bar{\gamma}_1+\bar{\mu}_1-u_{10})-a_{12}\bar{N}_1(\bar{\psi}_1+\bar{\psi}_2-u_{20})}{(\lambda_2-\lambda_1)a_{12}\bar{N}_1}(\mu_1-\lambda_1)e^{\lambda_1 t}$$
$$+\frac{(\mu_1-\lambda_1)(\bar{\gamma}+\bar{\mu}-u_{10})-a_{12}\bar{N}_1(\bar{\Psi}_1+\bar{\psi}_2-u_{20})}{(\lambda_1-\lambda_2)a_{12}\bar{N}_1}(\mu_1-\lambda_2)e^{\lambda_2 t}$$
$$+\bar{\psi}_1+\bar{\psi}_2\bar{e}^{a_4 t} \quad (27)$$

$$u_3 = u_{30},\ u_4 = \bar{\mu}_4(u_{40}-\bar{\mu}_4)e^{-a_4 t} \quad (28)$$

Where

$$\bar{\mu}_4 = \frac{a_{43}k_4u_{30}}{a_4},\ \bar{\beta} = a_{12}a_{24}\bar{N}_2(u_{40}-\bar{\mu}_4) \quad (29)$$

$$\bar{\alpha}_1 = (a_{12}a_{24}\bar{N}_2\bar{\mu}_4 - \mu_2 a_{13}u_{30})\bar{N}_1 \quad (30)$$

$$\bar{\gamma}_1 = \frac{\bar{\alpha}}{\mu_1\mu_2 + a_{12}a_{21}\bar{N}_1\bar{N}_2} \quad (31)$$

$$\bar{\mu}_1 = \frac{\bar{\beta}_1}{a_4^2 + (\mu_1+\mu_2)a_4 + \mu_1\mu_2 + a_{12}a_{21}\bar{N}_1\bar{N}_2} \quad (32)$$

$$\bar{\psi}_1 = \frac{a_{13}\bar{N}_1u_{30}+\mu_1\bar{\gamma}_1}{a_{12}\bar{N}_1},\ \bar{\psi}_2 = \frac{\mu_1\bar{\mu}_1 + a_4\bar{\mu}_1}{a_{12}\bar{N}_1} \quad (33)$$

And the solution curves are illustrated in Figures-10 to 13.

Case (a): If the roots λ_1 and λ_2 noted to be negative. Hence the state is neutrally stable.

Case (i): If $u_{10} < u_{20} < u_{40} < u_{30}$ and $a_4 < a_1 < a_2 < \mu_3$

In this case the natural birth rates of the host (S_4) of S_2, prey (S_1), predator (S_2) and the host (S_3) of S_1 are in ascending order. Initially the host (S_4) of S_2 dominates over the predator (S_2), prey (S_1) till the time instant t^*_{24}, t^*_{14}, respectively and thereafter the dominance is reversed. Further we notice that u_4 is asymptotic to u^*_4 which is evident from the equation (28).

Case (ii): If $u_{20} < u_{40} < u_{30} < u_{10}$ and $\mu_3 < a_4 < a_2 < a_1$

In this case the host (S_4) of S_2 has the least natural birth rate. Initially it is dominated over by the predator (S_2) till the time instant t^*_{24} and thereafter the dominance is reversed. Also the prey (S_1) dominates over the host (S_3) of S_1 till the time instant t^*_{31} and the dominance gets reversed thereafter.

Case (b): If one root (λ_1) is negative while the other root (λ_1) is positive. Hence the state is unstable.

Case (i): If $u_{30} < u_{40} < u_{10} < u_{20}$ and $a_1 < a_2 < a_4 < \mu_3$

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In this case the natural birth rates of the host (S_4) of S_1, host (S_3) of S_1, prey (S_1) and the predator (S_2) are in ascending order. Initially the host (S_4) of S_2 dominates over the host (S_3) of S_1 till the time instant t_{34}^* and thereafter the dominance is reversed.

Case (ii): If $u_{40} < u_{20} <_{10} < u_{30}$ and $a_2 < a_1 < \mu_3 < a_4$

In this case the host (S_4) of S_2 has the least natural birth rate. Initially the host (S_3) of S_1 dominates over the prey (S_1), predator (S_2) till the time instant t_{13}^*, t_{23}^*, respectively and thereafter the dominance is reversed.

6. TRAJECTORIES OF PERTUBATIONS

The trajectories in the $u_2 - u_4$ plane given by:

$$(a_4 + \mu_3)u_4 - a_{43}k_4u_3 = cu_3^{\frac{-a_4}{\mu_3}} \qquad (34)$$

where c is an arbitrary constant.

7. PERTUBATION GRAPHS

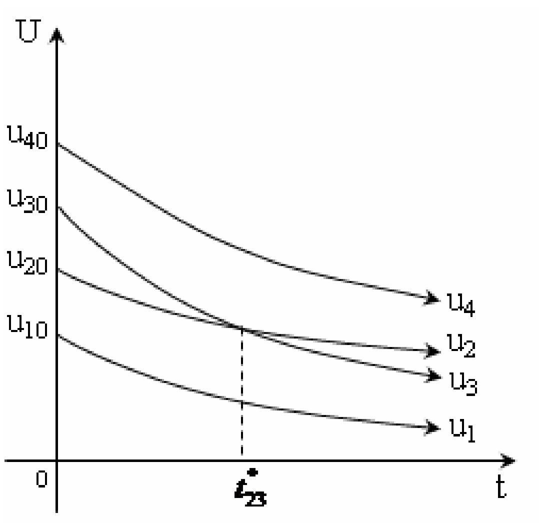

Figure-2. Graph of $u_{10} < u_{20} < u_{30} < u_{40}$; $a_1 < \mu_3 < a_2 < a_4$.

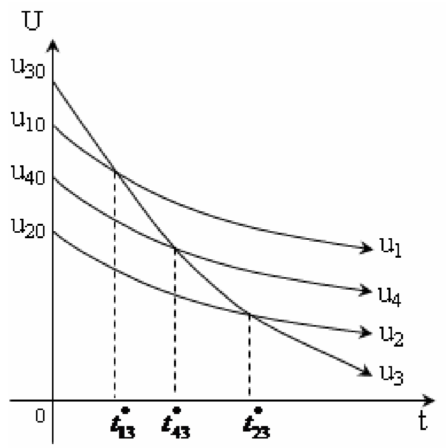

Figure-3. Graph of $u_{20} < u_{40} < u_{10} < u_{30}$; $\mu_3 < a_2 < a_4 < a_1$.

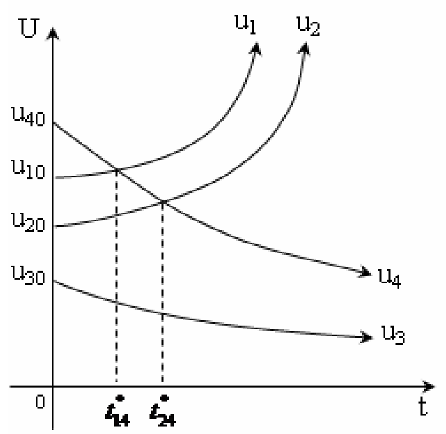

Figure-4. Graph of $u_{30} < u_{20} < u_{10} < u_{40}$; $a_2 < \mu_3 < a_1 < a_4$.

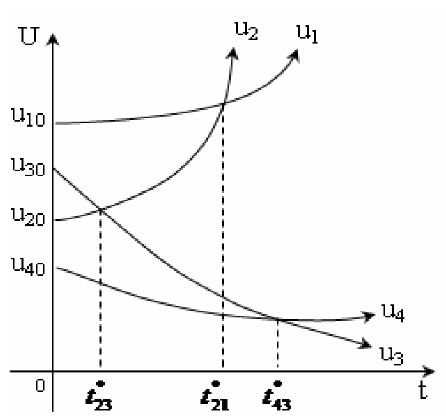

Figure-5. Graph of $u_{40} < u_{20} < u_{30} < u_{10}$; $\mu_3 < a_1 < a_4 < a_2$.

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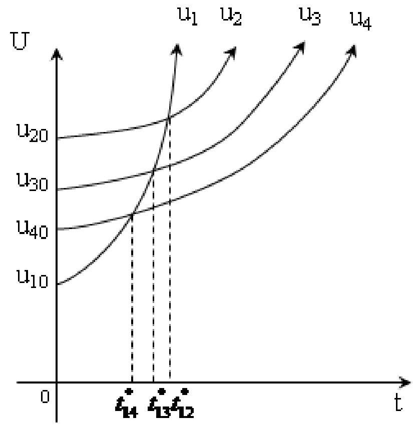

Figure-6. Graph of $u_{10} < u_{40} < u_{30} < u_{20}$;
$a_4 < \mu_3 < a_2 < a_1$.

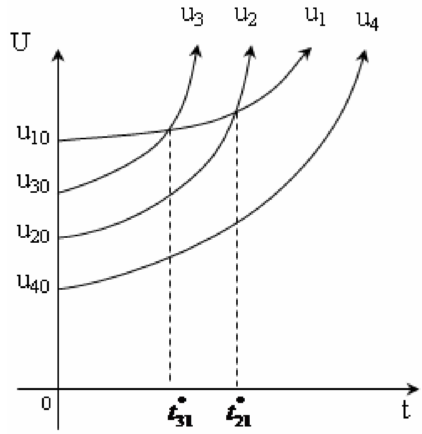

Figure-9. Graph of $u_{40} < u_{20} < u_{30} < u_{10}$;
$a_4 < a_1 < a_2 < \mu_3$.

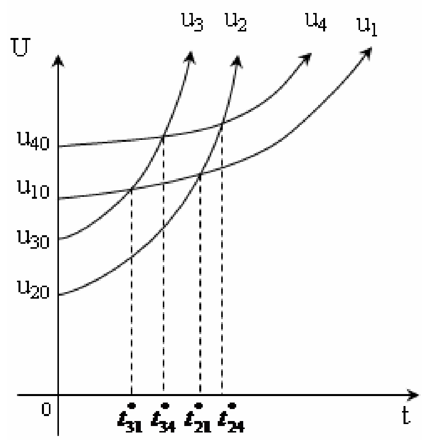

Figure-7. Graph of $u_{20} < u_{30} < u_{10} < u_{40}$;
$a_1 < a_4 < a_2 < \mu_3$.

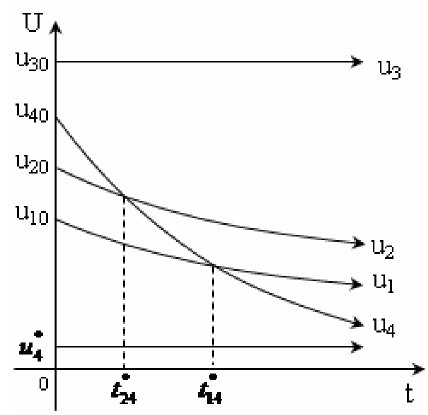

Figure-10. Graph of $u_{10} < u_{20} < u_{40} < u_{30}$;
$a_4 < a_1 < a_2 < \mu_3$.

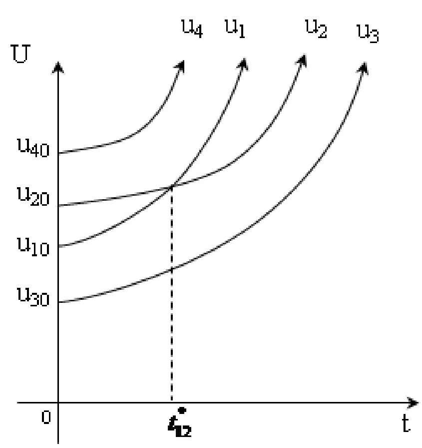

Figure-8. Graph of $u_{30} < u_{10} < u_{20} < u_{40}$;
$\mu_3 < a_2 < a_1 < a_4$.

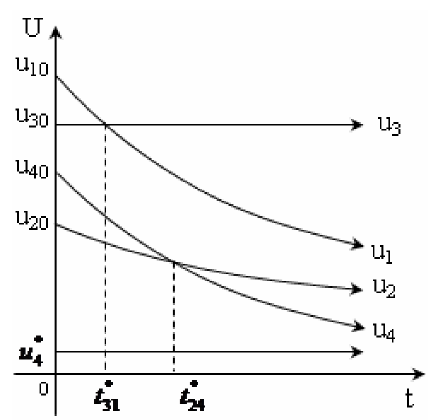

Figure-11. Graph of $u_{20} < u_{40} < u_{30} < u_{10}$;
$a_4 < a_2 < a_1 < \mu_3$.

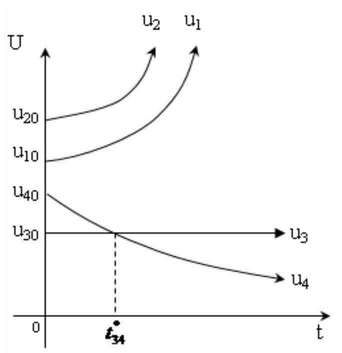

Figure-12. Graph of $u_{30} < u_{40} < u_{10} < u_{20}$; $a_1 < a_2 < a_4 < \mu_3$.

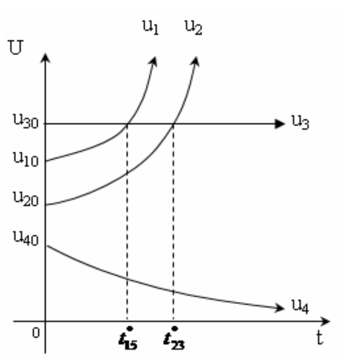

Figure-13. Graph of $u_{40} < u_{20} < u_{10} < u_{30}$; $a_2 < a_1 < \mu_3 < a_4$.

8. CONCLUSIONS

It is observed that the host of S_1 washed out state is conditionally stable. The stability of the other equilibrium states were already investigated and communicated to several International Journals.

9. FUTURE WORK

Investigate some relation-chains between the species such as Prey-Predation, Neutralism, Commensalism, Mutualism, Competition and Ammensalism between four species (S_1, S_2, S_3, S_4) with the population relations.

S_1 a Prey to S_2 and Commensal to S_3, S_2 is a Predator living on S_1 and Commensal to S_4, S_3 a Host to S_1, S_4 a Host to S_2 and S_3 a Prey to S_4, S_4 a Predator to S_3.

The present paper deals with the study on stability of the host of S_1 washed out states only of the above problem. The numerical solutions for the growth rate equations can also computed employing Runge Kutta fourth order method.

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