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REFORMULATION OF THE ELLIPTICAL FLOW GOVERNING EQUATION FOR A MORE COMPLETE WELL TEST DATA INTERPRETATION IN HORIZONTAL WELLS

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ABSTRACT

Before year 2000, elliptical flow regime was considered as a transition period found in a horizontal well transient test. It is recognized by a 0.36-slope (or 0.35-slope) tendency on the pressure derivative plot. Few researches have been conducted on the transient analysis for such flow regime. Since the first model does not provide a practical way of obtaining the reservoir horizontal permeability, a new model which is function of the reservoir length along the *x*-direction, reservoir thickness, horizontal wellbore length, well radius and horizontal reservoir permeability, was presented in 2004. Our experience, however, has indicated to us that this model sometimes fails to provide accurate values of horizontal permeability. Therefore, the first model presented in the literature, which depends upon the permeabilities in the horizontal direction, has been retaken in this study with a slight modification and successfully tested in many scenarios. The conventional straight-line and *TDS* methods are used as interpretation techniques for crude and gas flow. Both real time and pseudotime were implemented for gas flow.

Keywords: horizontal well, elliptical flow, radial, anisotropy, pseudorradial, permeability, intersection points, characteristic lines.

1. INTRODUCTION

When permeability varies from one direction to another, the reservoir is considered to be anisotropic. Permeability measured parallel to a bedding plane may be several times larger than that measured perpendicular to the bedding plane. It is normally accepted that most reservoir rocks have a lower vertical permeability than horizontal permeability. This anisotropy has been recognized and is often factored into the analysis of partially completed vertical well tests, for example. While it is agreed that horizontal permeability anisotropy can occur, it is routinely neglected in vertical well test analysis but it is more common in horizontal well test analysis.

Considerations on the elliptical flow in horizontal wells are fairly new. The first work of this kind showed up in 2000, Isaaka *et al*. (2000), in which they introduced the elliptical flow regime in horizontal wells. Their pressure derivative governing equation was a function of the areal permeability and the k_x/k_y ratio. They found out that the horizontal permeability anisotropy has no influence on the duration of this flow period but it affects its starting time. They did not use non-linear regression analysis as the interpretation technique, but use a point on the pressure derivative during the elliptical flow to estimate the permeability anisotropy ratio with an expression they provided in their work. They stated that the slope of the pressure derivative during such flow regime had a value of 0.35 which is slightly different to the value of 0.36 as taken by Tiab (1994) for vertical wells and and Escobar *et al*. (2004) for horizontal wells.

Chacon *et al*. (2004) introduced a new pressure derivative equation which depends upon the reservoir length along the *x*-direction, reservoir thickness, horizontal wellbore length, well radius and horizontal reservoir permeability. This flow regime corresponds to the transition period between early linear and pseudorradial flow regimes. It is characterized for a 0.36-slope line on the pressure derivative log-log plot (Figure-1). Escobar *et al*. (2004) obtained the governing pressure equation by integrating the model presented by Chacon *et al*. (2004). They applied the *TDS*, Tiab (1993), methodology for characterizing the elliptical flow regime so new equations using characteristic lines and points found on the pressure and pressure derivative plot were developed to obtain areal permeability, $(k_xk_y)^{0.5}$, the reservoir length along the *x*-direction, h_x, permeability in the *y*-direction, k_y, and the geometrical skin factor, $s_{Ell,}$ caused by the convergence from early-linear flow to elliptical flow regime.

The most recent research on this issue was presented by Escobar and Montealegre (2008). Using the model developed by Chacon *et al*. (2008), they implemented the conventional-straight line method for characterizing the elliptical flow regime, so the reservoir areal permeability and elliptical skin factor can be estimated.

However, in any of the above two cases, i.e., *TDS* or conventional analysis, it has been noted that these expressions fail sometimes to provide accurate values of the parameters, then, a new formulation is presented here. Therefore, the model proposed by Isaaka *et al*. (2000) was used for a better characterization of the elliptical-flow by means of both the *TDS* technique and conventional analysis. However, a slight variation of the model was introduced. The 0.35 exponent was changed for 0.36. Practical examples are given to demonstrate the accuracy of the presented methodology.

2. FUNDAMENTAL EQUATIONS

The dimensionless quantities used in this work for the oil phase are defined as,

$$P_D = \frac{\sqrt{k_x k_y}\, h_z}{141.2 q \mu B} \Delta P \quad (1)$$

$$t_D = \frac{0.0002637\sqrt{k_x k_y}}{\phi \mu c_t r_w^2} t \quad (2)$$

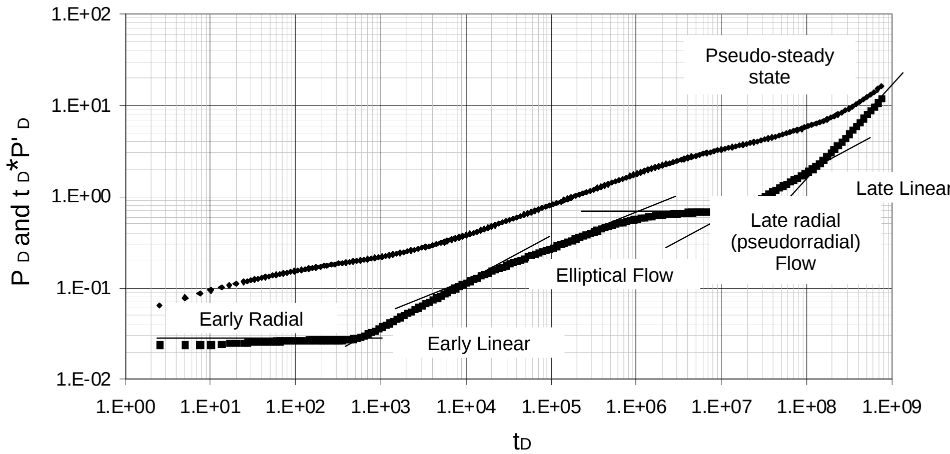

Figure-1. Identification of the early-radial, early-linear, elliptical and pseudorradial flow regimes on the pressure and pressure derivative plot.

Pressure is implicit in Equation (2) since for gases, both viscosity and compressibility are functions of pressure. In fact, that is why pseudotime was introduced. For gas flow, the definition of the pseudopressure function, Al-Hussainy *et al*. (1966), is given by:

$$m_D(P) = \frac{\sqrt{k_x k_y}\, h \Delta m(P)}{1422.52 q_{sc} T} \quad (3)$$

To account for variations of the total compressibility-viscosity product in gas flow behavior, Agarwal (1979) introduced the pseudotime concept, as follows:

$$t_{Da} = \left(\frac{0.0002637\sqrt{k_x k_y}}{\phi r_w^2} \right) t_a(P) \quad (4)$$

3. MATHEMATICAL DEVELOPMENT

There are two types of elliptical flow regimes that may occur in horizontal well. The first one is in the vertical plane at early stage which transit into early radial as can be observed in Figure-2. It is not practical since it cannot be observed in any field test because of time and wellbore storage. The second elliptical flow occurs in a region before the pseudorradial flow regime. This depends upon the horizontal well length compare with reservoir width in the direction of the horizontal length, and the horizontal permeability. This type of elliptical flow which we dwelled on will occur if the horizontal length is small compared with the reservoir width in the horizontal well direction. The index 0.35 or 0.36 will vary depending on this feature. It is not a fixed index. This elliptical flow is purely in horizontal plane and depends upon horizontal length compare with the reservoir width and horizontal permeability anisotropy. For such case, Chacon *et al*. (2004) proposed the following pressure governing equation:

$$P_D = \left(\frac{h_x h_z}{L_w r_w} \right)^{0.72} \frac{t_D^{\ 0.36}}{985.05 \times \pi^5} + S_{Ell} \quad (5)$$

This model, however, fails sometimes to provide accurate values of the estimated parameters, then, the model proposed by Isaaka *et al*. (2000) was used using an index of 0.36 to be conservative with the original work of Tiab (1994) and Escobar *et al*. (2004),

$$t_D * P_D' = 0.7241 \left(\sqrt{\frac{k_x}{k_y}}\ t_D \right)^{0.35} \quad (6)$$

As seen in Equation 6, the pressure derivative is a function of the horizontal anisotropy, k_y/k_x, and the areal permeability, $(k_x k_y)^{0.5}$, making difficult to solve for permeability in any direction, *x* or *y*, then, a new dimensionless pressure equation is presented here:

$$P_D = 2.1225 \left(\frac{r_w}{L_w} \right)^{0.72} \left(\sqrt{\frac{k_x}{k_y}} t_D \right)^{0.36} + s_{Ell} \quad (7)$$

After plugging Equations 1 y 2 into Equation 7 yields,

$$\Delta P_{wf} = \frac{15.427 k_x^{0.36} q \mu^{0.64} B}{\sqrt{k_x k_y}\, h_z (\phi c_t)^{0.36} L_w^{0.72}} t^{0.36} + \frac{141.2 q \mu B}{\sqrt{k_x k_y}\, h_z} s_{Ell} \quad (8)$$

$$\Delta P_{wf} = m_{Ell}\ t^{0.36} + b_{Ell}$$

For pressure buildup testing, after applying the superposition concept, the following expression is obtained:

$$\Delta P_{ws} = \frac{15.427 k_x^{0.36} q \mu^{0.64} B}{\sqrt{k_x k_y} h_z (\phi c_t)^{0.36} L_w^{0.72}} \left[(t_p + \Delta t)^{0.36} - \Delta t^{0.36} \right] \quad (9)$$

Equation 8 and 9 suggests that a Cartesian plot of ΔP vs. $t^{0.36}$ (for drawdown) or ΔP vs. $[(t_p+\Delta t)^{0.36} - \Delta t^{0.36}]$ (for buildup) gives a linear trend which slope and intercept allow for the estimation of then horizontal anisotropy and elliptical skin factor, such as:

$$\sqrt{k_x k_y} = \frac{15.427 q \mu^{0.64} B k_x^{0.36}}{h_z (\phi c_t)^{0.36} L_w^{0.72} m_{Ell}} \quad (10)$$

$$s_{Ell} = \frac{\sqrt{k_x k_y} h_z b_{Ell}}{141.2 q \mu B} \quad (11)$$

Characteristics points and lines

a) Taking derivative to Equation 7 with respect to dimensionless time yields:

$$t_D * P'_D = 0.7641 \left(\frac{r_w}{L_w} \right)^{0.72} \left(\sqrt{\frac{k_x}{k_y}} t_D \right)^{0.36} \quad (12)$$

After plugging Equations 1 y 2 into Equation 12 and solving for the horizontal permeability yields,

$$\sqrt{k_x k_y} = \frac{5.554 q \mu^{0.64} B k_x^{0.36}}{h_z (\phi c_t)^{0.36} L_w^{0.72} (t * \Delta P')_{Ell}} t_{Ell}^{0.36} \quad (13)$$

We can also solve for the horizontal well length, L_w:

$$L_w = \left[\frac{5.554 q \mu^{0.64} B k_x^{0.36}}{h_z (\phi c_t)^{0.36} (k_x k_y)^{0.5} (t * \Delta P')_{Ell}} t_{Ell}^{0.36} \right]^{\frac{1}{0.72}} \quad (14)$$

b) The skin factor caused by the elliptical flow regime results from dividing the dimensionless pressure equation, Equation 7, by the dimensionless pressure derivative, Equation 12. After plugging the dimensionless quantities, Equations 1 and 2, into the above expression and solving for the skin factor we obtain:

$$s_{Ell} = \left[\frac{\Delta P_{Ell}}{(t * \Delta P')_{Ell}} - \frac{1}{0.36} \right] \frac{1}{25.424} \left(\frac{k_x t_{Ell}}{\phi \mu c_t L_w^2} \right)^{0.36} \quad (15)$$

Being ΔP_{Ell} and $(t*\Delta P')_{Ell}$ are the pressure and pressure derivative values read on the elliptical flow line at any convenient time, t_{Ell}. The total skin factor is the summation of the mechanical skin (s_m), and the geometrical skin factors: the *x*-direction pseudoskin, the *z*-direction pseudoskin and the skin caused by the elliptical flow regime.

c) An equation for the *z*-direction permeability results from the intersection point of the early radial line with the elliptical lines is given as:

$$k_z = \left[\frac{h_z k_x^{0.14}}{0.07866 L_w^{0.28}} \left(\frac{\phi \mu c_t}{t_{i,er-ell}} \right)^{0.36} \right]^2 \quad (16)$$

d) Permeability in the *x*-direction can be estimated from the intersection point between the early- linear and elliptical flow regime:

$$k_x = 9.279 \left(\frac{\phi \mu c_t L_w^2}{t_{i,el-ell}} \right) \quad (17)$$

e) The intersection point of the elliptical flow regime and the pseudorradial flow regime, or late-radial flow regime, also provides an expression to estimate horizontal well length:

$$L_w = \left[0.07866 \left(\frac{k_x t_{i,ell-pr}}{\phi \mu c_t} \right)^{0.36} \right]^{\frac{1}{0.72}} \quad (18)$$

f) The last intersection point corresponds to that of the late linear and elliptical flow regimes. This also provides an expression to estimate the *x*-direction permeability:

$$k_x = 9.279 \frac{L_w^{0.72}}{h_x} \left(\frac{\phi \mu c_t}{t_{i,ell-ll}} \right) \quad (19)$$

The expressions for gas flow with both rigorous time and pseudo time are provided in appendix A.

4. APPLICATIONS

4.1. Example 1

Goode and Thambynayagam (1987) presented an example of a pressure buildup test of a horizontal oil well completed in the center of a semi-infinite anisotropic reservoir. Relevant information is given in Table-1. Determine *x*-, *y*- and *z*-direction permeabilities and skin factor.

Solution

Three well-defined flow regimes are observed in Figure-2: early radial, early linear and elliptical. The pseudorradial flow regime is seen only at the final period of time and it is not well defined. The following information was read from Figure-2:

$(t*\Delta P')_{ell}$ = 49.35 psi t_{ell} = 10.79 hr t_{er-ell} = 0.112 hr

$Tel\text{-}ell$ = 0.879 hr t_{er-el} = 0.18 hr t_{ell-pr} = 0.18 hr

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Tel-pr = 28 hr

Determine k_z and k_x from the intersection times of the linear line with either early-radial or pseudorradial straight lines.

$$k_z = 301.77\phi\mu c_t \frac{h_z^2}{t_{i,er-el}} = (301.77)(0.1)(1.5)(3x10^{-5})\frac{60^2}{0.18} = 27.16 \text{ md}$$

$$k_x = \frac{301.77\phi\mu c_t L_w^2}{t_{i,el-pr}} = \frac{(301.77)(0.1)(1.5)(3x10^{-5})(1000^2)}{28} = 48.49 \text{ md}$$

Estimate $(k_x k_y)^{0.5}$ using Equation 13,

$$\sqrt{k_x k_y} = \frac{5.554 q \mu^{0.64} B k_x^{0.36}}{h_z (\phi c_t)^{0.36} L_w^{0.72} (t*\Delta P')_{Ell}} t_{Ell}^{0.36}$$

$$\sqrt{k_x k_y} = \frac{5.554(3000)(1.5)^{0.64}(1.5)(48.49)^{0.36}}{(60)\left[(0.1)(3x10^{-5})\right]^{0.36}(1000)^{0.72}(49.35)} 10.79^{0.36} = 70.15 \text{ md}$$

Verify k_z and L_w from the intersection times of the elliptical line with either early radial line or pseudorradial line, $t_{i,er-ell}$ (Equation 16) and $t_{i,ell-pr}$ (Equation 18).

$$k_z = \left[\frac{h_z k_x^{0.14}}{0.07866 L_w^{0.28}}\left(\frac{\phi\mu c_t}{t_{i,er-ell}}\right)^{0.36}\right]^{2\frac{1}{0.64}}$$

$$k_z = \left[\frac{(60)(48.49)^{0.14}}{0.07866(1000)^{0.28}}\left(\frac{(0.1)(1.5)(3x10^{-5})}{0.112}\right)^{0.36}\right]^2 = 24.64 \text{ md}$$

$$L_w = \left[0.07866\left(\frac{k_x t_{i,ell-pr}}{\phi\mu c_t}\right)^{0.36}\right]^{\frac{1}{0.72}}$$

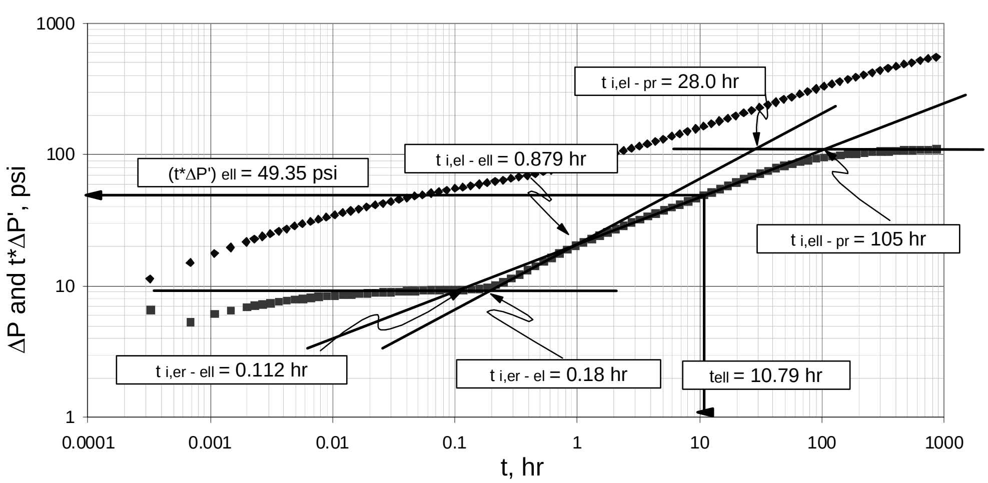

Figure-2. Pressure and pressure derivative for example 1.

$$L_w = \left[0.07866\left(\frac{(48.49)(105)}{0.1(3\times10^{-5})(1.5)}\right)^{0.36}\right]^{\frac{1}{0.72}} = 984 \text{ ft}$$

Verify k_x from Equation 17,

$$k_x = 9.279\left(\frac{\phi\mu c_t L_w^2}{t_{i,el-ell}}\right)$$

$$k_x = 9.279\left(\frac{(0.1)(3x10^{-5})(1.5)(1000)^2}{0.879}\right) = 47.5 \text{ md}$$

The mechanical skin factor is found from the early radial flow period:

$$s_m = \frac{1}{2}\left[\frac{\Delta P_{er}}{(t*\Delta P')_{er1hr}} - \ln\left(\frac{\sqrt{k_y k_z} t_{er}}{\phi\mu c_t r_w^2}\right) + 7.43\right] = \frac{1}{2}\left[\frac{55.27}{9.41} - \ln\left(\frac{\sqrt{(100x25)}(0.1)}{(0.1)(1.5)(0.354)^2(3\times10^{-5})}\right) + 7.43\right] = -1.35$$

Estimate the skin factor from the linear-flow regime,

$$s_m + s_z = \frac{0.029}{h_z}\sqrt{\frac{k_z t_{el}}{\phi\mu c_t}}\left[\frac{\Delta P_{el}}{(t*\Delta P')_{el}} - 2\right] = \frac{0.029}{60}\sqrt{\frac{(25)(0.53)}{(0.1)(1.5)(3\times10^{-5})}}\left[\frac{73.84}{15.405} - 2\right] = 2.32$$

The geometrical skin factor caused by the elliptical flow regime is found from Equation 15:

$$s_{Ell} = \left[\frac{\Delta P_{Ell}}{(t*\Delta P')_{Ell}} - \frac{1}{0.36}\right]\frac{1}{25.424}\left(\frac{k_x t_{Ell}}{\phi\mu c_t L_w^2}\right)^{0.36}$$

$$S_{Ell} = \left[\frac{157.25}{46.72} - \frac{1}{0.36}\right]\frac{1}{25.424}\left(\frac{(47.5)(9.28)}{(0.1)(1.5)(3\times10^{-5})(1000)^2}\right)^{0.36} = 0.12$$

4.2. Example 2

Determine $(k_x k_y)^{0.5}$ and skin factors for the previous example using the conventional method.

Solution

As shown in Figure-2, the elliptical flow characterized by a 0.36-slope line on the pressure derivative trend indicates that this flow exists approximately between 2 and 50 hrs. At the same period of time, the slope and intercept of the linear plot of Figure-3 were m_{Ell} = 58.33 psi/hr$^{0.36}$ and b_{Ell} = 27.1 psi.

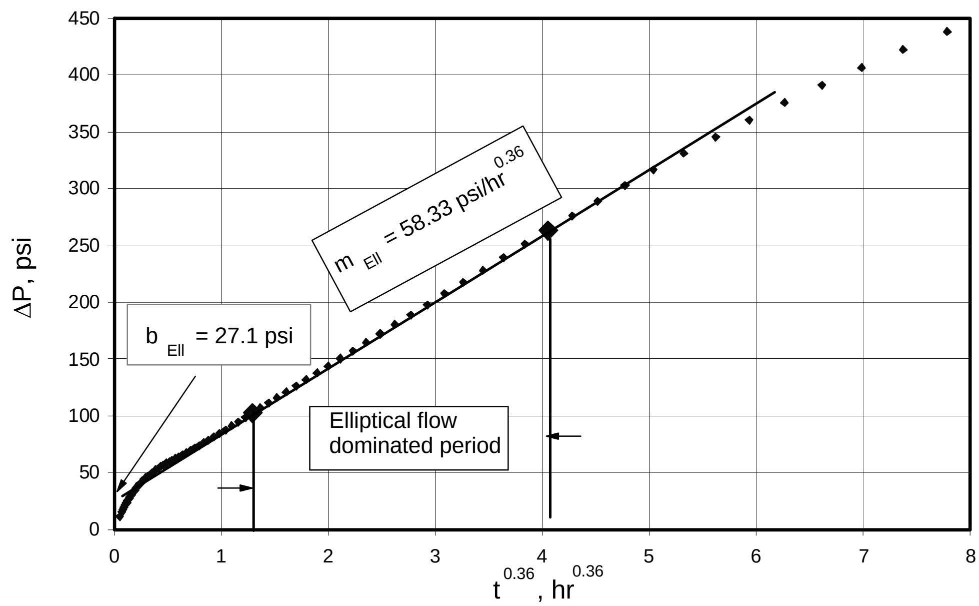

Figure-3. Cartesian plot of pressure drop vs. time to the power 9/25 for example.

(Kxky) $^{0.5}$ is obtained From Equation 10,

$$\sqrt{k_x k_y}=\frac{15.427 q\mu^{0.64} B k_x^{0.36}}{h_z(\phi c_t)^{0.36} L_w^{0.72} m_{Ell}}=\frac{15.427(3000)(1.5)^{0.64}(1.5)(50)^{0.36}}{(60)\left[(0.1)(3\times10^{-5})\right]^{0.36}(1000)^{0.72}(58.33)}=70.8\text{ md}$$

The skin factor caused by the elliptical flow regime is obtained from Equation 11,

$$s_{Ell}=\frac{\sqrt{k_x k_y}\,h_z b_{Ell}}{141.2 q\mu B}=\frac{(70.8)(60)(27.1)}{141.2(3000)(1.5)(1.5)}=0.121$$

4.3. Example 3

Ozkan *et al*. (1989) presented a well pressure test for a horizontal oil well completed at the center of a semi-infinite anisotropic reservoir. Table-1 shows information of reservoir and well properties. Determine *x*-, *y*- and *z*-direction permeabilities.

Solution

The pressure and pressure derivative curves of Figure-4 exhibit five well-defined flow regimes as follows: early radial, early linear, elliptical, pseudorradial and pseudosteady state. The following information is read from Figure-4,

$(t^*\Delta P')_{ell}$ = 8.4 psi	t_{ell} = 30.2 hr	$t_{er\text{-}ell}$ = 1.6 hr
Tell-pr = 860 hr	$(t^*\Delta P')_{pr}$ = 27.7 psi	$(t^*\Delta P')_{er}$ = 2.9 psi
Tel-pr = 240 hr	$t_{er\text{-}el}$ = 2.5 hr	$t_{el\text{-}ell}$ = 8.0 hr

Determine k_z and k_x from the intersection times of the linear line with either early-radial or pseudorradial straight lines.

$$k_z=301.77\phi\mu c_t\frac{h_z^2}{t_{i,er-el}}=(301.77)(0.24)(1.2)\left(5\times10^{-5}\right)\frac{84^2}{2.5}=12.26\text{ md}$$

$$k_x=\frac{301.77\phi\mu c_t L_w^2}{t_{i,el-pr}}=\frac{(301.77)(0.24)(1.2)\left(5x10^{-5}\right)(2626)^2}{240}=124.85\text{ md}$$

Estimate $(k_x k_y)^{0.5}$ using Equation 13,

$$\sqrt{k_x k_y}=\frac{5.554 q\mu^{0.64} B k_x^{0.36}}{h_z(\phi c_t)^{0.36} L_w^{0.72}(t^*\Delta P')_{Ell}}t_{Ell}^{0.36}$$

$$\sqrt{k_x k_y}=\frac{5.554(5000)(1.2)^{0.64}(1.12)(124.85)^{0.36}}{(84)\left(0.24x5x10^{-5}\right)^{0.36}(2626)^{0.72}(8.4)}30.2^{0.36}=195.89\text{ md}$$

Verify $(k_x k_y)^{0.5}$ from the pseudorradial flow regime, Engler and Tiab (1996),

$$\sqrt{k_x k_y}=\frac{70.6 q\mu B}{h_z(t^*\Delta P')_{pr}}=\frac{70.6(5000)(1.2)(1.12)}{(84)(27.7)}=203.9\text{ md}$$

Estimate k_y from Early radial flow regime

$$k_y = \left[\frac{70.6 q\mu B}{\sqrt{k_z} L_w (t^*\Delta P')_{er1hr}}\right]^2 = \left[\frac{70.6(5000)(1.2)(1.12)}{\sqrt{12.26}(2626)(2.9)}\right]^2 = 316.48 \text{ md}$$

Verify k_z and L_w from the intersection times of the elliptical line with either early radial line or pseudorradial line, $t_{i,er-ell}$ (Equation 16) and $t_{i,ell-pr}$ (Equation 18).

$$k_z = \left[\frac{h_z k_x^{0.14}}{0.07866 L_w^{0.28}}\left(\frac{\phi\mu c_t}{t_{i,er-ell}}\right)^{0.36}\right]^{2\frac{1}{0.64}}$$

$$k_z = \left[\frac{(84)(124.85)^{0.14}}{0.07866(2626)^{0.28}}\left(\frac{(0.24)(1.2)(5x10^{-5})}{1.6}\right)^{0.36}\right]^2 = 12.48 \text{ md}$$

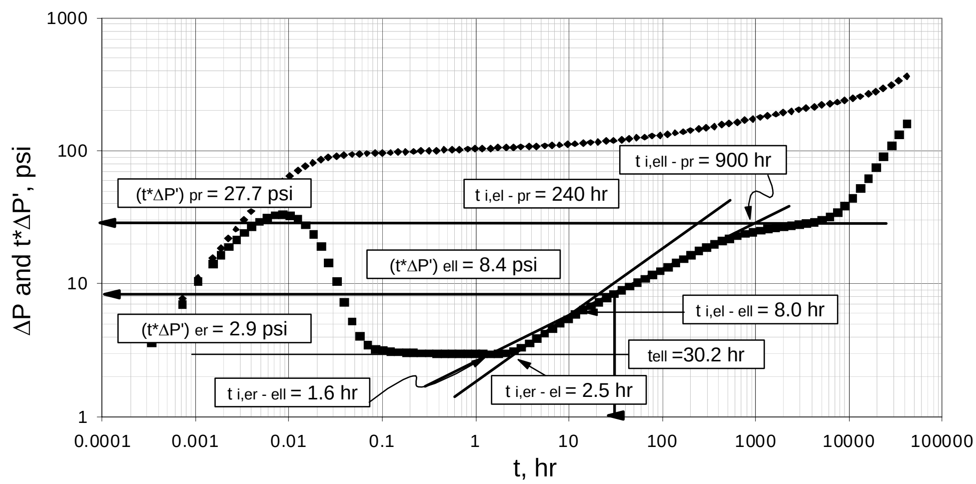

Figure-4. Pressure and pressure derivative for example 3.

$$L_w = \left[0.07866\left(\frac{k_x t_{i,ell-pr}}{\phi\mu c_t}\right)^{0.36}\right]^{\frac{1}{0.72}}$$

$$L_w = \left[0.07866\left(\frac{(124.85)(900)}{0.24(5\times10^{-5})(1.2)}\right)^{0.36}\right]^{\frac{1}{0.72}} = 2584 \text{ ft}$$

Verify k_x from Equation 17,

$$k_x = 9.279\left(\frac{\phi\mu c_t L_w^2}{t_{i,el-ell}}\right)$$

$$k_x = 9.279\left(\frac{(0.24)(5x10^{-5})(1.2)(2626)^2}{8.0}\right) = 115.18 \text{ md}$$

4.4. Example 4

A well pressure test was simulated in a gas reservoir using the information from Table-1. Pressure and pressure derivative data are reported in Figure-5. Determine *x*-, *y*- and *z*-direction permeabilities.

Solution

The log-log plot of pseudopressure and pressure derivative against pseudotime is given in Figure-5. From that plot the following information was read:

$(t_a^*\Delta m(P)')_{ell} = 69331$ psi²/cp $t_{a,ell} = 9.75x10^5$ psi hr/cp $t_{a,er-ell} = 9000$ psi hr/cp
$t_{a,el-ell} = 150000$ psi hr/cp $t_{a,er-el} = 19000$ psi hr/cp $t_{a,ell-pr} = 1.8x10^7$ psi hr/cp
$t_{a,el-pr} = 5x10^6$ psi hr/cp

Determine k_z and k_x from the intersection times of the linear line with either early radial or pseudorradial lines,

$$k_z = 301.77\phi\frac{h_z^2}{t_a(P)_{i,er-el}} = (301.77)(0.25)\frac{50^2}{19000} = 9.93 \text{ md}$$

$$k_x = \frac{301.77\,\phi\, L_w^2}{t_a(P)_{i,el-pr}} = \frac{(301.77)(0.25)(1200^2)}{5\times10^6} = 21.73 \text{ md}$$

Estimate $(k_x k_y)^{0.5}$ using the Equation A.7,

$$\sqrt{k_x k_y} = \frac{57.4009 q_g T k_x^{0.36}}{h_z \phi^{0.36} L_w^{0.72}\left(t_a(P)^*\Delta m(P)'\right)_{Ell}} t_a(P)_{Ell}^{0.36}$$

$$\sqrt{k_x k_y} = \frac{57.4009(1000)(250+460)(21.73)^{0.36}}{(50)(0.25)^{0.36}(1200)^{0.72}(69331)}(9.75x10^5)^{0.36} = 50.98 \text{ md}$$

Estimate k_y from early radial flow regime,

$$k_y = \left[\frac{711.26 q_g T}{L_w\sqrt{k_z}\left(t_a(P)^*\Delta m(P)'\right)_{er}}\right]^2 = \left[\frac{711.26(1000)(250+460)}{\sqrt{9.93}(1200)(12500)}\right]^2 = 114.14 \text{ md}$$

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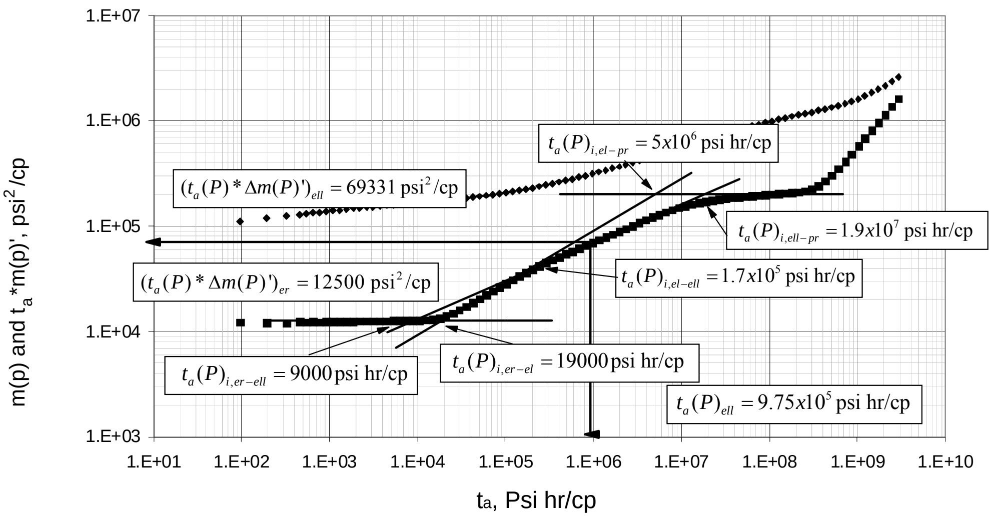

Figure-5. Pseudopressure and Pseudopressure derivative for example 4.

Verify k_x from Equation A.10,

$$k_x = 9.279\left(\frac{\phi L_w^2}{t_a(P)_{i,el-ell}}\right)$$

$$k_x = 9.279\left(\frac{(0.25)(1200)^2}{1.7x10^5}\right) = 19.65 \text{ md}$$

Verify k_z and L_w from the intersection times of the elliptical straight line with either early-radial or pseudorradial line, $t_{i,er-ell}$ (Equation A.9) and $t_{i,ell-pr}$ (Equation A.11).

$$k_z = \left[\frac{h_z k_x^{0.14}}{0.07866 L_w^{0.28}}\left(\frac{\phi}{t_a(P)_{i,er-ell}}\right)^{0.36}\right]^2$$

$$k_z = \left[\frac{(50)(19.65)^{0.14}}{0.07866(1200)^{0.28}}\left(\frac{0.25}{9000}\right)^{0.36}\right]^2 = 9.2 \text{ md}$$

$$L_w = \left[0.07866\left(\frac{k_x t_a(P)_{i,ell-pr}}{\phi}\right)^{0.36}\right]^{\frac{1}{0.72}}$$

$$L_w = \left[0.07866\left(\frac{(19.65)1.9\times10^7}{(0.25)}\right)^{0.36}\right]^{\frac{1}{0.72}} = 1131 \text{ ft}$$

5. ANALYSIS OF RESULTS

The results obtained from the worked problems agree very well with either those from the literature sources or the input values used for simulation implying the accuracy of the new proposed governing equation, therefore, there is no need of simulating examples 1 and 3 for verification.

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Table-1. Reservoir and well parameters for examples.

Parameter	Value, oil flow		Value, gas flow
	Example 1 and 2	Example 3	Example 4
k_x, md	50	122	22
k_y, md	100	315	120
k_z, md	25	12	10
q, BPD	3000	5000	1000 MSCF/D
μ, cp	1.5	1.2	0.0242
B, bbl/STB)	1.5	1.12	-
h_s, ft	30	42	25
L_w, ft	1000	2626	1200
ϕ, %	10	24	25
c_t, (psi^{-1}	3.0x10^{-5}	5.0x10^{-5}	1.4693x10^{-4}
h_z, ft	60	84	50
h_x, ft	+13500	-	-
r_w, ft	0.354	0.35	0.5
s_m	- 1.4	5.0	1.5
T, °F	-		250

Nomenclature

B	Oil formation factor, bbl/STB
c_t	Compressibility, 1/psi
h	Formation thickness, ft
k	Permeability, md
k_h	Areal permeability $(k_x k_y)^{0.5}$, md
L_w	Horizontal well length, ft
$m(P)$	Pseudopressure function, psi^2/cp
P	Pressure, psi
P_D'	Dimensionless pressure derivative
P_D	Dimensionless pressure
q	Flow rate, bbl/D. For gas reservoirs the units are Mscf/D
r_w	Well radius, ft
s	Skin factor
s_m	Mechanical skin factor
s_x	Geometrical skin factor or x-direction pseudoskin factor
s_z	Geometrical skin factor or z-direction pseudoskin factor
T	Reservoir temperature, ºR
t	Time, hr
$t*\Delta P'$	Pressure derivative function, psi
$t*\Delta m(P)'$	Pseudopressure derivative function, psi^2/cp
$t_a(P)$	Pseudotime function, psi h/cp
$t*\Delta m(P)'$	Pseudopressure derivative function with respect to rigorous/real time
$t_a*\Delta m(P)'$	Pseudopressure derivative function with respect to pseudotime
t_D	Dimensionless time

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Greek

Δ	Change, drop
Δt	Flow time, hr
ϕ	Porosity, fraction
μ	Viscosity, cp

Suffices

1 *hr*	Time of 1 hr
D	Dimensionless
el	Early linear flow period
ell	Elliptical flow period
er	Early radial flow period
g	Gas
h	Horizontal
i	Intersection
ll	Late linear flow period
pr	Pseudoradial flow period
o	Oil
sc	Standard conditions
t	Total
w	Well
x	*x*-direction index
y	*y*-direction index
z	*z*-direction index

6. CONCLUSIONS

An existing governing equation for elliptical flow presented in horizontal wells was utilized and slightly modified to provide a more complete *TDS* and conventional methodologies for the estimation of the horizontal permeability, *x*-, and *z* direction permeabilities, elliptical skin factor and horizontal wellbore length are accurately estimated for oil and gas reservoirs. These equations were successfully tested with examples provided in the literature and one synthetic example.

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APPENDIX A. GAS RESERVOIRS' EQUATIONS

- **Dimensionless pressure and pressure derivative**

$$m(P)_D = 2.1775\left(\frac{r_w}{L_w}\right)^{0.72}\left(\sqrt{\frac{k_x}{k_y}}t_D\right)^{0.36} + s_{Ell} \tag{A.1}$$

$$t_D * m(P)'_D = 0.7839\left(\frac{r_w}{L_w}\right)^{0.72}\left(\sqrt{\frac{k_x}{k_y}}t_D\right)^{0.36} \tag{A.2}$$

- **Conventional method**

$$\sqrt{k_x k_y} = \left(\frac{159.447 q_g k_x^{0.36} T}{h_z\left[\phi\left(\mu c_t\right)_i\right]^{0.36} L_w^{0.72} m_{Ell}}\right) \tag{A.3}$$

$$s_{Ell} = \frac{\sqrt{k_x k_y} h_z b_{Ell}}{1422.52 q_g T} \tag{A.4}$$

- ***TDS* rigorous time**

$$\sqrt{k_x k_y} = \frac{57.4009 q_g T k_x^{0.36}}{h_z\left[\phi\left(\mu c_t\right)_i\right]^{0.36} L_w^{0.72}\left(t * \Delta m(P)'\right)_{Ell}} t_{Ell}^{0.36} \tag{A.5}$$

$$s_{Ell} = \left[\frac{\Delta m(P)_{Ell}}{\left(t * \Delta m(P)'\right)_{Ell}} - \frac{1}{0.36}\right]\frac{1}{25.424}\left(\frac{k_x t_{Ell}}{\phi \mu c_t L_w^2}\right)^{0.36} \tag{A.6}$$

- ***TDS* pseudotime**

$$\sqrt{k_x k_y} = \frac{57.4009 q_g k_x^{0.36} T}{h_z \phi^{0.36} L_w^{0.72}\left(t_a(P) * \Delta m(P)'\right)_{Ell}} t_a(P)_{Ell}^{0.36} \tag{A.7}$$

$$s_{Ell} = \left[\frac{\Delta m(P)_{Ell}}{\left(t_a(P) * \Delta m(P)'\right)_{Ell}} - \frac{1}{0.36}\right]\frac{1}{25.424}\left(\frac{k_x t_a(P)_{Ell}}{\phi L_w^2}\right)^{0.36} \tag{A.8}$$

$$k_z = \left[\frac{h_z k_x^{0.14}}{0.07866 L_w^{0.28}}\left(\frac{\phi}{t_a(P)_{i,er-ell}}\right)^{0.36}\right]^2 \tag{A.9}$$

$$k_x = 9.279\left(\frac{\phi L_w^2}{t_a(P)_{i,el-ell}}\right) \tag{A.10}$$

$$L_w = \left[0.07866\left(\frac{k_x t_a(P)_{i,ell-pr}}{\phi}\right)^{0.36}\right]^{\frac{1}{0.72}} \tag{A.11}$$

$$k_x = 9.279\frac{L_w^{0.72}}{h_x}\left(\frac{\phi}{t_a(P)_{i,ell-ll}}\right) \tag{A.12}$$