STUDIES ON FORCED CONVECTION NANOFLUID FLOW IN CIRCULAR CONDUITS

Harikrishna Vishwanadula and Emmanuel C. Nsofor
Department of Mechanical Engineering and Energy Processes, Southern Illinois University, Carbondale, USA
E-Mail: nsofor@engr.siu.edu

ABSTRACT
An experimental system was developed and used to study the nanofluid flow and heat transfer in circular conduits. Experiments were performed for a variety of nanofluid flow features in the system. Results obtained from the study show that the heat transfer rate for flow of the base fluid is less than that of the nanofluid used in the study. It was also found that the observed relationship between molecular diffusivity of momentum and the molecular diffusivity of thermal energy at the macroscale may not necessarily be the same at the nanoscale. A heat transfer correlation for turbulent forced convection flow in circular pipes was developed from the results in terms of Nusselt number, Reynolds number and Prandtl number. The correlation developed was compared to related correlations in the literature. Important factors that affect nanofluid flow and heat transfer in circular conduits were also determined. This type of study is essential for heat exchanger applications.

Keywords: nanofluid, convection, heat transfer, circular conduits, Nusselt number.

INTRODUCTION
The application of nanofluids or fluids containing suspensions of metallic nanoparticles to confront heat transfer problems in thermal management is one of the technological uses of nanoparticles that hold enormous promise today. Experiments have shown that nanofluids have improved thermal conductivities when compared to the base fluids. Technology is continually focusing on miniaturization of system processes resulting in higher power densities for a wide range of applications. This has resulted in the need for increased cooling capacities for compact high energy systems. The standard means of achieving increased cooling using extension fins and micro channels to increase available heat transfer surface areas are being exhaustively used and are becoming limited in effectiveness. A recent focus has been on the cooling fluid itself. Hence interest on the application of nanofluids in heat exchangers has been increasing.

The previous studies that are related to this one include Maiga et al., [1] which studied forced convection flow of water - aluminum oxide (Al₂O₃) and ethylene glycol nanofluids inside a uniformly heated tube subjected to constant and uniform heat flux at the wall. The study concluded that the use of nanoparticles increased the heat transfer at the tube wall considerably for both laminar and turbulent flow. Wen and Ding [2] experimentally studied the convective heat transfer of nanofluids flowing through a copper tube in the laminar flow regime. The results showed that there is considerable enhancement of convective heat transfer using the nanofluids. The study also found that the classical Shah equation failed to predict the heat transfer behavior of nanofluids. Xuan and Li [3] built an experimental system to investigate convective heat transfer and flow features of the nanofluid in a tube. The study used copper particles in water as the nanofluid and measured the convective heat transfer coefficient and friction factor of the nanofluid for turbulent flow and proposed a convective heat transfer correlation for the nanofluid heat transfer.

A Lattice Boltzmann model was proposed by Xuan and Yao [4] for simulating flow and energy transport processes inside nanofluids by accounting for the external and internal forces acting on the suspended nanoparticles and interactions among the nanoparticles and fluid particles. Yang et al., [5] reported on the investigation of the heat transfer properties of nanoparticle-in-fluid dispersions in laminar flow. The study measured the convective heat transfer coefficients of a number of nanoparticle-in-liquid dispersions under laminar flow in a horizontal tube heat exchanger and found that the graphite nanoparticles increased the static thermal conductivities of the fluid significantly at low weight fraction loadings. Mapa and Mazhar [6] performed a similar study using commercially available equipment. The results showed that a number of factors increase the thermal conductivity of the nanofluid and the presence of nanoparticles reduces the thermal boundary layer thickness.

The hydrodynamic thermal fields of water-γ Al₂O₃ nanofluid in a radial laminar flow cooling system was studied numerically by Roy et al., [7]. The study included the heat transfer capabilities of a nanofluid in the radial flow cooling system. The results showed that considerable heat transfer enhancement is possible. Deductions from the study showed that inclusion of nanoparticles in a traditional coolant can provide considerable improvement in heat transfer rates, even at small particle volume fractions. The wall shear stress was also found to increase with an increase in particle volume concentration. Maiga et al., [8] investigated laminar forced convection flow of nanofluids in a uniformly heated tube and a system of parallel, coaxial and heated disks. From results obtained numerically, it was shown that inclusion of nanoparticles in the base fluid produced a considerable enhancement of the heat transfer coefficient that clearly increases with an increase of the particle concentration. It
was found however that the presence of such particles also induced drastic effects on the wall shear stress which increased noticeably with the particle loading. Regarding tube flow, the results showed that, in general, the heat transfer enhancement also increased considerably with increased Reynolds number of the flow. Correlations were developed for computing the Nusselt number for the nanofluids considered. In the case of radial flow, the results showed that both the Reynolds number and the distance separating the disks do not appear to affect the heat transfer enhancement of the nanofluids (when compared to the base fluid at the same Reynolds number and distance) significantly.

Titanium oxide nanofluids with different particle sizes and concentrations were measured by He et al., [9] for their heat transfer and flow behavior for upward flow through a vertical pipe in both laminar and turbulent flow. It was found that with given flow Reynolds number and particle size, the convective heat transfer coefficient increased with nanoparticle concentration in both laminar and turbulent flow. It was also found that the effect of particle concentration seemed to be more considerable in the turbulent flow regime. The convective heat transfer coefficient did not seem to be sensitive to the average particle size under the conditions of the study with given volume fraction and Flow Reynolds number.

In this present study an experimental system was developed and used to investigate the flow characteristics of nanofluids. The nanofluid used consists of spherical 45 nm aluminum oxide nanoparticles dispersed in water. The experimental system was made up of the piping system with appropriate valves, a test section, pumps, data acquisition system, flow meters and two heat exchangers. Temperatures at appropriate points in the test section were measured with thermocouples connected to the data acquisition system. Measurements were made for different volume fractions of the nanofluid. Results obtained show that the heat transfer rate for the nanofluid flow in the pipe system is greater than that for the base fluid. Also, it increases with volume fraction of the nanofluid. A heat transfer correlation for turbulent forced convection flow in circular pipes was developed from the results in terms of Nusselt number, Reynolds number and Prandtl number. The correlation developed from this study was compared to related correlations in the literature. A number of important factors that affect nanofluid flow and heat transfer in pipes were also identified.

**BASIS OF THE ANALYSIS AND DESIGN OF THE EXPERIMENTAL SYSTEM**

The specific heat of the nanofluid was calculated based on the report by Maiga et al., [8]

\[
c_{p, nf} = (1 - \phi)c_{p, f} + \phi c_{p, p} \quad (1)
\]

Li and Peterson [10] reported the expression for the effective thermal conductivity of the nanofluid as

\[
k_{eff} = 1 + \frac{3(\beta - 1)\phi}{\beta + 2 - (\beta - 1)\phi} \quad (2)
\]

\[
\frac{k}{k_f} = \frac{k_{pf}}{k_f}
\]

For each experiment, the Nusselt number and the Reynolds number were obtained using the measured values of the local heat transfer coefficient \(h_s\) and the expression.

\[
Nu = \frac{1}{k_{nf}} \int_0^L h_x dx \quad (3)
\]

The thermal diffusivity for the nanofluid was determined using the thermal conductivity, density and specific heat through the equation.

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho c_{p})_{nf}} \quad (4)
\]

The Prandtl number and the Reynolds number were obtained using the following equations:

\[
Pr = \frac{\nu_{nf}}{\alpha_{nf}} \quad (5)
\]

\[
Re = \frac{U_m D}{\nu_{nf}} \quad (6)
\]

For each experiment, the Nusselt number and the Reynolds number were determined. Using the values obtained, a Nusselt number correlation was developed which best represents the experimental results in the following form

\[
Nu = a Re^b Pr^c \quad (7)
\]

where a, b and c are constants. The nonlinear regression function in the Excel Scientific Graph System was used for the curve fitting.

**EXPERIMENTAL SETUP AND PROCEDURE**

Figure-1 shows an outline or schematic illustrating the arrangement of the experimental system. The setup basically consists of three sections, i.e., the nanofluid loop, the water heating loop, and the cooling loop. The water heating loop consists mainly of piping, a flow meter and a pump for liquid circulation in the loop, the cutoff valve to control the flow rate of the hot fluid, a water tank, another cut off valve for draining water from the loop and the heater.

The nanofluid loop consists of piping through which the nanofluid flows, a tank to store the nanofluid, a pump to circulate the nanofluid, a recirculation loop to...
reduce the load on the pump, a throttle valve to control the flow rate of the nanofluid, an extra piping to bleed air bubbles from the loop, a strainer, a flow meter and a test section which is a double pipe heat exchanger with twelve thermocouples located at appropriate positions. The nanofluid used in the experimental system consists of spherical 45 nm aluminum oxide nanoparticles dispersed in water. The thermocouples were used to measure the temperature of the nanofluid at inlet and outlet and at a number of positions at the surface along the length of the pipe. Four cut-off valves were used in the system with one located for filling the nanofluid into the nanofluid tank. The second one was located for draining the nanofluid from the loop while the third one was located and used in the recirculation loop. The fourth cut off valve was used in the extra piping. All the thermocouples and flow meters were connected to the Data acquisition (DAQ) system. The cooling loop consists also of a double-pipe heat exchanger similar to the heat exchanger in the nanofluid loop. It has an inlet and an outlet for the cooling water which flows in the outer pipe while the nanofluid flows in the inner pipe. The thermocouples and flow meters were calibrated before the experiments were performed using suitable calibrators even though they were calibrated by the manufacturers.

The experimental setup was first tested before the actual experiments with the nanofluid were performed. Experiments were first performed with water instead of the nanofluid in the nanofluid loop. Following satisfactory results with water, the nanofluid loop was drained and the water was replaced with the nanofluid of volume fraction 0.5%. Measurements were made under steady state conditions. To check for consistency of the results, each experiment was repeated twice for each volume fraction. After performing the experiment for the 0.5% volume fraction, the nanofluid was drained from the system. The procedure was repeated as indicated earlier and measurements were made using nanofluid with different volume fractions of 2.1%, 2.7%, 3.1%, 4%, and 4.5%. Experiments were also conducted using the same range of volume fractions but at different flow rates.

Uncertainties in the experimental measurements were estimated at 95% confidence level according to the methods described by Coleman and Steele [11] and Moffat [12]. The experimental result $R$, which is a function of $N$ variables, $V_i$ was described as:

$$ R = R\left(V_1, V_2, V_3, \ldots, V_N\right) $$  \hspace{1cm} (8)

The precision limit $P_R$ for the experimental result $R$ was expressed as:

$$ P_R = \sum_{i=1}^{N} \phi_i^2 P_i^2 $$  \hspace{1cm} (9)

where $P_i$ is the precision error in the measured variable $V_i$ and $\phi_i = \frac{\partial R}{\partial V_i}$. The bias uncertainty $B_R$ in the experimental measurement of each individual variable was expressed as:

$$ B_R = 2 \sum_{i=1}^{N} \phi_i^2 B_i^2 $$  \hspace{1cm} (10)

where $B_i$ is the bias error in the values of the variable. The uncertainty ($U_R$) in the experimental result was obtained using the root-sum-square model as:
\[ U_R = \sqrt{\left( B_R^2 + P_R^2 \right)} \]  \hspace{1cm} (11)

RESULTS AND DISCUSSIONS

The results from the study are shown in Figures 2, 3, 4, 5 and 6. These results were correlated in terms of Nusselt number, Prandtl number and Reynolds number. The nonlinear regression function in the Microsoft Excel Scientific Graph System was used for the curve fitting. This function fits equations to data which are nonlinear functions of their parameters, and determines the best parameters that minimize the sum of the squares of differences between the dependent variables in the equation and the observed results. The equation obtained based on the experimental results was:

\[ Nu = 1.752 \text{ Re}^{1.618} \text{ Pr}^{-0.819} \] \hspace{1cm} (12)

The correlation coefficient which is a measure of the strength of the correlation was 0.984. The inverse proportionality of the Prandtl number in the correlation obtained in this study shows that the relationship of the molecular diffusivity of momentum to the molecular diffusivity of thermal energy at the macroscale may not necessarily hold the same as at the nanoscale. The values used for the viscosity were obtained using classical expressions for the two-phase mixture [13].

Comparison of the correlation predictions with experimental results are shown in Figure-2. This means that the figure shows the prediction obtained using the correlation developed from the entire study compared to actual measurements from the experimental run. The uncertainty bands are also indicated in the figure. By combining the bias and precision limits with experimental data at 95% confidence level, the maximum uncertainty in the measurement of the Nusselt number (\( Nu \)) was determined to be in the range of ±4.6%. As described in the section on the basis of the design and system equations, the bias and precision errors in the determination of the variables which affect the Nusselt number were taken into consideration. From the figure, it can be seen that as much as possible, the correlation represents the experimental data. It can also be seen that the heat transfer increases with Reynolds number.

The effect of the volume fraction of the nanofluid on the Nusselt number for heat transfer in the pipe flow is shown in Figure-3. It can be seen that the Nusselt number increases with the volume fraction. This implies that the heat transfer rate for the nanofluid has some dependency on the particle concentration or volume fraction and it increases with the volume fraction. Experiments were also performed for different mass flow rates of fluid flow in the pipe from which the Reynolds and the Nusselt Numbers were variously determined. The relationship between the Nusselt number and the Reynolds number for the same volume fraction for nanofluid flow in the pipe is shown in Figure-4.

The straight line in the figure is the best fit for this set of experimental data. The line illustrates the trend in the relationship between \( Nu \) and \( Re \). For the same volume fraction, the results indicate that the heat transfer in the nanofluid increases as the Reynolds number increases. Although the nanofluid is a two-phase solid-liquid mixture, this shows that heat transfer is also enhanced by turbulence. The enhanced convective heat transfer of the nanofluid compared to the base fluid is possibly due to the contribution of the increased thermal conductivity of the solid nanoparticles. It may also be due to the disorderly movement of the solid nanoparticles thus...
increasing the energy exchange process in the nanofluid. It should be noted here that effects of factors such as Brownian force, friction force between the fluid and the solid nanoparticles, agglomeration, and gravity may also play some parts in the flow of the nanofluid.

Comparison of the heat transfer in the nanofluid and the heat transfer in the base fluid is illustrated by Figure-5. It can be seen from the figure that for the same value of Reynolds number for flow in the pipeline, the Nusselt number for the nanofluid heat transfer rate is more than 20% higher than that for the base fluid. Thus within the limits of the range represented by the experiments it can be concluded that the nanofluid transfers more heat in pipe flow compared to using only the base fluid. This is opposite to a conclusion reached by Pak and Cho [13] whose study found that the convective heat transfer coefficient of the dispersed fluid was 12% smaller than that of pure water which is the base fluid used in the study.

The Nusselt number correlation developed from this study was compared to a number of previous correlations in the literature. Related correlations in the literature include Pak and Cho [13], Xuan and Li [3] and Dittus-Boelter developed the correlation:

\[
Nu_{\text{Dittus-Boelter}} = 0.0256 \left(\frac{Re}{Pr}\right)^{0.8} \left(\frac{Pr}{Re}\right)^{0.4}
\]

Comparison of the heat transfer in the base fluid with that in the nanofluid.

Figure-5. Comparison of heat transfer in the base fluid with that in the nanofluid.

The Nusselt number correlation best represents the heat transfer performance of water-Aluminum oxide nanofluid in a circular conduit has been studied experimentally. It was found that suspended nanoparticles in the base fluid causes a remarkable increase in the heat transfer performance in pipe flow compared to only the base fluid. From the results it was also seen that as the volume fraction of the nanofluid increases, the Nusselt number increases. This means that within the range of the study, increased concentration of nanoparticles in the base fluid increases the heat transfer performance of the nanofluid. For the same volume fraction, it was found that as the Reynolds number increases, the Nusselt number for the heat transfer increases.

A heat transfer correlation was developed from this study and compared to related ones in the literature. It was also seen that within the limits of the experiments, this correlation best represents the heat transfer performance of the nanofluid from the lowest Reynolds number range to the highest Reynolds number range. This correlation gives the best representation.

CONCLUSIONS

The convective heat transfer performance of water-Aluminum oxide nanofluid in a circular conduit has been studied experimentally. It was found that suspended nanoparticles in the base fluid causes a remarkable increase in the heat transfer performance in pipe flow compared to only the base fluid. From the results it was also seen that as the volume fraction of the nanofluid increases, the Nusselt number for the convection increases. This means that within the range of the study, increased concentration of nanoparticles in the base fluid increases the heat transfer performance of the nanofluid. For the same volume fraction, it was found that as the Reynolds number increases, the Nusselt number for the heat transfer increases.

Based on the report by Vasu et al., [14], Dittus-Boelter developed the correlation:

\[
Nu = 0.0256 \left(\frac{Re}{Pr}\right)^{0.8} \left(\frac{Pr}{Re}\right)^{0.4}
\]

Comparison of the correlation from this present study with the three correlations described here is shown in Figure-6. This is in the form of a graph of Nusselt number versus Reynolds number. It can be seen from the figure that the correlation developed from this study agrees fairly well with these other correlations. However, looking at the heat transfer in the nanofluid starting from the lowest Reynolds number range to the highest, this correlation gives the best representation.
found that the convective heat transfer coefficient of the dispersed fluid was 12% smaller than that of water, the base fluid used in the study.

**Nomenclature**

- \( A \) = area
- \( B_i \) = bias error
- \( B_R \) = bias uncertainty
- \( c_p \) = specific heat
- \( d \) = diameter of the nanoparticles
- \( D \) = inner diameter of the pipe
- \( h \) = convection heat transfer coefficient
- \( k \) = thermal conductivity
- \( Nu \) = Nusselt number
- \( Pi \) = precision error
- \( PR \) = precision limit
- \( Pr \) = Prandtl number
- \( R \) = experimental result
- \( Re \) = Reynolds number
- \( t \) = time
- \( T \) = temperature
- \( U \) = uncertainty
- \( u \) = velocity
- \( V \) = variable
- \( x \) = axial dimension
- \( \chi \) = thermal diffusivity
- \( \nu \) = kinematic viscosity
- \( \rho \) = density
- \( \phi \) = volume fraction

**Subscripts**

- \( eff \) = effective
- \( f \) = base fluid
- \( i \) = measured variable
- \( nf \) = nanofluid
- \( p \) = particle

**ACKNOWLEDGMENTS**

The authors are very grateful to Tim Attig, Eric White, Derek Perry and Jason Willenborg for valuable contributions in the initial development and setup of the experimental system. The authors are also grateful to James Mathias and Kanchan Mondal for valuable advice.

**REFERENCES**


