OPTIMUM DESIGN OF SWITCHED RELUCTANCE MACHINE USING ADAPTIVE PARTICLE SWARM OPTIMIZATION

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ABSTRACT
This paper presents swarm intelligence based Adaptive Particle Swarm Optimization (APSO) technique to determine optimum design of Switched Reluctance Machine (SRM). In APSO technique, the inertia weight factor is made adaptive on the basis of objective functions of the current and best solutions to avoid premature convergence. The SRM design is treated as nonlinear multivariable constrained optimization problem. The objective functions for obtaining desired design are maximizing torque density, minimizing torque ripple and minimizing copper loss with stator and rotor pole arc as design variables. The potential of the proposed approach is tested on 8/6 four-phase, 5 HP, 1500 rpm SRM and the results are compared with those obtained from Genetic Algorithm (GA) and classical PSO technique. The results demonstrate that the proposed method is superior in terms of solution quality, accuracy, robustness and computational efficiency.

Keywords: adaptive particle swarm optimization, genetic algorithm, switched reluctance machine, torque density, torque ripple.

INTRODUCTION
There has been a growing interest towards SRM drives because of its simple and robust structure, high efficiency and fault tolerability (Lawrenson et al., 1980). The main drawback in SRM is higher torque ripple which contributes to acoustic noise and vibration. The torque pulsation in SRM is due to highly non-linear and discrete nature of torque production mechanism (Iqbal Husain and Mehrdad Ehsani, 1994). Sahin (2000) has proposed a neural network based approach to determine optimum geometry to minimize torque ripple. Generalized regression neural network based optimization of SRM to maximize average torque and minimize torque ripple has been discussed by (Sahraoui et al., 2007). An optimum design approach for a two-phase SRM drive using GA is proposed by (Yoshiaki Kano, 2010). Optimization algorithms such as Genetic Algorithm (GA) have been used in the optimal design of SRM (Mirzaeian et al., 2002; Nabeta 2008) to minimize torque ripple. From the literature it is evident that computational intelligence techniques like genetic algorithm and artificial neural network have been successfully applied for design optimization of SRM. In recent years PSO (Kennedy and Eberhart, 1995) method has gained popularity over its competitors. Compared with GA, PSO has some attractive characteristics. PSO has memory, so knowledge of good solutions is retained by all particles, whereas in GA, previous knowledge of problem is destroyed once the population changes. Recent research has identified certain deficiencies in GA based optimization (Fogel, 2000), particularly for problems in which variables are highly correlated. Premature convergence degrades the performance of GA and increases possibility of convergence to a local optimum solution. Due to its simplicity, superior convergence characteristics and high solution quality, PSO has gained attention and wide application in different fields. However, the performance of the classical PSO greatly depends on its parameters and it often suffers from the problem of being trapped in local optima (Shi and Eberhart, 1999). To overcome the above problems, Adaptive Inertia Weight Factor (AIWF) is employed (Liu et al., 2005) to control the global search and convergence to the global best solution. In this paper APSO is employed for solving switched reluctance motor design optimization problem. The performance of APSO algorithm is compared with GA based optimization and classical PSO. The results show that APSO based approach performs better in terms of solution quality, accuracy and robustness. The organization of paper is as follows. First the problem formulation is explained, while the APSO algorithm is briefly introduced in the next section followed by numerical simulations and comparisons. Finally, conclusions are given in the last section.

PROBLEM FORMULATION
The structure of 8/6 SRM is presented in Figure-1. The problem of determining optimal pole arc is formulated to provide trade off solutions between torque density,
torque ripple and copper loss. The stator and rotor pole arc are considered as design variables.

\[ x_1 = \text{Rotor Pole arc} \ (\beta_r) \]
\[ x_2 = \text{Stator Pole arc} \ (\beta_s) \]

The remaining design parameters are treated as fixed. The objective function is defined as:

\[ \text{Minimize } F = -(f_1 + f_2 + f_3) \]

\[ f_1(x) = \text{Maximization of torque density} \]
\[ f_2(x) = \text{Maximization of inductance ratio (to minimize torque ripple)} \]
\[ f_3(x) = \text{Minimization of copper loss} \]

In view of the fact that the torque density and inductance ratio of the motor is to be maximized, minus sign is introduced in the fitness function.

The following are the constraints imposed on the design optimization problem according to the rules of feasible triangle.

\[ x_1 \geq x_2 \]
\[ \frac{2\pi}{N_s} - x_1 > x_2 \]
\[ x_2 > \varepsilon \]

The constraints are taken into account by penalizing the fitness proportionally to the constraint violations.

**Torque density calculation**

Several methods such as Finite Element Method (FEM) (Wei Wu et al., 2003; Arkadan et al., 1994), Magnetic Equivalent Circuit (MEC) method (Moallem et al., 1998), and piecewise linear model (Miller et al., 1990) have been reported for the analysis of the SR motor, FEM is applied for accurate prediction of the machine parameters and performances. This method requires large modeling and computational time. In this work analytical method described by (Krishnan 2001) is used to evaluate the performance of the machine.

The average torque is given by:

\[ T_{\text{ave}} = \frac{\delta W_m N_s N_r}{4\pi} \]

\[ \delta W_m = W_{m \text{ aligned}} - W_{m \text{ unaligned}} \]

A comprehensive program is written in Matlab to compute the difference of co energies at aligned and unaligned position. The aligned co energy is calculated with trapezoidal integration algorithm. Once \( \delta W_m \) is determined, the average torque is calculated using equation (5). The motor lamination volume is calculated as

\[ V = V_s + V_r \]

where \( V_s \) represents the volume of stator lamination and \( V_r \) represents the volume of rotor lamination. Consequently, the average torque per motor lamination volume is determined as:

\[ T_v = \frac{T_{\text{ave}}}{V} \]

**Torque ripple calculation**

Torque ripple expected from SRM is evaluated from the torque dips in T-i-θ characteristics. Torque dip is the difference between the peak torque of a phase and the torque at an angle where two overlapping phases produce equal torque at equal levels of current. This is due to the deficiency of the incoming phase in supplying the necessary torque in those rotor positions (Iqbal Husain, 2002). Figure-2 shows the torque dip present in the initial design. The effect of pole arc variation on mean torque and torque dip can be evaluated from Inductance overlap ratio \( K_L \) given by equation (9). Inductance overlap ratio gives a direct measure of torque overlap of adjacent phases.

\[ K_L = 1 - \frac{\varepsilon}{\min(\beta_s, \beta_r)} \]

From equation (9) it is evident that by widening the stator and rotor poles, torque overlap can be increased. The higher the \( K_L \), the lower will be the torque dip and the higher will be the mean torque as well.

**Copper loss calculation**

The copper loss at rated current is given by:

\[ P_{cu} = i_p^2 R_s \]

The resistance of a single phase is calculated as:

\[ R_s = \frac{0.0177 * l_m T_{ph}}{a_c} \]
The mean length of the winding turn is given as:
\[ l_m = \left( 2L_{aw} + 4W_i + 4r_2 \sin \left( \frac{\beta_i}{2} \right) \right) \]  
(12)

**Particle swarm optimization (PSO)**

PSO, developed by Kennedy and Eberhart (1995) is found to be reliable in solving non-linear problems with multiple optima. In PSO, a number of particles form a “swarm” that evolve or fly throughout the feasible hyperspace to search for fruitful regions in which optimal solution may exist. Each particle has two vectors associated with it, the position \(X_i\) and velocity \(V_i\) vectors. In N-dimensional search space, \(X_i = [x_{i1}, x_{i2}, \ldots, x_{iN}]\) and \(V_i = [v_{i1}, v_{i2}, \ldots, v_{iN}]\) are the two vectors associated with each particle i. During their search, members of the swarm interact with each others in a certain way to optimize their search experience. There are different variants of particle swarm paradigms but the most commonly used one is the gbest model where the whole population is considered as a single neighborhood throughout the flying experience (Chaturvedi et al., 2009; Clerc and Kennedy, 2002). In each iteration, particle with the best solution shares its position coordinates \(gbest\) information with the rest of the swarm. Each particle updates its coordinates based on its own best search experience \(pbest\) and \(gbest\) according to the following equations:

\[ v_i^{k+1} = w v_i^k + c_1 \cdot \text{rand}(pbest^k - x_i^k) + c_2 \cdot \text{rand}(gbest^k - x_i^k) \]  
(13)

\[ x_i^{k+1} = x_i^k + v_i^{k+1} \]  
(14)

where \(c_1\) and \(c_2\) are two positive acceleration constants, they keep balance between the particle’s individual and social behavior when they are set equal; \(\text{rand}\) and \(\text{rand}_2\) are two randomly generated numbers with a range of \([0,1]\) added in the model to introduce stochastic nature in particle’s movement; and \(w\) is the inertia weight and it keeps a balance between exploration and exploitation. In our case, it is a linearly decreasing function of the iteration index:

\[ w(k) = w_{\text{max}} - \left( \frac{w_{\text{max}} - w_{\text{min}}}{\text{iter}_{\text{max}}} \right) \times \text{iter} \]  
(15)

where \(\text{iter}_{\text{max}}\) is the maximum number of iteration, \(\text{iter}\) is the current iteration number, \(w_{\text{max}}\) is the initial weight and \(w_{\text{min}}\) is the final weight. In conclusion, an initial value of \(w\) around 1, with a gradual decline toward 0 is considered as a proper choice. The most important factor that governs the PSO performance in its optimal search is to maintain a balance between exploration and exploitation. Exploration is the PSO ability to cover and explore different areas in the feasible search space while exploitation is the ability to concentrate only on promising areas in the search space and to enhance the quality of potential solution in the fruitful region. Exploration requires bigger step sizes at the beginning of the optimization process to determine the most promising areas then the step size is reduced to focus only on that area. This balance is usually achieved through proper tuning of PSO key parameters. Recently, PSO developments and applications have been widely explored in engineering and science mainly due to its distinct favorable characteristics (Chaturvedi et al., 2009).

**Adaptive particle swarm optimization (APSO)**

In PSO, proper control of global exploration and local exploitation is vital in determining the optimum solution efficiently (Shi and Eberhart, 1998) the performance of PSO greatly depends on its parameters. It is clear that first part of equation (13) represents the influence of previous velocity, which provides the necessary momentum for the particles to roam across the search space. The inertia weight \(w\) is the modulus that controls the impact of previous velocity on the current one. The balance between exploration and exploitation in PSO is dictated by the value of ‘w’. Thus proper control of inertia weight is very important to find the optimum solution accurately and efficiently. To achieve trade off between exploration and exploitation, \(w\) is varied adaptively in response to the objective values of the particles (Liu et al., 2005). The adaptive inertia weight factor is determined as follows:

\[ w = \begin{cases} 
    w_{\text{max}} + \frac{(w_{\text{max}} - w_{\text{min}})(f - f_{\text{min}})}{f_{\text{avg}} - f_{\text{min}}} & , f < f_{\text{avg}} \\
    w_{\text{max}} & , f > f_{\text{avg}} 
\end{cases} \]  
(16)

where \(f\) is the current objective of the particle, \(f_{\text{avg}}\) and \(f_{\text{min}}\) are the average and minimum values of all particles, respectively.

**Implementation of APSO for optimal design of SRM**

In this design, APSO is used to find a set of design variables which ensure that the function \(F(x)\) has a minimum value and all the constraints are satisfied. The algorithm for design optimization process is given below:

a) Read specifications and performance indices of motor
b) Generate initial population of N particles (design variables) with random positions and velocities
c) Compute objective value and performance indices of motor
d) Evaluate the fitness of each particle
e) Update personal best: Compare the fitness value of each particle with its pbests. If the current value is better than pbest, then set pbest value to the current value
f) Update global best: Compare the fitness value of each particle with gbest. If the current value is better than gbest, set gbest to the current particle’s value
g) Update weight using equation (16)
h) Update velocities: Calculate velocities \(v^{k+1}\) using Equation (13)
i) Update positions: Calculate positions $X^{k+1}$ using Equation (14)

j) Return to step (d) until the current iteration reaches the maximum iteration number

k) Output the optimal design variables

The performance of the proposed method is tested on a 5HP motor. The specifications of the sample motor are given in Appendix-1. The algorithm is coded in Mat lab and executed using a Pentium IV based PC as the test platform. During the process the following parameter setting is used in APSO: Number of particles=30, acceleration factor $C1 = C2 = 1.5$, maximum iteration $\text{Itermax} = 100$.

<table>
<thead>
<tr>
<th>Table-1. Results of optimal design.</th>
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</thead>
<tbody>
<tr>
<td>Stator pole arc</td>
</tr>
<tr>
<td>18 deg</td>
</tr>
<tr>
<td>Rotor pole arc</td>
</tr>
<tr>
<td>Average torque</td>
</tr>
<tr>
<td>Torque density</td>
</tr>
<tr>
<td>Inductance ratio</td>
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<tr>
<td>Copper loss</td>
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<tr>
<td>Torque dip</td>
</tr>
</tbody>
</table>

Upon execution of the algorithm, an optimal structure with the configuration $\beta_s = 21.91$ and $\beta_r = 24.08$ is obtained. The performance parameters of the optimal motor design are given in Table-1. From the table it is clear that there is significant improvement in torque density and inductance ratio.

### Characterization using FEA

The optimized geometry was exposed to finite-element calculation. The flux lines at aligned position are shown in Figure-3. The static torque characteristics of the optimal machine at rated current of 13 A is shown in Figure-4. The optimal machine produced an average torque of 28.96 Nm with a torque dip of 4.46 Nm. The results of finite-element calculation confirm the application of optimization procedure for SRM design.

<table>
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<tr>
<th>Figure-3. Flux lines at aligned position.</th>
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<tr>
<td>Figure-4. Static torque characteristics of the optimal design.</td>
</tr>
<tr>
<td>Figure-5. Convergence characteristics of GA, PSO and APSO based methods</td>
</tr>
</tbody>
</table>

### Comparative studies

The performance of the optimization technique in terms of convergence with GA, PSO and APSO is shown in Figure-5. From the figure it is clear that APSO method converges earlier than the GA and PSO. In order to verify the robustness of the algorithms, simulations were carried out for 20 independent runs. From the results in Table-2 it is evident that the APSO method is more robust than the GA and DE as the standard deviation of the fitness values for 20 runs is very low in the APSO method.

<table>
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<tr>
<th>Table-2. Comparison of different optimization techniques</th>
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<tbody>
<tr>
<td>GA</td>
</tr>
<tr>
<td>Best solution</td>
</tr>
<tr>
<td>Worst solution</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Standard deviation</td>
</tr>
<tr>
<td>Execution time (sec)</td>
</tr>
</tbody>
</table>
CONCLUSIONS
This paper describes the design optimization procedure of SRM using APSO with the objective of maximizing torque density, minimizing torque ripple and copper loss. The results obtained by this approach show improvement in torque density and reduction in torque dip. The optimized geometry was exposed to finite-element calculation using MagNet software. The results of finite-element calculation confirm the application of APSO based optimization procedure for SRM design. When compared with GA and classical PSO, APSO algorithm is superior in terms of global exploration, robustness, fast convergence and statistical accuracy.

APPENDIX-1. Design data of the machine.

<table>
<thead>
<tr>
<th>Machine configuration</th>
<th>8/6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power output</td>
<td>5 hp</td>
</tr>
<tr>
<td>Stator pole arc</td>
<td>18 degrees</td>
</tr>
<tr>
<td>Rotor pole arc</td>
<td>22 degrees</td>
</tr>
<tr>
<td>Air gap length</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Outer stator diameter</td>
<td>190 mm</td>
</tr>
<tr>
<td>Bore diameter</td>
<td>100.6 mm</td>
</tr>
<tr>
<td>Stack length</td>
<td>200 mm</td>
</tr>
<tr>
<td>Shaft diameter</td>
<td>28 mm</td>
</tr>
<tr>
<td>Speed</td>
<td>1500 rpm</td>
</tr>
<tr>
<td>Height of stator pole</td>
<td>32.7 mm</td>
</tr>
<tr>
<td>Height of rotor pole</td>
<td>19.8 mm</td>
</tr>
<tr>
<td>Turns per phase</td>
<td>154</td>
</tr>
<tr>
<td>Rated current</td>
<td>13 A</td>
</tr>
</tbody>
</table>

REFERENCES


