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# FREQUENCY OFFSET ESTIMATION IN COHERENT OFDM SYSTEMS USING DIFFERENT FADING CHANNELS

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## ABSTRACT

The well known problem in an orthogonal frequency division multiplexing (OFDM) system is its sensitivity to frequency offset. Most of the coherent OFDM systems transmit pilot symbols on some of the subcarriers to estimate channel attenuation and also add a cyclic prefix (CP) to avoid intercarrier interference and intersymbol interference. An estimation algorithm based on the redundancy of both cyclic prefix and pilot subcarriers is proposed for the correction of frequency offset. If the frame timing is synchronised in advance, by considering the two kinds of redundancy simultaneously, the performance of the proposed hybrid algorithm achieves significant improvement under low SNR and short CP. As to high SNR and long CP, the performance of the hybrid algorithm is almost identical to that of CP-based algorithm. Some comparative simulations are given to illustrate the advantages of the proposed hybrid estimation scheme.

**Keywords:** orthogonal frequency division multiplexing system, cyclic prefix, subcarriers, frequency offset.

## INTRODUCTION

The orthogonal frequency division multiplexing (OFDM) system is capable of coping with the frequency selective fading and narrowband noise because each subscriber in OFDM system is narrow band with respect to coherent bandwidth. Sensitivity to synchronization errors, including both frequency and time is one of the main problems in OFDM systems. Compared to single carrier modulation schemes, OFDM is very sensitive to frequency offset. Minor carrier frequency offset makes the subcarriers of OFDM system to loose their orthoganality because all the subcarriers in OFDM systems are overlapped and orthogonal to each other[1]. Detection of carrier frequency and time offset is a difficult task in OFDM systems. Using of analogy of sample time offset in time domain causing Inter Symbol Interference (ISI), frequency offset significantly degrades the system performance and cause Inter Symbol Interference (ISI). Frequency and Time correction algorithms are classified into two types based on whether the additional data is required or not, if required what kind of additional data is be used. For the applications that require fast and reliable synchronization, data aided schemes are suitable because of the redundancy of the OFDM data frame. Two kinds of redundancy data which are usually used are training symbols (or pilot symbols) and pilot subcarriers. Training symbols are two or more consecutive and identical symbols, used to estimate frequency or time offset [1-3]. Pilot subcarriers are used to estimate channel attenuation in a single carrier system, as pilot-symbol assisted modulation (PSAM). [4] In case of multicarrier systems pilot subcarriers are used, to estimate frequency offset. [5, 6] Depending on nature structure of OFDM frames, non-data-aided schemes are used to estimate frequency offset. To obtain better performance null subcarriers are selected. [7] Based on the utilization of the correlation of received data samples, other blind methods are proposed. [8, 9] which uses an adaptive algorithm to reduce mean square error of frequency offset. One of the most popular non-data-aided synchronization schemes is based on using the cyclostationarity properties of the OFDM signals because of the insertion of cyclic prefix [10, 11]. The cyclic prefix is a part of a transmitted symbol frames and pilots are always inserted in some specific subcarriers in the modern specification of OFDM systems, such as 802.11a and HIPERLAN/2. An algorithm combining the two features, CP and pilot carriers, in one OFDM frame has good potential to provide better estimation performance. The presence of the cyclic prefix and the redundancy of the pilot sub carriers are considered in this paper. The hybrid maximum likelihood estimator for carrier frequency offset only is derived [12]. As the frequency synchronization is usually done after the correction of symbol time offset, the time offset is discarded.

## OFDM SYSTEM MODEL

In the design of frequency offset estimation, an OFDM system model with CP and pilot insertion is considered. In this system, there are $N_p$ pilot symbols inserted into total N subcarriers, and the length of the CP is assumed to be L. In case of multicarrier systems, the pilot symbols are scattered in time and frequency grids, the idea of PSAM is extended to 2D-PSAM. Therefore, the set of subcarriers indexing carrying pilots is denoted as $\gamma$ and the set of OFDM frames including pilots is denoted as $\Gamma$. By definition, the modulated symbol $X_n$ in frame i, where $i \in \Gamma$ and $n \notin \gamma$, is the data symbol transmitted on the nth subcarrier, and $P_n$ in frame 'i' is the intentionally inserted pilot symbol. The signal is separated into two parts; the one containing the $N$-$N_p$ subcarriers is denoted as s(k) and the other containing the $N_p$ subcarriers is represented by m (K), after the symbol frame 'i' passes

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through IDFT. For k∈ [0, N-1], s (k) and m (k) are defined as:

$$S(k) = \frac{1}{\sqrt{N}} \sum_{n \notin \gamma} X_n e^{\frac{j2\prod}{N}} \quad (1)$$

$$m(k) = \frac{1}{\sqrt{N}} \sum_{n \in \gamma} P_n e^{\frac{j2\prod}{N}} \quad (2)$$

The data symbol $X_n$ is random with average energy E $\{|X_n|^2\} = \sigma_x^2$. The transmitted signal m (k) is treated as a time-variant, because the pilot symbol $P_n$ is known with the assumption of E $\{|P_n|^2\} = \sigma_x^2$. The statistical properties of s (k) are simplified as a discrete time Gaussian random process with variance α $\sigma_x^2$ (where α = N-$N_p$/N), since the amount of the data-carrying subcarriers is reasonably large (N>>$N_p$). The CP is added after IDFT modulation in OFDM systems. Hence ,once the system uses the cyclic prefix, the tail L samples of the N-sample (N>L) transmitted signal s (k)+m (k) are copied and inserted in front of the original signal, that is s (k)= s (K+N) and m (k) = m (k+N) for k∈ [0, L-1]. Finally, the transmitted signal s (k) +m (k), becomes N+L samples and is not a white process anymore. Considering that additive white Gaussian noise (AWGN) channel is used, the time dispersion and time offset are not introduced. The received signal r (k) is modeled as

$$r(k)=(s(k)+m(K))e^{\frac{j2\prod k\varepsilon}{N}}+n(k) \quad (3)$$

where n(k) is a complex white zero-mean Gaussian noise with variance $\sigma_x^2$. The exponential term in (3) is used to model the presence of frequency offset, which is caused by the instability of local oscillators and Doppler effects. In a discrete time system ,frequency offset is introduced as a fraction of frequency spacing (the distance between each subcarrier) denoted as ε, where |ε|<0.5., so it is modelled as $e^{\frac{j2\prod kn}{N}}$ in equation (3) mathematically, assuming that all the subcarriers between transmitter and receiver experience the same frequency offset mismatch.

## ESTIMATION ALGORITHM

The ML function of frequency offset is derived step by step with the help of the received signal model in (3). First a simplified assumption about the statistical properties of the correlation of r (k) is considered. As the noise is zero mean Gaussian and m (k) is a deterministic signal known at the receiver, the modeled received signal r (k) is also a Gaussian process with the time varying mean m (k)$e^{\frac{j2\prod kn}{N}}$. As the numbers of pilots assumed are much smaller than that of data-carrying sub carriers, the correlation between successive frames is ignored. The autocorrelation of r (k) is calculated as:

$$E[r(k)\, r^*(l)] = \begin{cases} \alpha\sigma_x^2 + \sigma_n^2, & k = l \\ \alpha\sigma_x^2 e^{-j2\pi\varepsilon}, & k - l = -N \\ \alpha\sigma_x^2 e^{-j2\pi\varepsilon}, & k - l = N \\ 0, & otherwise \end{cases} \quad (4)$$

Therefore, the PDF of r (k) and the joint PDF of r (k) and r (k+N) is analyzed using the above correlation properties as:

$$F(r(k)) = \frac{1}{\pi(\sigma_s^2+\sigma_n^2)} \exp\left( - \frac{\left| r(k) - m(k) e^{\frac{j2\pi\varepsilon}{N}} \right|^2}{\sigma_s^2+\sigma_n^2} \right) \quad (5)$$

and

f (r(k),r (k+N))

$$= \frac{1}{A} \exp\left(- \frac{|\Omega(k)|^2 - 2\rho Re\{\Omega(k)\Omega^*(k+N)e^{j2\prod\varepsilon}\} + |\Omega(k+N)|^2}{(\sigma_s^2+\sigma_n^2)(1-\rho^2)}\right) \quad (6)$$

To estimate the existing frequency offset ε the ML estimation scheme is used. The ML estimator of the frequency offset ε is derived by investigating the log-likelihood function of ε, which is written as:

$$\Lambda(\varepsilon) = \log f(r/\epsilon)$$

$$= \sum_{k=0}^{L-1} \log\left( \frac{f(r(k), r(k+N))}{f(r(k)) f(r(k+N))} \right) + \sum_{k=0}^{N+L-1} \log f(r(k)) \quad (7)$$

where f(.) represents for the corresponding probability density function. Therefore, the ML frequency offset estimate can be obtained by maximizing the log-likelihood function in (7) over all possible values of $\varepsilon$ , that is:

$$\varepsilon^{\hat{}}_{ML} = \arg max_{\varepsilon}\{(\varepsilon)\} \quad (8)$$

Maximizing the log- likelihood function *A (e)* in (7) is equivalent to maximizing the following function.

$$\rho\Lambda_{CP}(\varepsilon) + (1-\rho)\,\Lambda_p(\varepsilon) \quad (9)$$

The log likelihood function in (9) is solved by substituting (5) and (6) into (9). Due to the cyclic prefix insertion with m (k) = m (k+N), k∈ [0, L-1]. The term $\sum_0^{L-1} m(k)m^*(k+N)$ becomes a constant, $\sum_0^{L-1} |m(K)|^2$ and is not relevant to the maximizing argument of the log-likelihood function. Like another constant term$\sum_{k=0}^{L-1} \log(1-\rho)^2$, both of them are dropped during derivation. Finally the first term of (7) is proportional to:

$$\sum_{k=0}^{L-1} Re\{r(k)r^*(k+N)e^{j2\pi\varepsilon}\}$$

$$-\frac{\rho}{2} \sum_{k=0}^{L-1} [|r(k)|^2 + |r(k+N)|^2]$$

$$-(1-\rho) \sum_{k=0}^{L-1} Re\{[r^*(k) + r^*(k+N)e^{j2\pi\varepsilon}]m(k)e^{j2\pi\varepsilon k}\} \quad (10)$$

and the second term of (5) is proportional to;

$$\frac{1-\rho^2}{\rho} Re\left\{\sum_{k=0}^{N+L-1} r^*(k)m(k)e^{\frac{j2\pi\varepsilon k}{N}}\right\} \quad (11)$$

Where

$$\rho = \left| \frac{\alpha\sigma_x^2 e^{-j2\pi\varepsilon}}{\alpha\sigma_x^2+\sigma_n^2} \right| = \frac{\alpha SNR}{\alpha SNR+1} \quad (12)$$

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$$\Lambda_{CP}(\varepsilon)= \sum_{k=0}^{L-1} Re\{r(k)r^*(k+N)e^{j2\pi\varepsilon}\}$$
$$-\frac{\rho}{2}\sum_{k=0}^{L-1}[|r(k)|^2+|r(k+N)|^2] \quad (13)$$

$$\Lambda_p(\varepsilon)= (1+\rho)_{p1}(\varepsilon)-\rho_{p2}(\varepsilon) \quad (14)$$

$$\Lambda_{p1}(\varepsilon)=\sum_{k=0}^{N+L-1} Re\left\{r^*(k)m(k)e^{\frac{j2\pi\varepsilon k}{N}}\right\} \quad (15)$$

$$\Lambda p2(\varepsilon)=$$
$$\sum_{k=0}^{L-1} Re\left\{\left[r^*(k)+r^*(k+N)e^{j2\pi\varepsilon}\right]m(k)e^{\frac{j2\pi\varepsilon k}{N}}\right\} \quad (16)$$

The signal to noise ratio in (12) is SNR=$\sigma_x^2/\sigma_n^2$. The function in (9) uses the linear combinations of the information from the cyclic prefix and from pilot insertion to estimate the frequency offset. The important weighting factor ρ defined in (12), is determined by the SNR of the signals and the coefficient α, which is the ratio of the number of data carrying sub carriers (N-Np) to the total number of sub carriers (N). ρ will approach 1 if the SNR is high and the proposed hybrid estimator will be dominated by the first term $\Lambda cp$ in (9). The estimator depends more on the last term $\Lambda_p(\varepsilon)$ of (9) for a low SNR. However, more the pilot subcarriers, an OFDM frame has, more the term $\Lambda_p(\varepsilon)$ contributes. In special cases, if the transmitted signal does not contain any pilot sub carrier, then Np=0, ρ=SNR/SNR+1 and the function in (9) only exploits the cyclic prefix redundancy. Then the proposed hybrid estimator in (9) is similar in form to the estimator to the symbol timing offset in [9]. Furthermore it is important to investigate the two function terms $\Lambda_{CP}(\varepsilon)$ and $\Lambda_p(\varepsilon)$ in (9). As the summation range of $\Lambda_{CP}(\varepsilon)$ is from 0 to L-1 only, the information of frequency offset in $\Lambda_{CP}(\varepsilon)$ is contributed by the redundancy in the received signal due to cyclic prefix. The likelihood function proposed in [12] depends on the frequency offset information provide by the CP but it discussed the frequency and time offset together in one function Hence if time offset is set to 0 in [12], its maximum likelihood function and (13) will become same. The function $\Lambda_P(\varepsilon)$ in (14) contains the frequency offset information given by the redundancy in the received signal due to the pilot carriers. As defined in (15) and (16) this term can be separated into two parts $\Lambda_{p1}(\varepsilon)$ and $\Lambda_{p2}(\varepsilon)$ the information of pilot subcarriers contained in $\Lambda_{p1}(\varepsilon)$ is provided by whole symbol and the information in $\Lambda_{p2}(\varepsilon)$ is provided by CP. $\Lambda_{p1}(\varepsilon)$ is same as the corresponding ML function proposed by [5], as these two estimators use ML criteria and the pilot subcarriers. Moreover [5] only contains general OFDM symbol frame without involving CP redundancy so the term $\Lambda_{p2}(\varepsilon)$ never appears in its ML function.

The proposed estimation algorithm uses the pilot subcarriers and cyclic prefix together to estimate the frequency offset. The proposed method uses special features in one OFDM symbol hence named as hybrid synchronization scheme. The proposed hybrid estimator is the weighting combination of estimator proposed in [12] and that in [5], which in turn depends on value of ρ. Hence the performance comparison of the simulation results for the three different estimators (the proposed hybrid estimator, the CP based estimator and the pilot based estimator) is reported in the following section under different fading channels.

## COMPARISIONS

### Effect of CP length

To observe the performance of the proposed hybrid estimation algorithm, 4000 symbol frames are used for the simulation and the mean square error of estimated frequency offset is calculated. The timing offset is assumed to be zero in comparative simulations, frequency offset is assumed to be 0.18 and the channel is one tap with an additive white Gaussian noise. The 16-QAM mapping scheme is used and no coding technique is applied. The pilot symbols are needed to be inserted to the subcarriers, they are inserted as per the specification of IEEE i.e., 802.11a, where the number of subcarriers is N=64. By using the MSE of estimated frequency offset, the performance of three different algorithms are evaluated and normalized with the range 0-0.5. The three maximum likelihood functions are written below:

**i. Proposed:** $\rho\Lambda_{CP}(\varepsilon)+(1-\rho)\Lambda_p(\varepsilon)$

**ii. CP-based:**

$$\Lambda_{CP}(\varepsilon)= \sum_{k=0}^{L-1} Re\{r(k)r^*(k+N)e^{j2\pi\varepsilon}\}$$
$$-\frac{\rho}{2}\sum_{k=0}^{L-1}[|r(k)|^2+|r(k+N)|^2]$$

**iii. pilot-based:**

$$\Lambda_{PB}(\varepsilon)=\sum_{k=0}^{N-1} Re\left\{r^*(k)m(k)e^{\frac{j2\pi\varepsilon k}{N}}\right\}$$

The effect of CP length is observed clearly Figures 1 and 2 show the mean square error against various lengths of CP with different SNR's, under AWGN channel. Since the pilot-based algorithm does not take CP into consideration, its MSE is almost constant shown in Figures 1 and 2. In Figure-1, the proposed algorithm outperforms the other two techniques, but the MSE difference between the proposed and CP-based algorithms becomes smaller as the length of the CP is increased. With the increase of SNR from 4 to 14 dB, that is from Figure-2, the simulation results of the CP-based and proposed algorithms become closer to each other when L≥2. This makes sense because the estimation accuracy of the proposed hybrid estimation technique achieved excellent performance with some insignificant degradation in terms of MSE while operating on a multipath channel. Figure-3 to 6 shows the performance of the system under Rician and Rayleigh channels. Results shows that with the increase in SNR, the performance of rician fading channel is far better compared to Rayleigh fading channel.

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## RESULTS

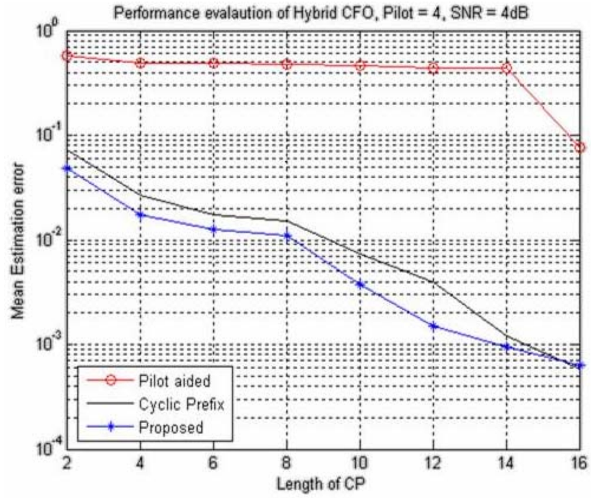


**Figure-1.** Varying cp with pilot = 4 and SNR = 4dB.

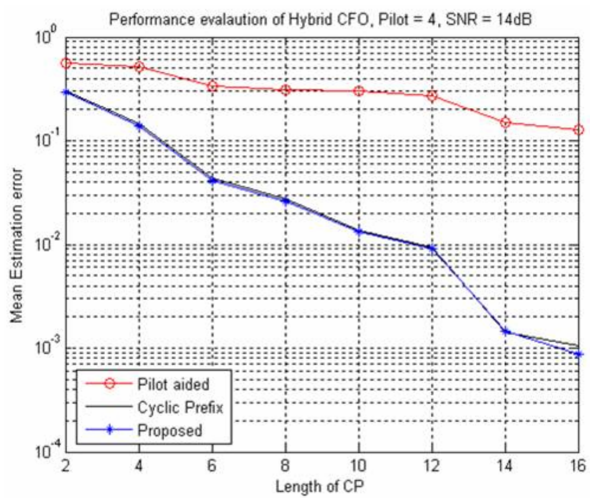


**Figure-2.** Varying cp with pilot = 4 and SNR = 14dB.

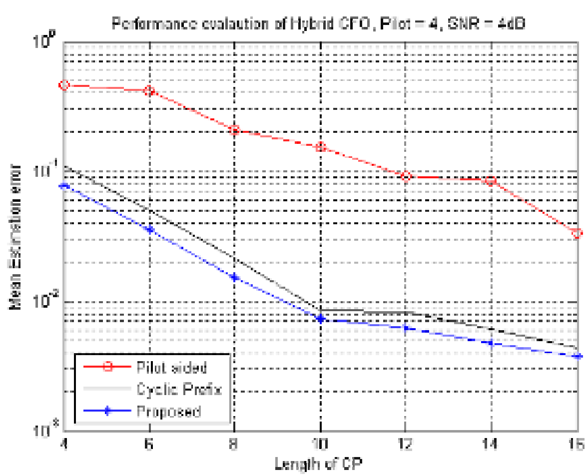


**Figure-3.** Varying cp in rician channel with pilot = 4 SNR = 4dB.

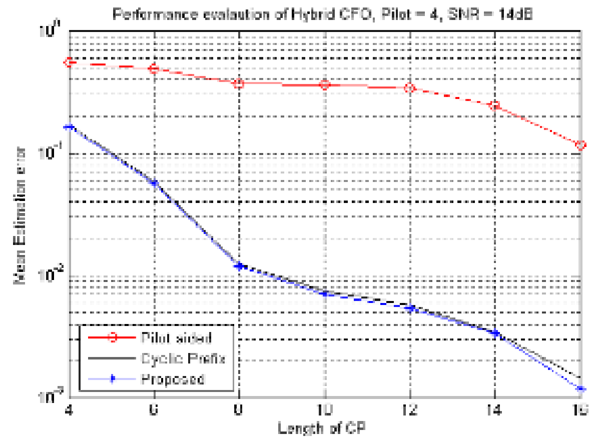


**Figure-4.** Varying cp in rician channel with pilot = 4 SNR = 14dB.

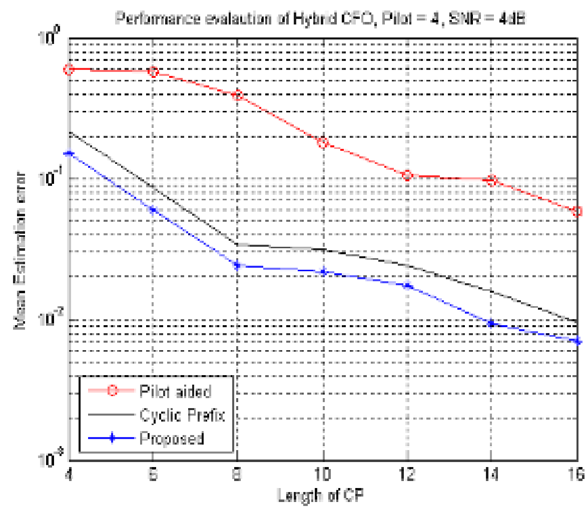


**Figure-5.** Varying cp in rayleigh channel with pilot = 4 SNR = 4dB.

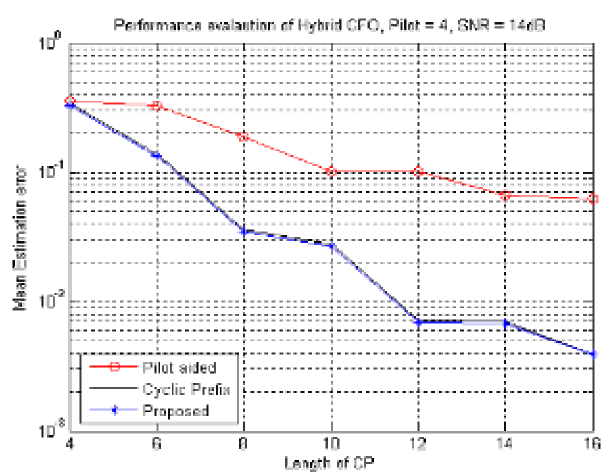


**Figure-6.** Varying cp in rayleigh channel with pilot = 4 SNR = 14dB.

The number of pilots used is set as four in the simulation experiment, following the specification of IEEE 802.11a. The mean square errors which are against normalized carrier frequency offset for different SNR values is shown in Figure-1. It is observed that the MSE performance of the pilot based algorithms is almost independent of the actual value of frequency offset for different SNR. The difference in MSE performance for the CP based and proposed algorithm is smaller for SNR = 5dB and almost identical for frequency offset in the range 0.3-0.45. For SNR =15dB insignificant performance results. The performance of the proposed hybrid algorithm is superior for SNR = 5dB than the other two algorithms, pilot-based ($N_p$=4) and CP-based (L=N/32) schemes, for SNR=15dB. Figures 2 and 3 shows the MSE performance against frequency offset under Rician and Rayleigh channels. MSE performance of the pilot based algorithms is almost independent of the actual value of frequency offset for different SNR in both the channels. MSE performance for proposed model for SNR = 5dB and 15dB are almost identical with insignificant improvement for SNR = 5dB for $\varepsilon$ ranging from 0.4- 0.45. Figure-3 shows the MSE performance for SNR = 15dB improves considerably for the proposed model. Finally the MSE performance against $\varepsilon$ under AWGN channel for SNR= 5dB outperforms compared to CP based and pilot-based algorithm.

## CONCLUSIONS

The performance of all the three estimated schemes is discussed. The performance in terms of MSE degrades considerably if a Rayleigh fading channel is considered with high Doppler frequency $\omega$. In the proposed model time offset is not considered, only frequency offset is considered. A new model needs to be proposed where time offset needs to be taken into consideration along with frequency offset in further research work. With the increase in SNR the performance of CP-based and proposed algorithms become closer and almost identical.

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