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A CASE OF SIMILARITY SOLUTIONS FOR UNSTEADY LAMINAR BOUNDARY LAYER FLOW IN CURVILINEAR SURFACE

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ABSTRACT
In this paper, a study is made of the unsteady laminar natural convection boundary layer equations on a vertical curvilinear surface to establish necessary and sufficient conditions under which the similarity solutions are possible. The free parameter method is used to obtain similarity solutions. One of the cases of possible similarity solutions is discussed analytically and numerically.

Keywords: similarity solutions, unsteady, free convection, curvilinear surface, Nacthsheim-Swigert iteration technique.

INTRODUCTION
An analysis is made of three dimensional unsteady laminar boundary layer equations for free convection flow around a curvilinear surface, in order to establish necessary and sufficient conditions under which similarity solutions are possible. The concept of 'similarity' initially introduced by Blasius has become a useful tool now-a-day. On the basis of similarity transformations and finally the reduction of the set of partial differential equations to a set of ordinary differential equations have now reached stage of any great extent. It is often difficult and even impossible to find the solution of partial differential equation with usual classical method. So applied mathematicians and engineers devote themselves to develop the ways and means for their solutions with simplifying assumptions. Similarity solution is one of the means, where the reduction of number of independent variables into one being done successfully.

The theoretical, experimental and numerical analysis has been carried out extensively by among others [1], [2], [3], [4], etc for the natural convection boundary layer flow about isothermal. Johnson and Cheng [5] examined the necessary and sufficient condition under which similarity solution exist for free convection boundary layers adjacent to flat plates in porous media. The solutions obtained in their work were more general than those appearing in the previous studies. Later Merkin [6] studied the similarity solutions for free convection vertical plate where the (non-dimensional) plate temperature and the (non-dimensional) surface heat flux were taken to be x^{λ} and $-x^{\mu}$, respectively. He also discussed the conditions for which the solution became valid $(\lambda, \mu) \geq 1$. Next, Pop and Takhar [7] investigated the free convection flow over a non-isothermal two dimensional body shape geometrical configuration which permitted similarity solution. A comprehensive study of similarity solutions for free convection boundary layer flow over a permeable wall in a fluid saturated porous medium was carried out by Chaundhury *et al*., [8] which shown that the system depends on the power low exponent and the dimensionless surface mass transfer rate. Jayaraj *et al*., [9] discussed elaborately the analysis of thermophoresis in natural convection flow with variable fliuid properties above a vertical cooled plate.

Williams *et al.,* [10] studied the unsteady free convection flow over a vertical flat plate under the assumption of variations of the wall temperature with time and distance. They found possible semi-similar solutions for a verity of classes of wall temperature distributions. Kumari *et al.,* [11] observed that the unsteadiness in the flow field was caused by the time dependent velocity of the moving sheet. The constant temperature and the constant heat flux conditions were consideration in their investigation. Slaouti *et al*., [12] investigated the temperature and surface heat transfer were changed in a small interval of time for the unsteady free convection flow in the stagnation-point region of a three dimensional body. The surface heat transfer parameter increased with the increase of Prandtl number while the surface skin friction parameters decreased with the increase of Prandtl number. The possible similarity cases were discussed in tabulated form for ΔT -variations in addition to those of exterior velocity components tabulated by Hansen and Ohio [13].

The theoretical studies on laminar free convection on vertical plates and cylinders [14] have received wider attention, especially in dealing with non-uniform surface temperature and heat-flux distributions. However, available in the literature, are only a few exact solutions, which have all been derived by using the technique of similarity solution. In such technique, the pertinent boundary layer equation, under a suitable transformation, are reduced to a set of set of ordinary differential equations in terms of a similarity variable, which is a function of the original independent variables. Then these simultaneous ordinary equations with boundary conditions are solved numerically, yielding velocity and temperature profiles, from which important boundary-layer characteristics are determined. However, because of the nature of the transformation, these similarity solutions are only valid for certain specific surface conditions.

In the present study we deal with the similarity solution of unsteady free convective laminar incompressible flow over a heated vertical curvilinear surface. With systematic analyses the governing partial differential equations are transformed into a set of ordinary differential equations. Finally similarity requirements are exhibited for T, h_1, h_2, U_F and V_F variations. Numerical results are presented to predict flow characteristics for different values of the controlling parameters involved in the similarity transformation. The rest of the paper is arranged as follows. In section two, a complete set of boundary layer equations governing the flow and temperature fields in general orthogonal curvilinear co-ordinates are formulated. The transformation leading to similarity and a brief description of the possible similarity case are given in section three and four respectively. Section five gives a numerical study to demonstrate the influence of the controlling parameters on the flow field and the temperature distribution as well as on the skin-friction and heat transfer factors. In section six the conclusion are drawn from our present research.

GOVERNING EQUATIONS

We consider the flow direction along the ξ-axis and η-axis and be defined in the surface over which the boundary layer is flowing. For simplicity $h_3(\xi,\eta)=1$ has been set such that ζ represents actual distance measured normal to the surface. The body force is taken as the gravitational force $\bar{g}\left(g_\xi(\xi,\eta), g_\eta(\xi,\eta), 0\right)$. The fluid property variations other than density variation in the buoyancy term of the momentum equation are ignored.

The governing boundary layer equations of the flow field in general orthogonal curvilinear co-ordinates are:

Continuity equation

$$\frac{\partial}{\partial \xi}(h_2 u)+\frac{\partial}{\partial \eta}(h_1 v)+\frac{\partial}{\partial \zeta}(h_1 h_2 w)=0 \tag{1}$$

u-momentum equation

$$\frac{Du}{Dt}+\frac{uv}{h_1 h_2}\frac{\partial h_1}{\partial \eta}-\frac{v^2}{h_1 h_2}\frac{\partial h_2}{\partial \xi}=-\frac{1}{h_1}\beta_T \Delta T \theta g_\xi + \nu\frac{\partial^2 u}{\partial \zeta^2} \tag{2}$$

v-momentum equation

$$\frac{Dv}{Dt}+\frac{uv}{h_1 h_2}\frac{\partial h_2}{\partial \xi}-\frac{u^2}{h_1 h_2}\frac{\partial h_1}{\partial \eta}=-\frac{1}{h_2}\beta_T \Delta T \theta g_\eta + \nu\frac{\partial^2 v}{\partial \zeta^2} \tag{3}$$

Energy equation

$$\frac{D\theta}{Dt}+\theta\left\{\frac{\partial}{\partial t}(\ln \Delta T)+\frac{u}{h_1}\frac{\partial}{\partial \xi}(\ln \Delta T)+\frac{v}{h_2}\frac{\partial}{\partial \eta}(\ln \Delta T)\right\} = \frac{\nu}{\Pr}\frac{\partial^2 \theta}{\partial \zeta^2} \tag{4}$$

where $\Pr=\frac{\mu C_p}{k}$ is the Prandtl number of the fluid.

The boundary conditions are:

$$u(t,\xi,\eta,0)=v(t,\xi,\eta,0)=0,\ \theta(t,\xi,\eta,0)=1$$
$$u(t,\xi,\eta,\infty)=v(t,\xi,\eta,\infty)=\theta(t,\xi,\eta,\infty)=0 \tag{5}$$

TRANSFORMATION LEADING TO SIMILARITY SOLUTION

Equations (1-4) are non-linear, simultaneous partial differential equations and the solutions of these equations are extremely difficult to obtain. Hence our aim is to reduce equations (2-4) to ordinary differential equations with the help of (1) which permits possible variations in $\Delta T, U_F, V_F, h_1$ and h_2 with respect to t, ξ and η.

Let us now change the variables t, ξ, η and ζ to a new set of variables τ, X, Y and $\bar{\phi}$, that is, $(t,\xi,\eta,\zeta)\rightarrow(\tau,X,Y,\bar{\phi})$ by the set of following equations:

$$t=\tau, \xi=X, \eta=Y \text{ and } \bar{\phi}=\frac{\zeta}{\gamma(\tau,X,Y)} \tag{6}$$

$\gamma(\tau,X,Y)$ is considered primarily here to be proportional to the square root of the local boundary layer thickness.

Thus the transformed momentum and energy equation:

$$\nu \bar{F}_{\bar{\phi}\bar{\phi}\bar{\phi}}+\frac{1}{2}(a_0+a_1-a_2)\bar{F}\bar{F}_{\bar{\phi}\bar{\phi}}+\frac{1}{2}(a_3+a_4-a_5) \bar{S}\bar{F}_{\bar{\phi}\bar{\phi}}-(a_6-\bar{\phi}a_7)\bar{S}\bar{F}_{\bar{\phi}\bar{\phi}}-a_8\bar{F}_{\bar{\phi}}^2-(a_9+a_{10})\bar{F}_{\bar{\phi}}\bar{S}_{\bar{\phi}} +a_{11}\bar{S}_{\bar{\phi}}^2-a_{12}\bar{F}_{\bar{\phi}}+a_{13}\bar{\theta}=0 \tag{7}$$

$$\nu \bar{S}_{\bar{\phi}\bar{\phi}\bar{\phi}}+\frac{1}{2}(a_3+a_4-a_5)\bar{S}\bar{S}_{\bar{\phi}\bar{\phi}}+\frac{1}{2}(a_0+a_1-a_2) \bar{S}_{\bar{\phi}\bar{\phi}}\bar{F}-(a_6-\bar{\phi}a_7)\bar{S}_{\bar{\phi}\bar{\phi}}-a_{14}\bar{S}_{\bar{\phi}}^2-(a_{15}+a_{16})\bar{F}_{\bar{\phi}}\bar{S}_{\bar{\phi}} +a_{17}\bar{F}_{\bar{\phi}}^2-a_{18}\bar{S}_{\bar{\phi}}+a_{19}\bar{\theta}=0 \tag{8}$$

and

$$\frac{\nu}{\Pr}\bar{\theta}_{\bar{\phi}\bar{\phi}}+\frac{1}{2}(a_0+a_1-a_2)\bar{F}\bar{\theta}_{\bar{\phi}}+\frac{1}{2}(a_3+a_4-a_5) \bar{S}\bar{\theta}_{\bar{\phi}}-(a_6-\bar{\phi}a_7)\bar{\theta}_{\bar{\phi}}-\left(a_{20}\bar{F}_{\bar{\phi}}+a_{21}\bar{S}_{\bar{\phi}}\right)\theta-a_{22}\bar{\theta}=0 \tag{9}$$

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where the constant a's and the differential equation involving the independent parameter τ, X and Y are given by the following differential equations:

$$\left(\frac{\gamma^2 U_F}{h_1}\right)_X = a_0, \quad \frac{\gamma^2 (h_2 U_F)_X}{h_1 h_2} = a_1, \tag{10}$$

$$\gamma^2 U_F h_2 \left(\frac{1}{h_1 h_2}\right)_X = a_2, \left(\frac{\gamma^2 V_F}{h_2}\right)_Y = a_3,$$

$$\frac{\gamma^2 (h_1 V_F)_Y}{h_1 h_2} = a_4, \ \gamma^2 V_F h_1 \left(\frac{1}{h_1 h_2}\right)_Y = a_5, \ w\gamma = a_6,$$

$$\gamma\gamma_\tau = a_7, \ \frac{\gamma^2}{h_1}(U_F)_X = a_8, \ \frac{\gamma^2}{h_2} V_F \left\{\frac{(U_F)_Y}{U_F}\right\} = a_9, \ \frac{\gamma^2}{h_2} V_F \frac{h_{1Y}}{h_1} = a_{10},$$

$$\frac{\gamma^2}{h_1 h_2} \frac{(V_F)^2}{U_F} h_{2X} = a_{11}, \quad \frac{\gamma^2 (U_F)_\tau}{U_F} = a_{12}, \quad \frac{\gamma^2}{h_1 U_F} \beta_T \Delta T g_X = a_{13},$$

$$\frac{\gamma^2}{h_2}(V_F)_Y = a_{14}, \quad \frac{\gamma^2 U_F}{h_1} \frac{(V_F)_X}{V_F} = a_{15}, \quad \frac{\gamma^2 U_F}{h_1} \frac{h_{2X}}{h_2} = a_{16},$$

$$\frac{\gamma^2}{h_1 h_2} \frac{(U_F)^2}{V_F} h_{1Y} = a_{17}, \quad \frac{\gamma^2 (V_F)_\tau}{V_F} = a_{18}, \ \frac{\gamma^2}{h_2 V_F} \beta_T \Delta T g_Y = a_{19},$$

$$\frac{\gamma^2 U_F}{h_1}(\ln \Delta T)_X = a_{20}, \quad \frac{\gamma^2 V_F}{h_2}(\ln \Delta T)_Y = a_{21}, \quad \gamma^2 (\ln \Delta T)_\tau = a_{22}.$$

The above differential equations are denoted by the equation number (10).

On simplification of expression for as in equation (10) and ignoring the suction or injection effects (i.e., $a_6 = 0$), we obtain the following relationships:

$$\frac{\gamma^2 U_F}{h_1} = a_0 X + A(Y, \tau) \tag{11}$$

$$\frac{\gamma^2 V_F}{h_2} = a_3 Y + B(\tau, X) \tag{12}$$

$$\gamma^2 = 2a_7 \tau + C(X, Y) \tag{13}$$

POSSIBLE SIMILARITY CASE

We consider here:

$$\begin{aligned} &\frac{\partial A(Y,\tau)}{\partial Y} = 0, \frac{\partial B(\tau, X)}{\partial \tau} \neq 0, \frac{\partial C(X,Y)}{\partial X} \neq 0, \\ &\frac{\partial C(X,Y)}{\partial Y} = 0, \frac{\partial A(Y,\tau)}{\partial \tau} \neq 0, \frac{\partial B(\tau, X)}{\partial X} \neq 0 \end{aligned} \tag{14}$$

Let us suppose that all above partial derivatives are constants and choose $h_1 = h_2$. Then by virtue of equations (14), we obtain:

$$\left.\begin{aligned} A(Y,\tau) &= \frac{l_5}{k_3}\tau + A_0 \\ B(\tau, X) &= k_2 l_2 + \frac{l_6}{k_1} X + B_0 \\ C(X,Y) &= k_3 l_3 X + C_0 \end{aligned}\right\} \tag{15}$$

where

$$\frac{U_F}{V_F} = k_1, \ \frac{V_F}{h_1} = k_2, \ \frac{h_1}{U_F} = k_3, \ 2a_7 + a_{18} - a_{24} = l_2,$$

$$a_0 - a_1 - a_2 = l_3,$$

$$2a_7 + a_{12} - a_{23} = l_5$$

and

$$a_0 - a_1 - a_2 - a_{11} + a_{15} = l_6.$$

For simplicity γ^2 is found to be

$$\gamma^2 = a_0 k_3 X + 2a_7 \tau + A_0 k_3 \tag{16}$$

Therefore, in view of equation (13) and (16), we obtain:

$$h_2 = h_1 = b_1 (a_0 k_3 X + 2a_7 \tau + A_0 k_3)^m \tag{17}$$

where $m = \dfrac{a_{16}}{a_0}$

$$U_F = \frac{b_1}{k_3}(a_0 k_3 X + 2a_7 \tau + A_0 k_3)^m \tag{18}$$

$$V_F = b_1 k_2 (a_0 k_3 X + 2a_7 \tau + A_0 k_3)^m \tag{19}$$

The similarity requirements furnish us with the relation between the constants (a's). The relations are, a_0, a_7 are arbitrary,

$$\left.\begin{aligned} &a_1 = 2ma_0, a_2 = -2ma_0, a_3 = a_4 = a_5 = a_6 = 0, \\ &a_8 = ma_0, a_9 = a_{10} = 0, a_{11} = \frac{ma_0}{k_1^2}, \ a_{12} = 2ma_7, \\ &a_{13} = \frac{k_3}{b_1^2}(a_0 k_3 X + 2a_7 \tau + A_0 k_3)^{1-2m} \beta_T \Delta T g_X, \\ &a_{14} = 0, a_{15} = ma_0, a_{16} = ma_0, a_{17} = 0, \ a_{18} = 2ma_7, \\ &a_{19} = \frac{1}{b_1^2 k_2}(a_0 k_3 X + 2a_7 \tau + A_0 k_3)^{1-2m} \beta_T \Delta T g_Y, \\ &a_{20} = (2m-1)a_0, a_{21} = 0, \quad a_{22} = (2m-1)a_7. \end{aligned}\right\} \tag{20}$$

Hence the general equations (7-9) reduce to:

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$$\nu \bar{F}_{\bar{\phi}\bar{\phi}\bar{\phi}} + \frac{4m+1}{2}\bar{F}\bar{F}_{\bar{\phi}\bar{\phi}} + a_7\bar{\phi}\bar{F}_{\bar{\phi}\bar{\phi}} - ma_0\bar{F}_{\bar{\phi}}^2 + \frac{ma_0}{k_1^2}\bar{S}_{\bar{\phi}}^2 - 2ma_7\bar{F}_{\bar{\phi}} + a_{13}\bar{\theta} = 0$$

$$\nu \bar{S}_{\bar{\phi}\bar{\phi}\bar{\phi}} + \frac{4m+1}{2}a_0\bar{S}_{\bar{\phi}\bar{\phi}}\bar{F} + a_7\bar{\phi}\bar{S}_{\bar{\phi}\bar{\phi}} - 2ma_0\bar{F}_{\bar{\phi}}\bar{S}_{\bar{\phi}} - 2ma_7\bar{S}_{\bar{\phi}} + a_{13}\bar{\theta} = 0$$

and

$$\frac{\nu}{\Pr}\bar{\theta}_{\bar{\phi}\bar{\phi}} + \frac{4m+1}{2}a_0\bar{F}\bar{\theta}_{\bar{\phi}} + a_7\bar{\phi}\bar{\theta}_{\bar{\phi}} - (2m-1)a_0\bar{F}_{\bar{\phi}}\theta - 2(2m-1) + a_7\bar{\theta} = 0$$

Subject to boundary conditions

$\bar{F}(0) = \bar{F}_{\bar{\phi}}(0) = 0,\ \ \bar{F}_{\bar{\phi}}(\infty) = 0,\ \bar{S}(0) = \bar{S}_{\bar{\phi}}(0) = 0,$

$\bar{S}_{\bar{\phi}}(\infty) = 0$ for the dimensionless stream function and $\bar{\theta}(0) = 1,\ \bar{\theta}(\infty) = 0$ for the dimensionless temperature function.

Let us now substitute $\bar{F} = \alpha f$, $\bar{S} = \alpha s$, $\bar{\phi} = \alpha\phi$, $\bar{\theta} = \theta$ in the above equation. Then putting $\left(\frac{4m+1}{2}\right)\frac{a_0\alpha^2}{\nu} = 1$ and writing $\frac{a_7}{a_0} = c,\ \frac{2m}{4m+1} = \beta$. Also $\frac{2}{4m+1}\frac{a_{13}}{a_0} = 1$ and $\frac{2}{4m+1}\frac{a_{19}}{a_0} = R$ for purely free convection. Finally we obtain,

$$f''' + f\,f'' + (2-4\beta)c\phi\, f'' - \beta\left(f'^2 - \frac{1}{k_1^2}s'^2 + 2cf'\right) + \theta = 0 \tag{21}$$

$$s''' + fs'' + (2-4\beta)c\phi s'' - 2\beta(f's' + s') + R\theta = 0 \tag{22}$$

and

$$\Pr^{-1}\theta'' + f\theta' + (2-4\beta)c\phi\theta' - (6\beta-2)f'\theta - (12\beta-4)c\theta = 0 \tag{23}$$

where prime denotes the differentiation with respect to ϕ.

The boundary conditions are

$$\left.\begin{aligned} &f(0) = f'(0) = 0, \qquad f'(\infty) = 0 \\ &s(0) = s'(0) = 0, \qquad s'(\infty) = 0 \\ &\theta(0) = 1, \qquad \theta(\infty) = 0 \end{aligned}\right\} \tag{24}$$

If we put $f = 0$ and adjust value of the controlling parameter, equations (21) to (23) with boundary conditions (24) coincide with the cases of the possible similarity solutions for laminar free convection on vertical plates studied by Yang [14].

We have, in this case, the similarity requirements are:

$h_1 \propto (\bar{x} + \bar{c}\bar{t})$, $U_F^2 \propto g_X\beta_T\Delta T$, $V_F^2 \propto g_X\beta_T\Delta T$,

$\Delta T \propto (\bar{x} + \bar{c}\bar{t})^{2m-1}$, where $\bar{x} = X + X_0$ and $\bar{t} = \tau + \tau_0$.

The similarity variable ϕ is:

$$\phi = \left(Gr_{\bar{x}\bar{t}}\right)^{1/4}\frac{z}{(\bar{x} + \bar{c}\bar{t})}$$

where $\left(Gr_{\bar{x}\bar{t}}\right)^{1/4} = \left[\frac{4m+1}{2h_1^2}\frac{g_X\beta_T\Delta T(\bar{x} + \bar{c}\bar{t})^3}{\nu^2}\right]$ is the modified Grashof number.

The velocity components

$u = U_F\, f'(\phi)$ where $U_F^2 = -g_X\beta_T\Delta TL_1$

$v = V_F\, s'(\phi)$ where $V_F^2 = -g_Y\beta_T\Delta TL_2$

and

$$w = \left\{\frac{2\nu}{k_3(4m+1)(\bar{x} + \bar{c}\bar{t})}\right\}^{\frac{1}{2}}\left[\frac{1}{2}\phi\, f' - \left(2m + \frac{1}{2}\right)f\right]$$

The skin friction coefficients are:

$$\tau_{w1} = \mu\left(\frac{\partial u}{\partial z}\right)_{z=0} = \frac{(4m+1)}{2}\rho U_F^2\frac{Gr_{xt}^{-\frac{1}{4}}}{h_1}f''(0)$$

$$\tau_{w2} = \mu\left(\frac{\partial v}{\partial z}\right)_{z=0} = \frac{(4m+1)}{2}k_1\rho V_F^2\frac{Gr_{xt}^{-\frac{1}{4}}}{h_1}s''(0)$$

Heat flux,

$$q_w = -k\left(\frac{\partial T}{\partial z}\right)_{z=0} = -k\Delta T\left(\frac{4m+1}{2\nu}\right)\left(\frac{a_0}{k_3}\right)^{\frac{1}{2}}(\bar{x} + \bar{c}\bar{t})^{\frac{1}{2}}\theta'(0)$$

NUMERICAL SOLUTIONS

The set of ordinary differential equations (21) to (23) with boundary conditions (24) are nonlinear and coupled. A standard initial value solver i.e., the shooting method is used to solve these equations numerically. For this purpose we applied the Nacthsheim-Swigert iteration technique (Nachtsheim and Swigert, 1965) [15], [16], [17]. In the process of iteration the velocity and temperature profile, the skin friction coefficients $(f''(0)s''(0))$ and the heat transfer factor $(\theta'(0))$ are evaluated. The numerical results obtained for several selected values of the established parameter are displayed in graphs and tables below.

The dimensionless velocity profiles are presented in Figures-1-8, whereas the dimensionless temperature profiles are presented in Figures- 9-12.

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Figures 1 and 2 represent the dimensionless velocity profiles along x and y-directions, respectively for fixed value of $\beta = 0.3, k_1 = 0.3, R = 1$ and $\Pr = 0.72$ with several value of c. It is observed that both the velocity profile decreases as the values.

Procedure for paper submission

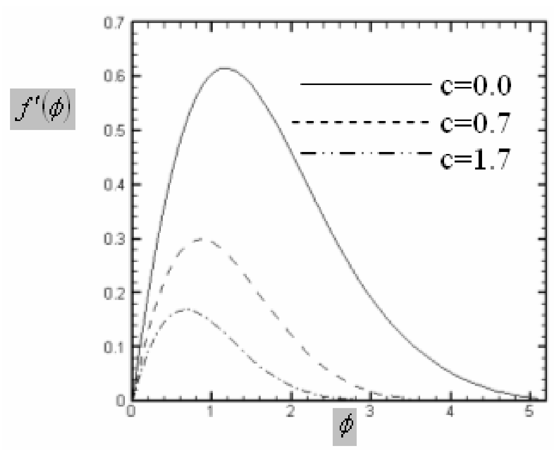

Figure-1. Variation of the dimensionless velocities against ϕ along x-direction for different values of c (β, k_1 R and Pr are fixed).

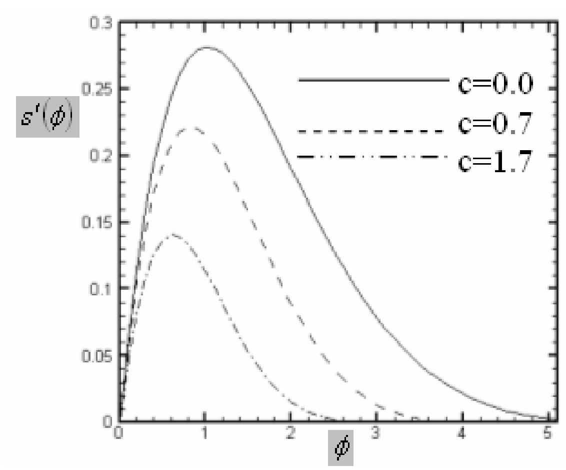

Figure-2. Variation of the dimensionless velocities against ϕ along y-direction for different values of c (β, k_1, R and Pr are fixed).

Figures 3 and 4 illustrate the dimensionless velocity profiles along x and y-directions, respectively for fixed value of $c = 0.0, k_1 = 0.3, R = 1$ and $\Pr = 0.72$ with several value of β. It is observed that both the velocity profile decreases with the increase of the parameter β.

Figures 5 and 6 display the variation of dimensionless velocity profiles along x and y-directions, respectively for fixed value of $c = 0.0, \beta = 0.3, R = 1$ and $\Pr = 0.72$ with several value of k_1. It is seen from this Figure that both the velocity profile increases with the increases of the parameter k_1 from which we conclude that the fluid velocity rises due to greater thermal-diffusion.

Figures 7 and 8 exhibits the behavior of dimensionless velocity profiles along x and y-directions, respectively for fixed value of $c = 0.0, k_1 = 0.3, \beta = 0.3$ and $\Pr = 0.72$ with some selected value of R. Here that both the velocity profile has the same displacement as the values of the parameter R increases and later velocity profile crosses the prior velocity profile.

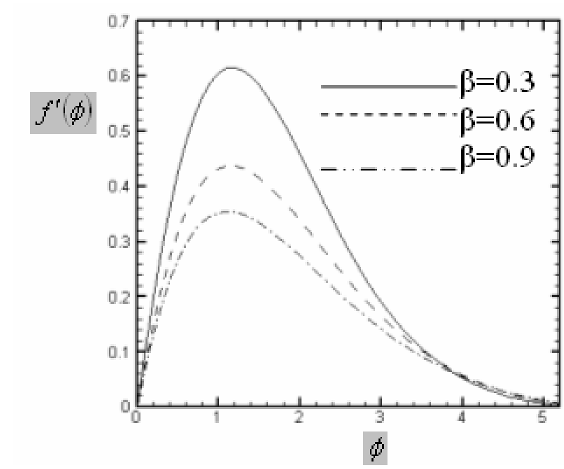

Figure-3. Variation of the dimensionless velocities against ϕ along x-direction for different values of β (c, k_1, R and Pr are fixed).

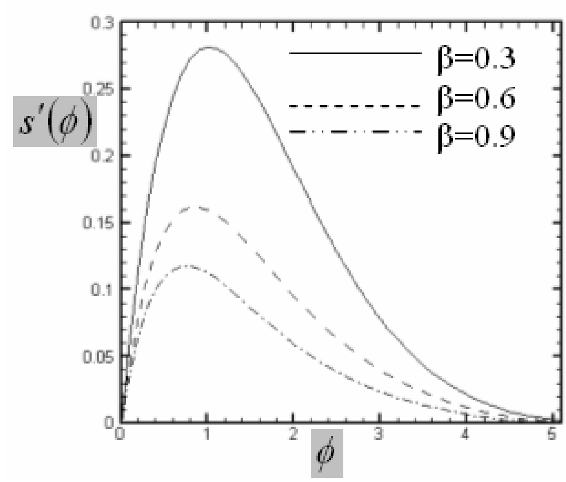

Figure-4. Variation of the dimensionless velocities against ϕ along y-direction for different values of β (c, k_1, R and Pr are fixed).

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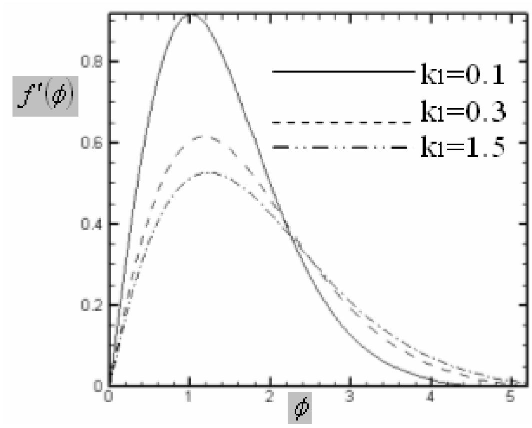

Figure-5. Variation of the dimensionless velocities against ϕ along x-direction for different values of k_1 (c, β, R and Pr are fixed).

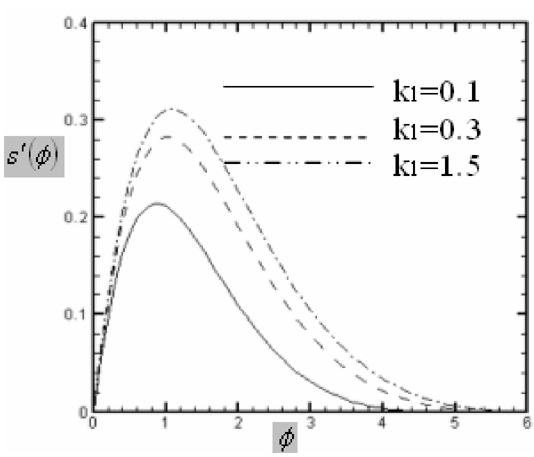

Figure-6. Variation of the dimensionless velocities against ϕ along y-direction for different values of k_1 (c, β, R and Pr are fixed).

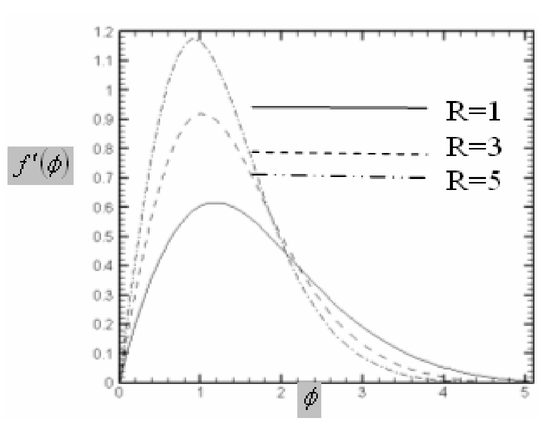

Figure-7. Variation of the dimensionless velocities against ϕ along x-direction for different values of R (c, β, k_1 and Pr are fixed).

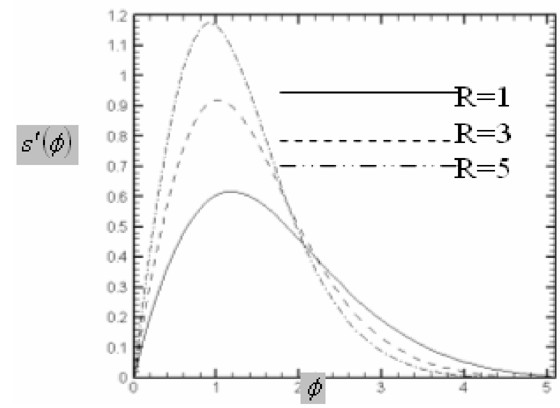

Figure-8. Variation of the dimensionless velocities against ϕ along y-direction for different values of R (c, β, k_1 and Pr are fixed).

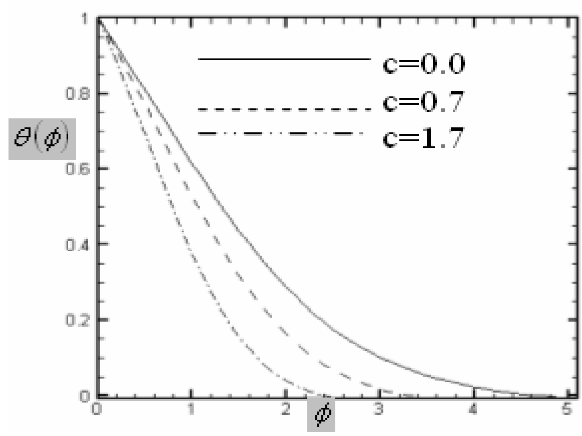

Figure-9. Variation of the dimensionless temperature against ϕ along x-direction for different values of c (β, k_1, R and Pr are fixed).

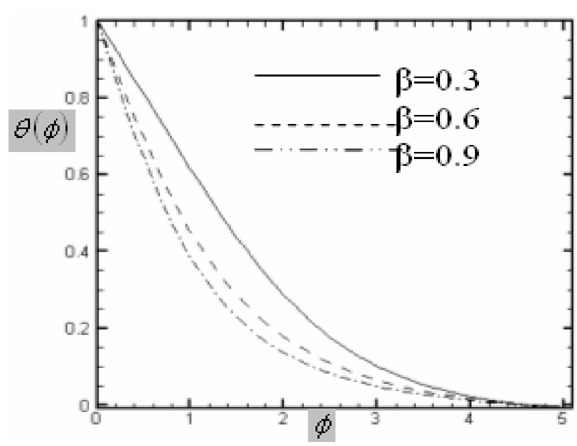

Figure-10. Variation of the dimensionless temperature against ϕ along x-direction for different values of β (c, k_1, R and Pr are fixed).

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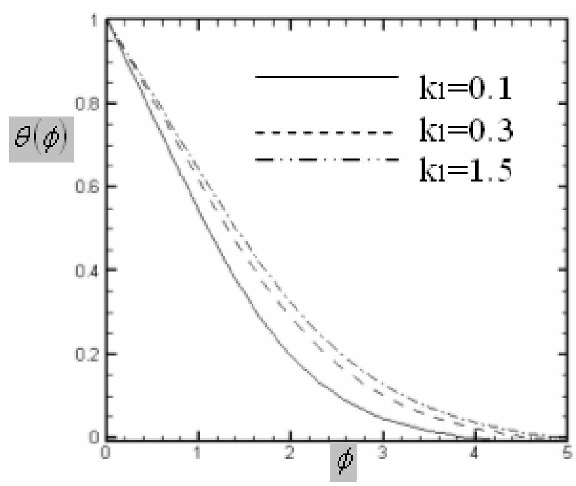

Figure-11. Variation of the dimensionless temperature against ϕ along x-direction for different values of k_1 (c, β, R and Pr are fixed).

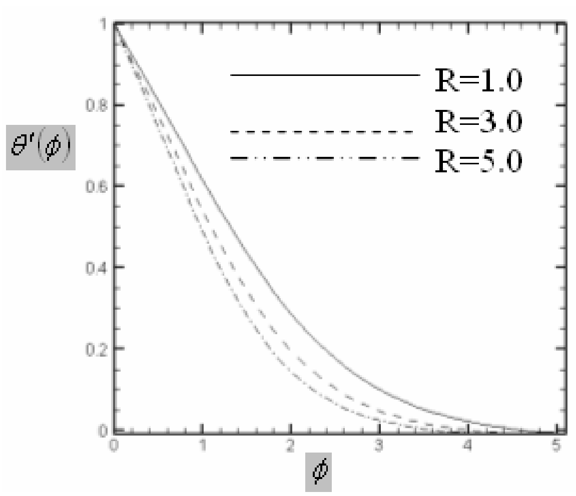

Figure-12. Variation of the dimensionless temperature against ϕ along x-direction for different values of R (c, β, k_1 and Pr are fixed).

Figure-9 presents the dimensionless temperature profiles for fixed value of $\beta = 0.3$, $k_1 = 0.3$, and $R = 1$ $\Pr = 0.72$ with selected value of c. We observe that the temperature profile decrease with the increasing values of the parameter c.

Figure-10 exhibits the behavior of dimensionless temperature profiles for β-variations for fixed value of $c = 0.0$, $k_1 = 0.3$, $R = 1$ and $\Pr = 0.72$. Here, the temperature profiles habitually decrease with increasing of β. The temperature near to the plate surface is large and away from the surface the temperature drops off asymptotically.

A reverse situation is observed in Figure-11 for the variation of the parameter k_1, a raise in temperature is examined as k_1 increases from 1.0 to 5.0.

Figure-12 displayed the variation of the dimensionless temperature profiles for fixed value of $c = 0.0$, $\beta = 0.3$, $k_1 = 1$ and $\Pr = 0.72$ with the values of the parameter R. We observe that the temperature profile decrease as the value of R increases.

Since the flow characteristics of the present problem are associated with the skin friction and heat transfer coefficients and are of practical interest, so the numerical results for $f''(0)$, $s''(0)$ and $\theta'(0)$ for variation of different parameters are presented in tabular forms. The variation of the coefficients of skin-friction along x and y-directions and the heat transfer coefficients with the variation of the parameters c, β, k_1 and R are displayed in Table-1 (a)-(b) and Table-2 (a)-(b), respectively.

From the Table-1 (a), we observe that the values proportional to the skin friction coefficients gradually decreases with increasing values of parameter c but the rate of decrease is more leading along y-direction $(s''(0))$ than x-direction $(f''(0))$. The variation of the dimensionless skin friction factors with the variation of the parameters β, k_1 and R along x and y-directions are also presented in Table-1 (b), Table-2 (a) and Table-2 (b) respectively, given below.

It can be observed from Table-2 (b) that the skin-friction coefficient in the x and y-direction increases as R increases. The values proportional to the variation of heat transfer coefficients $(-\theta'(0))$ with the variation of parameter c for fixed $\beta = 0.3$, $k_1 = 0.3$, $R = 1$, $\Pr = 0.72$ and β for fixed $c = 0.0$, $k_1 = 0.3$, $R = 1$, $\Pr = 0.72$ are shown in Table-1 (a)-(b), respectively.

Table-1. Variation of the coefficients of skin friction and heat transfer for different values of parameter (a) c (with $\beta = 0.3, k_1 = 0.3, R = 1$ and $\Pr = 0.72$) (b) β (with $c = 0.0, k_1 = 0.3, R = 1$ and $\Pr = 0.72$).

(a)

c	$f''(0)$	$s''(0)$	$\theta'(0)$
0.1	1.034560840	0.662554379	-0.378364551
0.3	0.924955131	0.652736787	-0.385210644
0.5	0.828371037	0.636187050	-0.398087688
0.7	0.745232677	0.614725437	-0.416365640
0.9	0.676796155	0.591539170	-0.437424159
1.1	0.620313035	0.567937456	-0.460609526
1.3	0.573194767	0.544644147	-0.485617789
1.5	0.535259504	0.523781561	-0.509151700
1.7	0.502662768	0.503741169	-0.533915911
1.9	0.475751275	0.486117761	-0.556691203

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(b)

β	$f''(0)$	$s''(0)$	$\theta'(0)$
0.1	1.640471340	1.173819440	0.449250465
0.2	1.264973090	0.811851171	-0.155709757
0.3	1.092851560	0.664281302	-0.377650067
0.4	0.988198088	0.581107820	-0.501164963
0.5	0.915786743	0.526328752	-0.584155744
0.6	0.861924680	0.486747057	-0.646150903
0.7	0.819615359	0.456343531	-0.695591060
0.8	0.785227425	0.431967110	-0.736804692
0.9	0.756436107	0.411810892	-0.772197621
1.0	0.731874099	0.394735891	-0.803313913

Table-2. Variation of the coefficients of skin friction and heat transfer for different values of parameter (a) k_1 (with $c = 0.0,\ \beta = 0.3,\ R = 1$ and $\Pr = 0.72$) (b) R (with $c = 0.0, \beta = 0.3,\ k_1 = 0.3$ and $\Pr = 0.72$).

(a)

k_1	$f''(0)$	$s''(0)$	$\theta'(0)$
0.1	1.640479980	0.584488045	-0.451787421
0.3	1.092851560	0.664281302	-0.377650067
0.5	1.008659520	0.682342271	-0.362830001
0.7	0.981658198	0.688812890	-0.357638941
0.9	0.969586539	0.691675487	-0.355401046
1.1	0.963428399	0.693234050	-0.354168905
1.3	0.959942394	0.694189279	-0.353399275
1.5	0.957552553	0.694742823	-0.352979020
1.7	0.955944997	0.695116411	-0.352695734
1.9	0.954812771	0.695380138	-0.352495921

(b)

R	$f''(0)$	$s''(0)$	$\theta'(0)$
1.0	1.09285156	0.66428130	-0.377650067
2.0	1.36080940	1.23987952	-0.416722268
3.0	1.64516365	1.75445705	-0.451264709
4.0	1.92708973	2.22992788	-0.480716950
5.0	2.20301222	2.67771952	-0.506223872
6.0	2.47309155	3.10461908	-0.528674340
7.0	2.73506986	3.51421759	-0.548954107
8.0	2.99110573	3.90986122	-0.567374103
9.0	3.24221630	4.29379992	-0.584244808
10.0	3.48943562	4.66777753	-0.599794620

The heat transfer factors with variation of parameters k_1 for fixed $c = 0.0,\ \beta = 0.3,\ R = 1,\ \Pr = 0.72$ and R for fixed $c = 0.0, \beta = 0.3,\ k_1 = 0.3,\ \Pr = 0.72$. are also presented in Table-2 (a)-(b), respectively.

CONCLUSIONS

An analysis is here made of unsteady laminar boundary Layer equations for free convection around a vertical heated curvilinear surface for establishing necessary and sufficient conditions under which similarity solution are possible. On the basis of the conditions one possible case has been derived and then solved numerically. The flow and temperature fields as well as the non-dimensional skin friction factors and heat transfer coefficients are determined for several selected values of the controlling parameter involved in equations (21) to (23). The physical behavior of the controlling parameters ($\Pr, c$ and $k_1 = U_F^2 / V_F^2$) used in equations (21) to (23) are, the Prandtle number (Pr) for air, c for the ratio between the changes of local boundary layer thickness with regard to position and time and $V_F^2 << U_F^2$ for a free convection flow. Further investigations are necessary to deal with the possible similarity cases in order to drawn overall remarks conclusively.

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