



## DIAGNOSIS OF FAULTS USING IMM ESTIMATOR

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### ABSTRACT

An approach to detection and diagnosis of multiple faults is proposed. It is based on Interacting Multiple Model (IMM) algorithm. The proposed approach provides means of detection, diagnosis of faults and state estimation. It is able to detect and diagnose the fault more quickly and reliably than other methods. In this paper the threshold values of various types of faults is also proposed. This is illustrated using an Aircraft example which has multiple failures-sensor, actuator and other component failures.

**Keywords:** multiple model estimation, fault diagnosis, state estimation, threshold, probability matrix.

### 1. INTRODUCTION

Fault Detection and Diagnosis (FDD) has become increasingly important in industrial processes, due to growing demands on operational reliability, safety and product quality. The general idea is to use a scheme based on measured process data to detect a fault occurrence in a physical process, e.g. an actuator fault, to detect and isolate the fault location in the process and to identify the magnitude and time of occurrence of the fault.

In flight control system, failures of actuator or sensor may cause serious problems and has to be detected and isolated as soon and as accurately as possible. Systems subjected to such failures cannot be modelled well by single set of state equations that vary continuously. A more appropriate mathematical model for such system is called stochastic hybrid model system. It differs from conventional system such that its states may jump as well as vary continuously. Hybrid systems have also been used in areas such as target tracking and control involving structural changes [1].

One of the effective methods is based on the Multiple Models (MM) estimation. Research on the multiple-model (MM) approach has attracted considerable interest in the last decades. The reason for this is the elegant solutions that the MM approach provides for estimation, control and modelling problems. A well known example to MM estimation is the target tracking problem. Another important estimation application of the MM framework is the fault detection and diagnosis (FDD) scheme.

The main motivation for using the MM framework for FDD is that it allows for a large class of fault conditions to be modelled. MM allows for the modelling of actuator, sensor as well as component faults. The reason for this is that each of the local models might have totally different dynamics. The basic steps of performing FDD with MM systems are as follows: a model set must be created that contains local models corresponding to different fault conditions and normal condition of the system. Multiple model has a bank of filters in parallel, each based on a model matching to a particular mode (i.e., normal and structural or behavioural pattern) of the system. The overall state estimate is

calculated by the probabilistically weighted sum of the outputs of all filters. MM algorithms have been developed for different application problems such as multiple hypothesis test detector and multiple model adaptive estimation (MMAE) algorithm [2]. In addition, an observer scheme which uses a bank of observers for fault detection and diagnosis of deterministic systems was devised in [3]. The above filter based approaches are based on the “non interacting” MM estimation: the single model based filters are running in parallel without mutual interaction (i.e., each filter operates independently at all times). This approach is effective in handling problems with an unknown structure or parameter but without structural or parametric changes. Since the system has structure or parameter changes when there is a sensor or actuator failure, this approach becomes ineffective.

A recent advance in MM estimation is the development of Interacting Multiple Model (IMM) estimation. It overcomes the disadvantage of non interacting MM approach by modelling the abrupt changes of the system by switching from one model to another in a probabilistic manner. Since structure of the system is considered, the IMM algorithm is more promising for the FDD scheme. The IMM differs from the noninteracting MM algorithm in that the single model based filters interact with each other and thus resulting in improved performance. The initial estimate at the beginning of each cycle for each filter is a mixture of all estimates from the single model based filters. The mixing of estimates helps in yielding a more fast and accurate estimate for the system states. The other feature is that the probability of each mode is calculated which clearly indicates the mode in effect and mode transition at each time. Its main advantage over the MM based FDD approach is that both single and multiple failures can be detected and identified more quickly and reliably. This is demonstrated using aircraft example with sensor and actuator failures.

This paper is organized as follows. Modelling of multiple failures are presented in section II. The FDD scheme based on IMM approach is presented in section III. In section IV the detection and diagnosis for sensor and actuator failures of aircraft model are discussed. Conclusion is given in section V.



## 2. MODELLING OF MULTIPLE FAULTS

### A. Hidden Markov chain model for system with failures

The model of the system with potential failures can be expressed as:

$$x(k+1) = F(k, m(k+1))x(k) + G(k, m(k+1))u(k) + \Xi(k, m(k+1))\varepsilon(k, m(k+1)) \quad (1)$$

$$z(k) = H(k, m(k))x(k) + \eta(k, m(k)) \quad (2)$$

Where

$x \in R^n$  is the state vector;  $z \in R^p$  is the measurement vector;  $u \in R^l$  is the control input vector;  $\varepsilon(k) \in R^n$  and  $\eta(k) \in R^p$  are independent discrete-time random process with mean  $\bar{\varepsilon}(k)$  and  $\bar{\eta}(k)$  co variances  $Q(k)$  and  $R(k)$ , representing system and measurement noises respectively. It is assumed that the initial state has a mean  $\bar{x}_0$  and a covariance  $\bar{P}_0$ , and they are independent from  $\varepsilon(k)$  and  $\eta(k)$ . The system (1)-(2) is known as a "jump linear system". It can be seen from (2) that the state observations are noisy and mode dependent. Therefore, the mode information is imbedded in the measurement sequence. In other words, the system mode sequence is an indirectly observed (or hidden) Markov chain.

Suppose that a discrete-time process which represents the possible system structural/parametric changes due to failures is represented by a first-order Markov chain with state  $m(k)$  taking values in a finite set  $S = 1, 2, \dots, s$ . At each time step, the transition probabilities of the chain can be defined by:

$$\pi_{ij}(k) = P\{m(k) = j | m(k-1) = i, i, j \in S\} \quad (3)$$

and

$$\sum_{j \in S} \pi_{ij}(k) = 1, \quad i = 1, 2, \dots, s \quad (4)$$

where  $P\{\cdot\}$  denotes the probability;  $m(k)$  is the discrete-valued modal state (i.e., the indicator of the normal or the fault mode) at time  $k$ ;  $\pi_{ij}$  is the transition probability from the mode  $i$  to the mode  $j$ ; the event that  $m_j$  is in effect at time  $k$  is denoted as  $m_j(k) \cong \{m(k) = j\}$ .

### B. Multiple model representation of system failures

The performance of an MM algorithm depends on the model set used. Based on the system model (1)-(2), it is possible to represent different failures in the system.

$$x(k+1) = (F(k) + \Delta F_j(k))x(k) + (G(k) + \Delta G_j(k))u(k) + \Xi_j(k)\varepsilon_j(k) = F_j(k)x(k) + G_j(k)u(k) + \Xi_j(k)\varepsilon_j(k) \quad (5)$$

$$z(k) = (H(k) + \Delta H_j(k))x(k) + \eta_j(k) = H_j(k)x(k) + \eta_j(k) \quad j = 1, \dots, N \quad (6)$$

Where

$\Delta F_j(k)$ ,  $\Delta G_j(k)$  and  $\Delta H_j(k)$  ( $j = 2, \dots, N$ ) represent the fault-induced changes in the system components, actuators and sensors, respectively. They should be null matrices when  $j = 1$  which denotes normal system. The subscript  $j$  denotes quantities pertaining to the model  $m_j \in M$ .  $M = [m_1, m_2, \dots, m_N]$  is a set of all system models representing the normal system and the system with all considered faults. Matrices  $F_j(k)$ ,  $G_j(k)$  and  $H_j(k)$  corresponds to the  $j$ th post fault models of the system.

Designing set of models is the key issue. This design should be done such that the models represent or cover all possible system modes at any time. This is the model set design. This design (i.e., the design of fault type, fault magnitude and duration) is crucial for the success in FDD. Design of good set of models requires a prior knowledge of the possible faults of the system. Faults can occur in sensors, actuators and other components of the system and may lead to failure of the whole system. They can be modelled by the abrupt changes of the components of the system. Failures can be of "total" or "partial" in nature.

## 3. FAULT DETECTION AND DIAGNOSIS SCHEME USING IMM ESTIMATOR

### A. IMM Estimator

For estimation, the following tasks should be completed: model set design, filter selection, estimate fusion and filter reinitialization [1].

Filter selection is to select each single model based recursive filter for each model, such as a Kalman filter for a linear system or an extended kalman filter for a nonlinear system. Estimate fusion combines the model-conditional estimates to yield the overall estimate. Three approaches are available: soft decision, hard decision and random decision. Re initialization of single model based filter is very important for estimation. This is done by using previous overall estimate and covariance of all filters.

The IMM algorithm is a recursive estimator with the following steps in each iteration:

- interaction of the model-conditional estimates
- model-conditional filtering
- mode probability update
- estimates combination

In the first step, the input to the filter matched to a certain mode is obtained by mixing the estimates of all filters from the previous iteration under the assumption that this particular mode is in effect at the present time; a bank of filters corresponding to different models is calculated in parallel in the second step; mode probability is then updated based on the model-conditional innovations and the likelihood functions; finally, the aggregated state estimate is obtained as a probability-weighted sum of the updated state estimates from all the filters.



The probability of the mode in effect plays a key role in determining the weights in the combination of state estimates and covariances for aggregated state estimate. The step-1 is unique for the IMM estimator and existing non interacting MM algorithms. It is because of this mixture of the estimates that makes the estimation for the state and identification for the system mode more responsive to the system changes, thus leads to significantly better FDD performance.

The IMM estimator [4] is generally considered to be one of the most cost-effective schemes for state estimation involving both continuous and discrete states. It

has been successfully used in a number of applications, e.g. maneuvering target tracking [5, 6], and FDD [7]. Figure-1 shows a block diagram of the IMM estimator for FDD.

### B. FDD Scheme

In active Fault Tolerant Control Systems (FTCS), timely and correct detection and diagnosis of a fault is crucial for good performance. Using the IMM estimator, it is effective to use the model probabilities to provide an indication of the mode in effect at a given time.

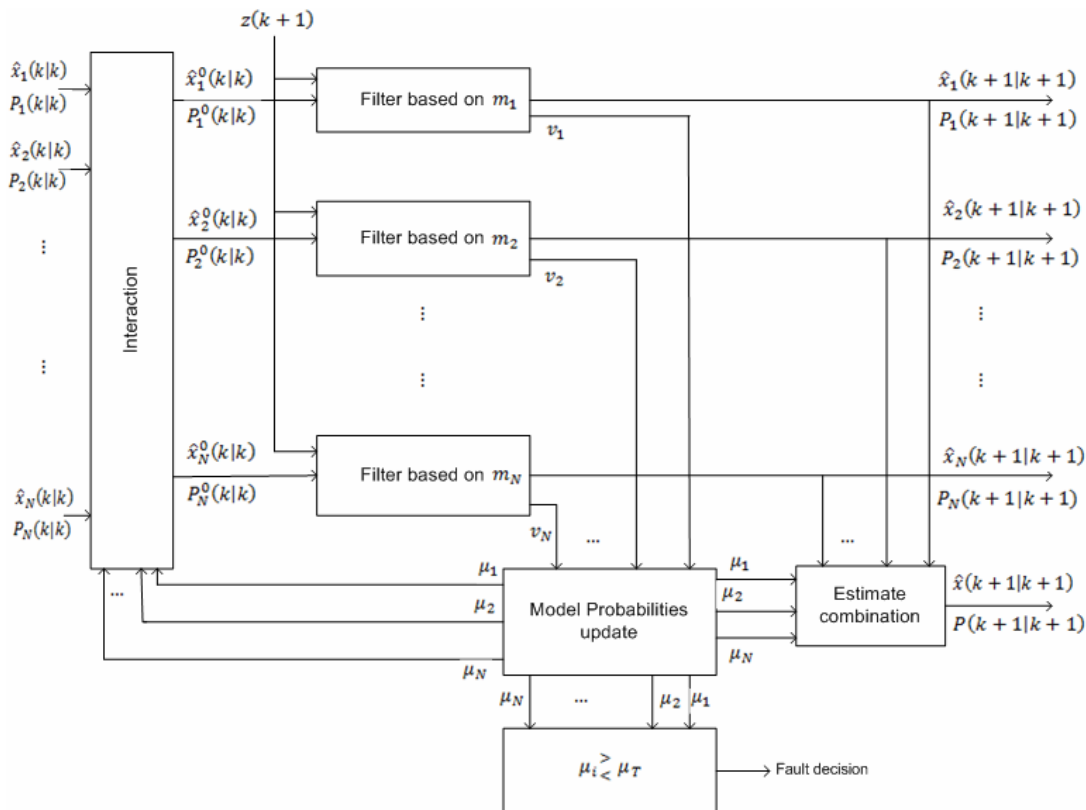


Figure-1. Block diagram of IMM estimator.

Hence, it can be used as an index for FDD. The fault detection decision can be made by the following rule:

$$\mu_j(k+1) = \max_i \mu_i(k+1) \quad (7)$$

$$\begin{cases} > \mu_T \rightarrow H_j : \text{fault } j \text{ occurred} \\ \leq \mu_T \rightarrow H_1 : \text{no fault} \end{cases}$$

Where

$\mu_T, 0 \leq \mu_T \leq 1$  is the detection threshold.

A complete cycle of the IMM-based FDD scheme with Kalman filters as its mode-matching filters is summarized in Table-1.

**Table-1.** One cycle of IMM based FDD scheme.

<b>1. Interaction /Mixing of the estimates (for <math>j = 1, \dots, N</math>)</b>	
Predicted mode probability:	$\mu_j(k+1 k) = P\{m_j(k+1 z^k)\} = \sum_i \pi_{ij} \mu_i(k)$
Mixing probability:	$\mu_{(i j)}(k) = P\{m_i(k) m_j(k+1), z^k\} = \frac{\pi_{ij} \mu_i(k)}{\mu_j(k+1 k)}$
Mixing estimate:	$\hat{x}_j^0(k k) = E\{x(k) m_j(k+1), z^k\} = \sum_i \hat{x}_i(k k) \mu_{(i j)}(k)$
Mixing covariance:	$\hat{P}_j^0(k k) = cov\{\hat{x}_j^0(k k) m_j(k+1), z^k\}$ $= \sum_i \{P_i(k k) + [\hat{x}_j^0(k k) - \hat{x}_i(k k)][\hat{x}_j^0(k k) - \hat{x}_i(k k)]'\} \mu_{(i j)}(k)$
<b>2. Model-conditional filtering (for <math>j = 2, \dots, N</math>)</b>	
Predicted state (from k to k+1):	
	$\hat{x}_j(k+1 k) = E\{x(k+1) m_j(k+1), z^k\} = F_j(k) \hat{x}_j^0(k k) + G_j(k) u(k) + T_j(k) \bar{e}_j(k)$
Predicted covariance:	
	$P_j(k+1 k) = cov\{\hat{x}_j(k+1 k) m_j(k+1), z^k\} = F_j(k) P_j^0(k k) F_j(k)' + T_j(k) Q_j(k) T_j(k)'$
Measurement residual: $v_j = z(k+1) - E\{z(k+1) m_j(k+1), z^k\}$	
	$= z(k+1) - H_j(k+1) \hat{x}_j(k+1 k) - \bar{\eta}_j(k+1)$
Residual covariance: $S_j = cov\{v_j m_j(k+1), z^k\}$	
	$= H_j(k+1) P_j(k+1 k) H_j(k+1)' + R_j(k+1)$
Filter gain: $K_j = P_j(k+1 k) H_j(k+1)' + S_j(k+1)^{-1}$	
Updated state:	
	$\hat{x}_j(k+1 k+1) = E\{x(k+1) m_j(k+1), z^{k+1}\} = \hat{x}_j(k+1 k) + K_j v_j$
Updated covariance: $P_j(k+1 k+1) = cov\{\hat{x}_j(k+1 k+1) m_j(k+1), z^{k+1}\}$	
	$= P_j(k+1 k) - K_j(k+1) S_j(k+1) K_j(k+1)'$
<b>3. Mode probability update and FDD logic (for <math>j = 2, \dots, N</math>):</b>	
Likelihood function: $L_j(k+1) = N\{v_j(k+1); 0, S_j(k+1)\}$	
	$= \frac{1}{\sqrt{( \Sigma  (2\pi) S_j(k+1))  \Sigma }} \exp\left\{-\frac{1}{2} v_j(k+1)' S_j^{-1}(k+1) v_j(k+1)\right\}$
Mode probability :	
	$\mu_j(k+1) = P\{m_j(k+1), z^{k+1}\} = \frac{\mu_j(k+1 k) L_j(k+1)}{\sum_i \mu_i(k+1 k) L_i(k+1)}$
Fault decision: $\mu_j(k+1) \underset{i \mu_i(k+1)}{\max} \equiv \begin{cases} \mu_T \rightarrow H_j : \text{fault } j \text{ occurred} \\ \leq \mu_T \rightarrow H_1 : \text{no fault} \end{cases}$	
<b>4. Combination of estimates</b>	
	$\hat{x}(k+1 k+1) = E\{x(k+1) z^{k+1}\} = \sum_j \hat{x}_j(k+1 k+1) \mu_j(k+1)$
Overall estimate:	
Overall covariance:	
	$P(k+1 k+1) = E\{[x(k+1) - \hat{x}(k+1 k+1)][x(k+1) - \hat{x}(k+1 k+1)]'   z^{k+1}\}$ $= \sum_j \{P_j(k+1 k+1) + [\hat{x}(k+1 k+1) - \hat{x}_j(k+1 k+1)][\hat{x}(k+1 k+1) - \hat{x}_j(k+1 k+1)]'\} \mu_j(k+1)$



### C. Design of Markov transition probability matrix

The design parameter for the IMM algorithm includes the transition probability matrix, the covariances of the process noise and measurement noise. The performance also depends on the type and magnitude of control input excitation used. However the design of transition probability matrix is unique and important for the IMM based approach.

A proper choice of the diagonal entries in the transition probability matrix is to match roughly the mean sojourn time of each mode [8],

$$\pi_{jj} = \max\left\{l_j, 1 - \frac{T}{\tau_j}\right\} \quad (8)$$

where  $\tau_j$  is the expected sojourn time of the  $j$  th mode;  $\pi_{jj}$  is the probability of transition from  $j$  th mode to itself and  $T$  is the sampling interval;  $l_j$  is a designed lower limit for the  $j$  th model transition probability. For example the “normal to normal” transition probability  $\pi_{11}$ , can be obtained by  $\pi_{11} = 1 - \frac{T}{\tau_1}$ , where  $\tau_1$  denotes the mean time between failures (MTBF). Normally  $T$  is much smaller than MTBF. The transition probability from the normal mode to a fault mode is equal to  $1 - \pi_{11}$ . Which particular fault mode it jumps to depend on the relative likelihood of the occurrence of the fault mode. “Fault to fault” transitions are normally disallowed except in the case where there is sufficient prior knowledge to believe that partial faults can occur one after another.

## 4. ILLUSTRATIVE EXAMPLE

### A. Aircraft model

The linearized model of the aircraft under the normal condition can be described as:

$$\begin{aligned} \dot{x}(t) &= Fx(t) + Gu(t) \\ z(t) &= Hx(t) \end{aligned} \quad (9)$$

Where, the state and the input vectors are:  $x = [p \ r \ \beta \ \varphi]^T$  and  $u = [\delta_a \ \delta_r]^T$ , respectively,  $p$  with representing the roll rate,  $r$  the yaw rate,  $\beta$  the sideslip angle,  $\varphi$  the bank angle,  $\delta_a$  the aileron deflection, and  $\delta_r$  the rudder deflection. The matrices are:

$$A_0 = \begin{bmatrix} -3.5980 & 0.1968 & -35.18 & 0 \\ -0.0377 & -0.3576 & 5.8840 & 0 \\ 0.0688 & -0.9957 & -0.2163 & 0.0733 \\ 0.9947 & -0.1027 & 0 & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 14.65 & 6.538 \\ 0.2179 & -3.087 \\ -0.0054 & 0.0516 \\ 0 & 0 \end{bmatrix}$$

Here only two out of four state variables sideslip and bank angle, are measurable. For simplicity, these two variables will be designated as the controlled variables. Hence the output matrices,  $H_j$  become:

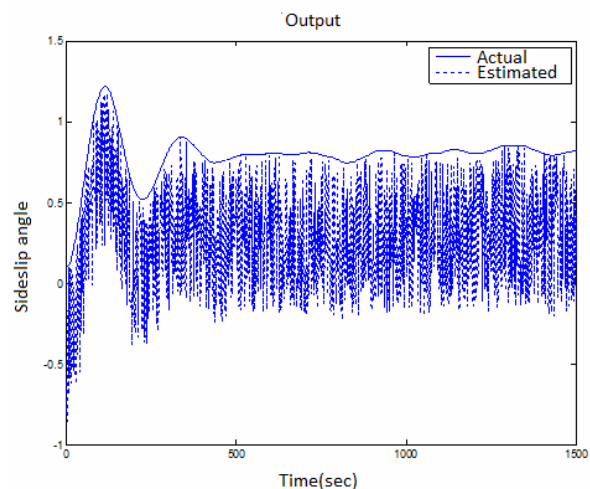
$$H_j = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad j = 1, \dots, N$$

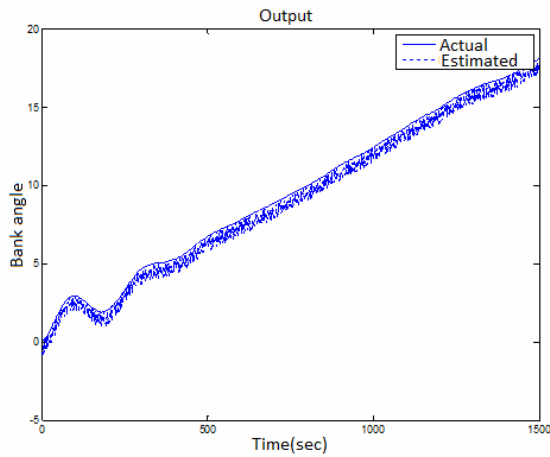
The specific faults are: 1) a system dynamic fault as a result of a partial loss of the rudder control surface, 2) a fault in either one of the two actuators, and 3) a fault in sideslip angle sensor. Therefore, there are total of 5 possible operating modes. In practice, if additional fault scenarios or the same fault type but with different severities need to be considered, more fault modes would have to be included in the model set. The above considered fault modes  $\Delta F_j, \Delta G_j, \Delta H_j, j = 1, \dots, 4$ .

## B. SIMULATION RESULTS

The actuator faults result in reduced values in the corresponding columns of the control matrix  $G$ , the sensor fault is represented also by a reduction in the corresponding row of the measurement matrix  $H$  and the loss of control surface is reflected as the changes in both  $F$  and  $G$  matrices.

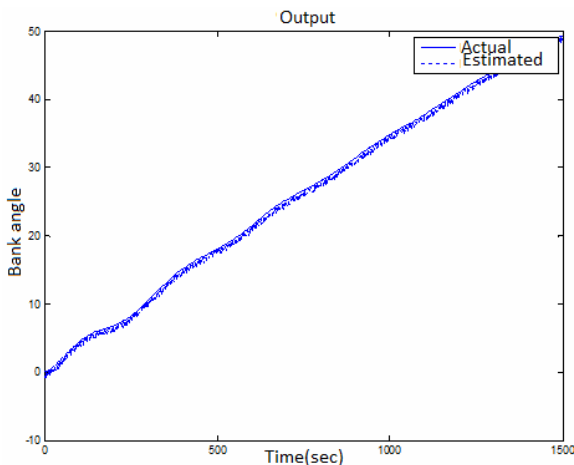
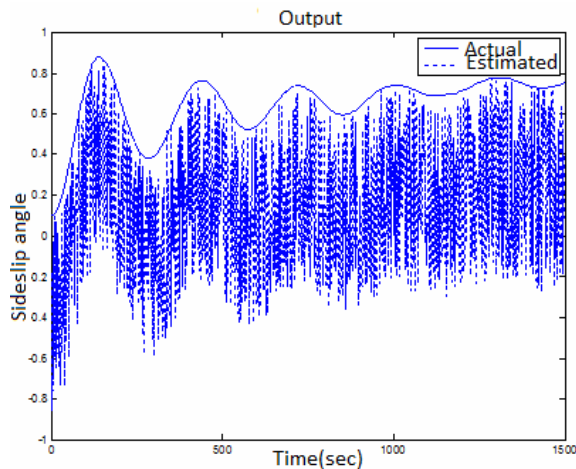
The system matrix is discretized and the IMM estimator is simulated for the normal node and the faulty cases. The normal mode output is shown in Figure-2. As seen in figure, the estimator tracks the normal mode output is continuously estimated and tracked. Thus any fault occurs in the system is detected and diagnosed immediately.





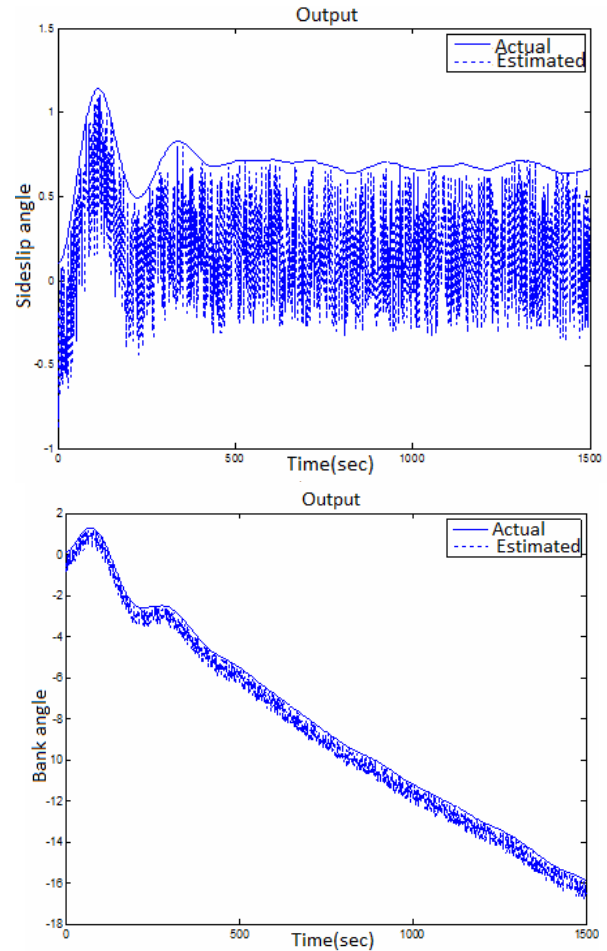
**Figure-2.** Normal mode output.

The possible faults in the system are simulated as shown here. First, the dynamic fault which is the partial loss of rudder control surface (50%) is shown in Figure-3. As seen, the dynamic fault results in increased amplitude of sideslip angle and bank angle.

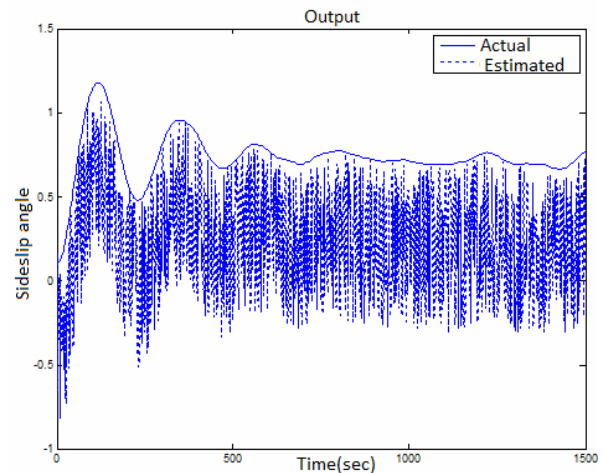


**Figure-3.** Normal mode output with continuous dynamic fault.

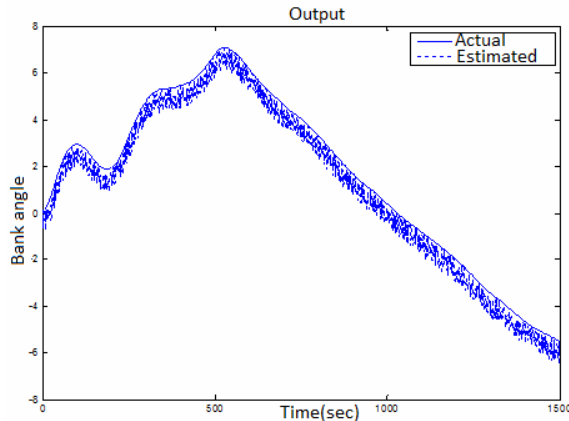
Next, the faults in actuators are shown in Figure-4. The faults in actuators usually represent the loss of effectiveness in rudder or aileron.



**Figure-4.** Normal mode output with continuous aileron fault.







**Figure-5.** Normal mode output with abrupt aileron fault introduced at  $t = 5$  sec.

The IMM estimator continuously monitors the system and estimates the states of the system. The actuator 1 fault (i.e., aileron fault) is introduced at time  $t = 5$  sec. The response of the system with fault is shown in Figure-5.

The decision rule (7) provides not only fault detection but also the information of the type (sensor or actuator), location (which sensor or actuator), size (total failure or partial fault with the fault magnitude) and fault occurrence time, i.e., simultaneous detection and diagnosis. In this approach, there is no need to set arbitrary threshold levels to balance false alarms against missed fault detections. Thus the mode probabilities of the normal and faulty modes are calculated from (7) which can be used as a detection threshold to find the type and time of occurrence of fault in the system.

The threshold values of various types of faults are shown.

**Table-2.** Faults and threshold values.

Faults	Threshold values
Dynamic fault	0.64259
Aileron fault	0.43986
Rudder fault	0.54047
Sensor fault	0.4121

From this, we can see that the dynamic fault and actuator fault are predominant and affects the system more. These faults have to be taken care before it results in catastrophic results.

## 5. CONCLUSIONS

In this paper, based on IMM estimation algorithm, a new FDD approach for the multiple failures in the system has been proposed. This helps in effective fault detection, diagnosis and state estimation. An aircraft example with normal and different fault conditions are demonstrated. This approach is significantly better in terms of robustness and timeliness in detecting faults than

other multiple model approaches. Future work includes the design of reconfigurable control for this multiple model approach.

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