



## PRESSURE AND PRESSURE DERIVATIVE ANALYSIS FOR SLANTED AND PARTIALLY PENETRATING WELLS

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### ABSTRACT

A well that penetrates the producing interval with a slant angle,  $\psi$  is called an inclined or slant well. If the well does not penetrate the entire formation thickness of the producing strata, it is called a partially penetrating or limited-entry well. For such cases, the slant angle, penetration length and penetration ratio significantly influence the characteristics of the pressure-transient responses. It is relevant the role of the ellipsoidal flow regime displayed at early time - before radial flow - which we recognized as a straight line exhibiting a -0.135 slope on the pressure derivative curve. Hence, this paper deals with the characterization of such flow regime and a methodology, using the pressure derivative, is introduced so the vertical and effective permeability is calculated. The proposed methodology was successfully applied to synthetic test data.

**Keywords:** well, transient pressure analysis, radial flow, ellipsoidal flow.

### 1. INTRODUCTION

The transient study of deviated wells was initiated by Roemershauer *et al.* (1955) who analyzed steady-state flow in an infinite reservoir producing through a slant well using an electrical model. They concluded that a fully-penetrated deviated well caused an increase in productivity since a negative skin factor was created due to the deviation angle and the formation thickness. This study was later complemented by Cinco-Ley *et al.* (1975), who developed an analytical solution for the dimensionless pressure distribution and concluded that three flow regimes are presented in those systems: early radial, transition and pseudoradial. Before year 2000, several researchers were conducted on the factors affecting the behavior of deviated and partially penetrated wells. To name a few of them: Kuchuk and Kirwan (1987), Besson (1990) and Larsen (1993). Ozkan and Raghavan (2000) presented an analytical solution in the Laplace-transform domain solutions which are the basis for the present work.

In this study, the pressure derivative behavior for the system under consideration is analyzed. Unique features are identified to determine several reservoir parameters. The most outstanding issue is the identification of the ellipsoidal flow regime which is observed to exhibit a -0.135 slope on the pressure derivative plot. An equation was developed for this flow so that vertical permeability is determined. Using this and the horizontal permeability, the effective reservoir permeability is then estimated. Verification of the developed expressions was achieved by their application to synthetic cases.

### 2. SIMULATION EXPERIMENTS

Aiming to observe the early transient behavior of a deviated and partially penetrated well several simulation runs were performed using the analytical solution provided by Ozkan and Raghavan (2000) so pressure and pressure

derivative curves were generated with the input data reported in Table-1.

The penetration length,  $h_w$ , varies in the range of  $10 \leq h_w \leq 200$ , the deviation angle,  $\psi$  varies in the range  $0 \leq \psi \leq 90$  and the permeability anisotropy,  $k_z/k_r$ , varies in the range  $10 \leq k_z/k_r \leq 90^\circ$ . For a better understanding of the well geometry refer to Figure-1. Results for three cases are reported in Figures 2 to 4. Most of the observations found on these plots were already reported by Kamal (2009). Among some observations we can point out that several flow regimes may be developed depending mainly upon well inclination. Notice that the deviation angle is measured with respect to the vertical axis, therefore, a wellbore lies horizontally for  $\psi = 90^\circ$  and vertically for  $\psi = 0^\circ$ . Hence, as the well length becomes larger and the angle is closed to  $90^\circ$ , an early radial flow regime is seen since the well behaves as horizontal. As the well length becomes shorter and shorter the well behaves as a source point and spherical flow is observed.

An especial condition is experimented for ranges of well penetration ratios ( $h_w/h$ ) approximately between 20 and 70% where a -0.135 slope on the pressure derivative curve is displayed. This feature was seen in almost all the simulations and it is known in the literature as ellipsoidal flow. This flow regime is hardly seen in a pressure tests, however, it normally occurs at very early time of flow around a perforation as well explained by Lea *et al.* (1992), see Figure-5. Since early radial and spherical flow regimes have already been characterized, this paper will focused on the characterization of ellipsoidal flow regime. Wellbore storage effects ought to be mitigated in order to observe such flow regime.

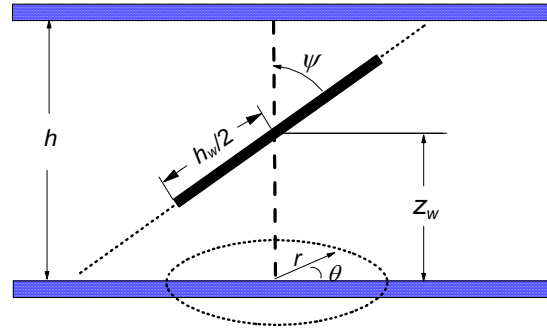


**3. MATHEMATICAL FORMULATION**

At early time, the dimensionless quantities are conveniently given as a function of the ellipsoidal permeability (tridimensional),  $k_{elp}$ :

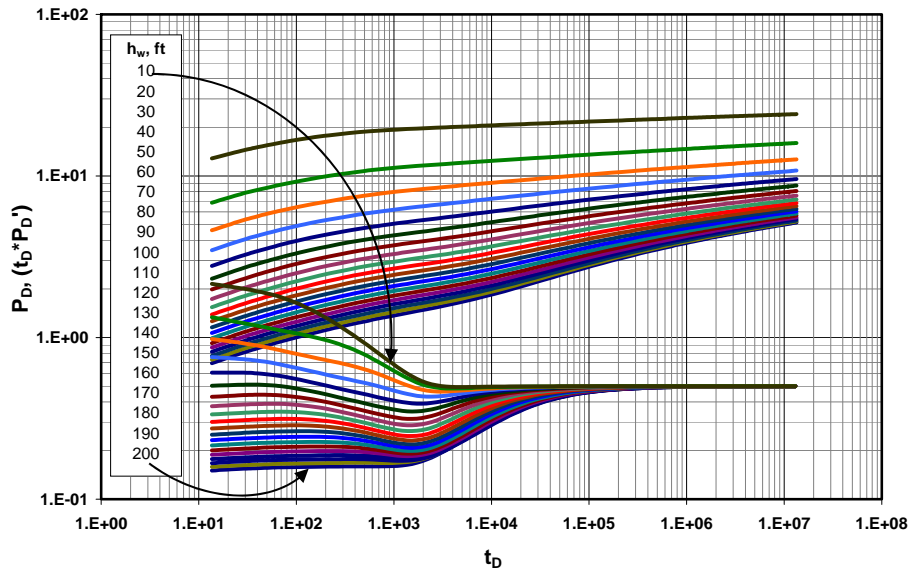
$$t_D * P_D' = \frac{k_{elp} h (t * \Delta P')}{141.2 q \mu B} \tag{1}$$

$$t_D = \frac{0.0002637 k_{elp} t}{\phi \mu c_i h_w^2} \tag{2}$$



**Figure-1.** Schematic of the general-slanted well model, after Kamal (2009).

The pressure derivative governing equation was found to be;



**Figure-2.** Effect of the deviation angle,  $\psi$  permeability ratio,  $k_z/k_r$ , and penetration length,  $h_w$  on the pressure and pressure derivative behavior.  $\psi = 80^\circ$  and  $k_z/k_r = 10$ .

**Table-1.** Data for simulation input and examples.

Parameter	Value	Synthetic example 1	Synthetic example 2	Field example
$k_h$ , md	33.33	100	220	-----
$h$ , ft		200	120	111
$k_z$ , md	-----	10	73.34	-----
$\phi$ , %		22	10	20
$c_i$ , psi <sup>-1</sup>	1.7E-5	3x10 <sup>-6</sup>	1x10 <sup>-5</sup>	1.32x10 <sup>-5</sup>
$P_i$ , psia		5000	3200	4433
$r_w$ , ft		0.4	0.25	0.25
$q$ , bpd	500	300	120	391
$\mu$ , cp	0.54	3	8	0.718
$B$ , rb/STB	1.6	1.2	1.1	1.3
$h_w$ , ft		60	50	55.5
$\psi$ , °	----	30	55	30

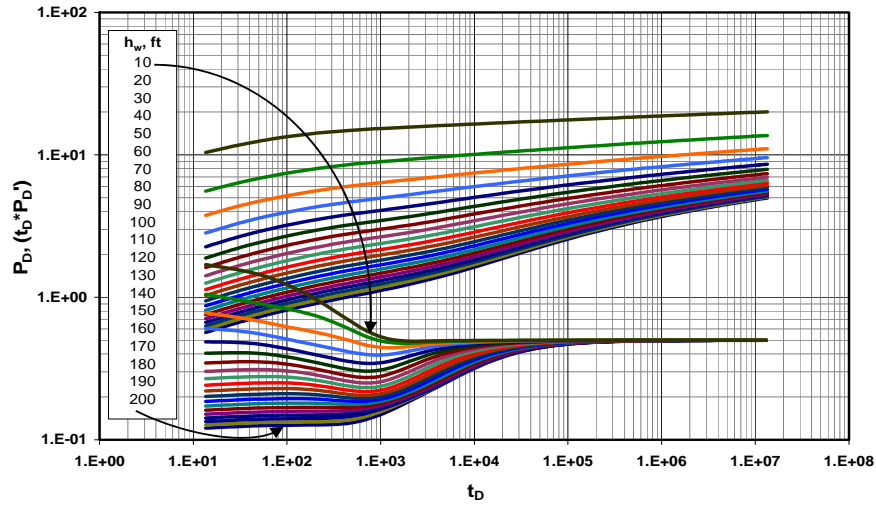


Figure-3. Effect of the deviation angle,  $\psi$  permeability ratio,  $k_z/k_r$ , and penetration length,  $h_w$  on the pressure and pressure derivative behavior.  $\psi = 60^\circ$  and  $k_z/k_r = 20$ .

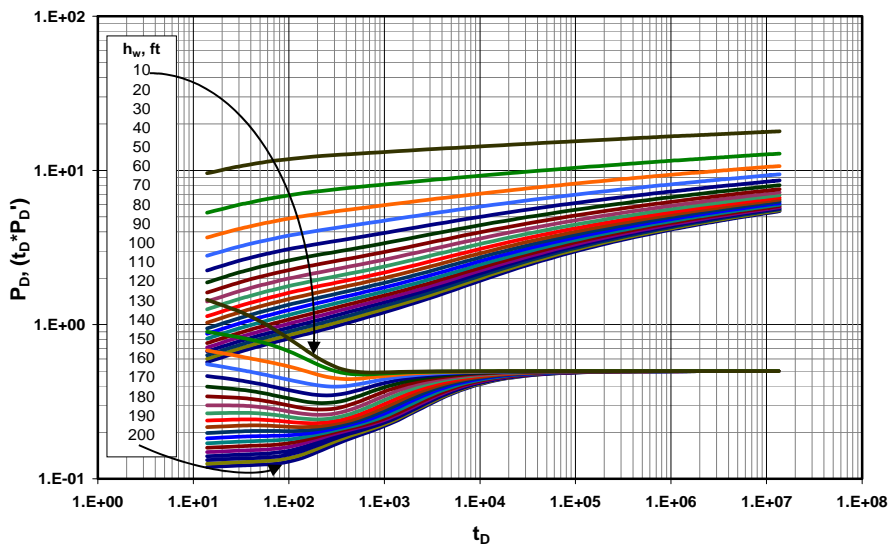


Figure-4. Effect of the deviation angle,  $\psi$  permeability ratio,  $k_z/k_r$ , and penetration length,  $h_w$  on the pressure and pressure derivative behavior.  $\psi = 30^\circ$  and  $k_z/k_r = 60$ .

$$\log(t_D * P_D')_{elp} = -0.135 \log(t_D)_{elp} + \log \Xi \quad (3)$$

For a dimensionless time of one, the value of the intercept  $\Xi$  can be found as:

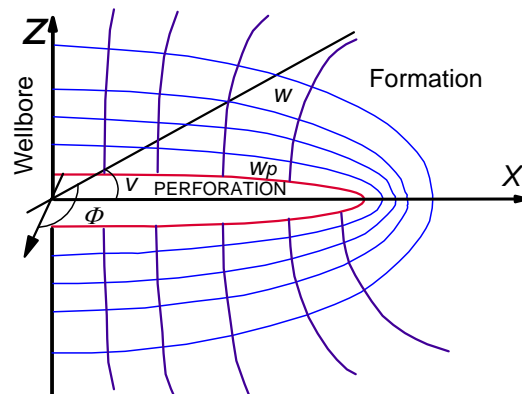


Figure-5. 2D ellipsoidal flow geometry ( $w$ ,  $v$  and  $\psi$  coordinates), after Lea *et al.* (1992).



$$\Xi = \frac{k_{elp} h (t^* \Delta P')_{elp1hr}}{141.2 q \mu B} \quad (4)$$

Replacing Equations (1) and (2) into Equation (3) and rearranging for the tridimensional permeability yields:

$$k_{elp} = \frac{3792.2 \phi \mu c_t h_w^2 \left( \frac{(t^* \Delta P')_{elp1hr}}{(t^* \Delta P')_{elp}} \right)^{7.407}}{t_{elp}} \quad (5)$$

This can be used to estimate the radial/horizontal permeability using any arbitrary time,  $t_{elp}$ , on the ellipsoidal flow regime. The ellipsoidal permeability can be approximated by:

$$k_{elp} = \sqrt{k_v k_h^2} \quad (6)$$

Equation (3) can be rewritten as:

$$(t_D^* P_D')_{elp} = \frac{\Xi}{(t_D^{-0.135})_{elp}} \quad (7)$$

During radial flow regime, the value of the dimensionless pressure derivative is one half. Therefore, at the intercept point of the straight line of the radial flow with ellipsoidal flow regime, the pressure derivative takes the value of 0.5. Then, taking into account that the dimensionless parameters are governed by the horizontal permeability and replacing Equations (2) and (4) into Equation (7) and solving for the radial permeability yields:

$$k_h = 37.9123 \mu \left( \frac{h (t^* \Delta P')_{elp1hr}}{q B} \right)^{-1/0.865} \left( \frac{t_{relpi}}{\phi \mu c_t h_w^2} \right)^{0.156} \quad (8)$$

As seen, the vertical permeability can be obtained from Equation (6). The horizontal or radial permeability may be also estimated from equation (9) - from the radial flow regime - according to Tiab (1993):

$$k_h = \frac{70.6 q \mu B}{h (t^* \Delta P')_r} \quad (9)$$

Tiab (1993) also introduced an expression to calculate the mechanical skin factor,  $s$ , by reading the

pressure drop,  $\Delta P_r$ , value at any arbitrary time,  $t_r$ , on the radial-flow-regime straight line, such as:

$$s = 0.5 \left[ \frac{(\Delta P')_r}{(t^* \Delta P')_r} - \ln \left( \frac{k_h t_r}{\phi \mu c_t r_w^2} \right) + 7.43 \right] \quad (10)$$

According to Ozkan and Raghavan (2000), the effective permeability is found from:

$$k_w = k_h \cos^2(\psi) + k_z \sin^2(\psi) \quad (11)$$

#### 4. EXAMPLES

##### Synthetic example 1

A pressure test was simulated with the data from the third column of Table-1. Pressure and pressure derivative data are reported in Figure-6. It is requested to find the areal and vertical permeabilities.

##### Solution

The following data are read from Figure-6:

$$\begin{aligned} t_{elp} &= 0.142 \text{ hr} & (t^* \Delta P')_{elp} &= 8.99 \text{ psi} & (t^* \Delta P')_{elp1hr} &= 8.99 \text{ psi} \\ t_r = t_{relpi} &= 130.1 \text{ hr} & (t^* \Delta P')_r &= 3.81 \text{ psi} & \Delta P_r &= 168.32 \text{ psia} \end{aligned}$$

Using Equation (5),  $k_{elp}$  is found to be 49.7 md. Then, a  $k_h$  value of 100.9 md is found using Equation (8). These permeability values are used in Equation (6) to find a value of  $k_z$  of 12.3 md. The horizontal permeability is recalculated with Equation (9) to be 100.1 md and a mechanical skin factor of 13.6 is determined by means of Equation (10). Finally, an effective permeability of 78.8 md is found with Equation (11). Noticed that the spherical flow regime can also be used to estimate, among other parameters, the vertical permeability as shown by Moncada *et al.* (2005).

##### Synthetic example 2

The Pressure and pressure derivative log-log plot of a pressure test simulated with the data from the fourth column of Table-1 is given in Figure-7. It is requested to find the areal and vertical permeabilities.

##### Solution

The following data are read from Figure-7:

$$\begin{aligned} t_{elp} &= 0.03 \text{ hr} & (t^* \Delta P')_{elp} &= 5.902 \text{ psi} & (t^* \Delta P')_{elp1hr} &= 4.05 \text{ psi} \\ t_{relpi} &= 27 \text{ hr} & (t^* \Delta P')_r &= 2.744 \text{ psi} & \Delta P_r &= 93.86 \text{ psia} \quad t_r = 10.1 \text{ hr} \end{aligned}$$

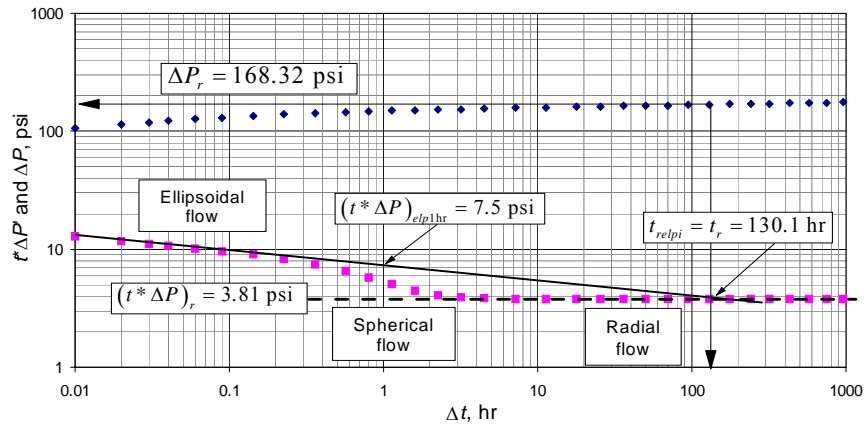


Figure-6. Pressure and pressure derivative log-log plot for the synthetic example 1.

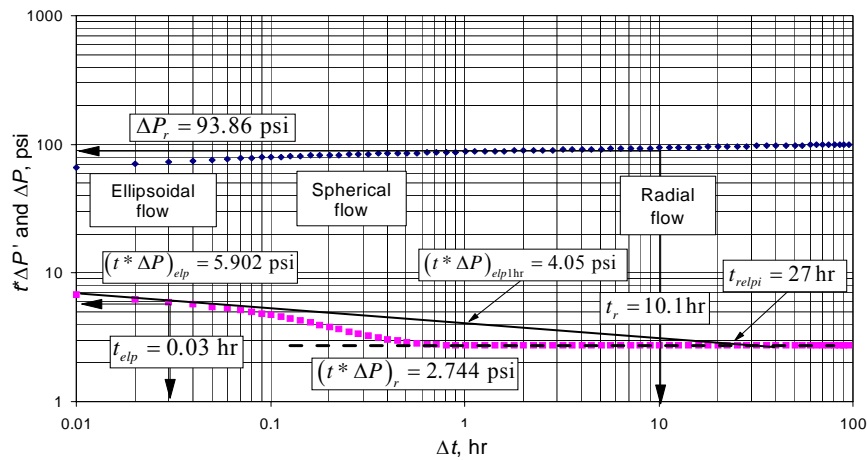


Figure-7. Pressure and pressure derivative log-log plot for the synthetic example 2.

Table-2. Summary of results.

Parameter	Equation	Synthetic example 1	Synthetic example 2
		Value	
$k_{elp}$ , md	5	49.7	155.4
$k_h$ , md	8	100.9	206.9
$k_{h2}$ , md	9	100.1	226.4
$k_z$ , md	6	12.3	79.9
$k_w$ , md	11	78.8	124.9
$s$	10	13.6	9.72

A similar procedure as outlined for the synthetic example 1 was performed for synthetic example 2. Results are reported in Table-2.

**Field example**

The pressure and pressure derivative log-log plot for a buildup pressure test run in a deviated and partially

penetrated well located in the Superior Valley of the Magdalena River in Colombia is shown in Figure-8. Relevant data for this test are found in the fifth column of Table-1.

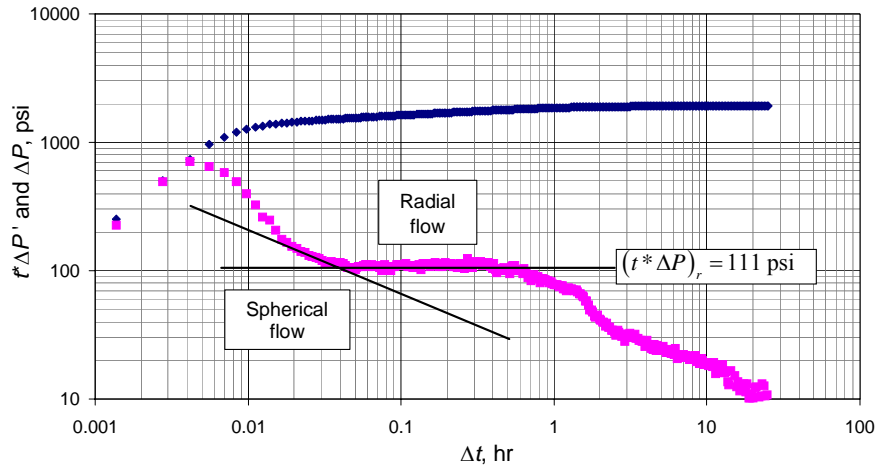


Figure-8. Pressure and pressure derivative log-log plot for the field example.

**Solution**

This test cannot be interpreted as proposed in this methodology since the ellipsoidal flow is not seen because the wellbore storage masks it. Spherical flow is slightly observed. The radial flow regime is clearly defined. From this a value of  $(t^* \Delta P')_r = 111$  psi allows for the estimation of a radial permeability of 4.72 md with Equation 9. We unsuccessfully tried to run deconvolution but the algorithm seeks for the radial flow as the objective function. We present this example to point out that it is feasible to have such system. However, for this particular case wellbore storage effects were not mitigated.

$$\begin{aligned}
 t_{ebs} &= 0.142 \text{ hr} & (t^* \Delta P')_{ebs} &= 8.99 \text{ psi} & (t^* \Delta P')_{ebs} &= 8.99 \text{ psi} \\
 t_w &= t_{webs} = 130.1 \text{ hr} & (t^* \Delta P')_w &= 3.81 \text{ psi} & (t^* \Delta P')_w &= 3.81 \text{ psi} \\
 \Delta P_w &= 168.32 \text{ psia}
 \end{aligned}$$

**5. ANALYSIS OF RESULTS**

From the synthetic examples it is possible to observe that the formulated methodology works quite well. The input data used to generate the transient pressure test agrees very well with the obtained results. However, it is strongly recommended to adequately draw the -0.135-slope straight line which has 7.1 time log cycles and one log cycle on the pressure axis. Notice that Equation 5 includes a ratio of pressure derivatives to the power 7.407 which leads to a big difference with a slight error in reading.

A governing equation for dimensionless pressure can be derived, so the geometrical skin factor caused by the well geometry and inclination can also be developed. However, it is not shown in here.

**CONCLUSIONS**

The governing equation for the pressure derivative behavior of ellipsoidal flow -recognized by a slope of -0.135 on the derivative curve- presented in slanted and partially penetrated wells is presented along with an interpretation methodology which was

successfully tested with analytical tests so vertical and horizontal permeability are readily obtained.

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**Nomenclature**

B	Oil formation factor, rb/STB
$c_t$	Total system compressibility, 1/psi
h	Formation thickness, ft
$h_w$	Penetration/perforation length, ft
k	Permeability, md
P	Pressure, psi
$P_D$	Dimensionless pressure
$P_i$	Initial reservoir pressure, psi
$P_{wf}$	Well flowing pressure, psi
q	Flow rate, bbl/D
$r_w$	Well radius, ft
s	Skin factor
t	Time, hr
$t_D$	Dimensionless time
$t_D^* P_D'$	Dimensionless pressure derivative
w, v, $\Phi$ $Z_w$ , ft	Ellipsoidal coordinates Midpoint of the penetrated/perforated interval

**Greek**

$\Xi$	Intercept of the dimensionless pressure derivative straight line
$\psi$	Inclination angle of the well, degrees
$\phi$	Porosity, fraction
$\mu$	Viscosity, cp



### Suffices

<i>D</i>	Dimensionless
<i>i</i>	Intersection or initial conditions
<i>h</i>	Radial/horizontal/areal
<i>elp</i>	Ellipsoidal
<i>elp1hr</i>	Time of ellipsoidal-flow line at one hour
<i>relpi</i>	Intersection of radial with ellipsoidal lines
<i>r</i>	Radial
<i>w</i>	Well
<i>z</i>	Vertical direction

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