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NUMERICAL SOLUTION FOR A RADIAL COMPOSITE RESERVOIR MODEL WITH A NON-NEWTONIAN/NEWTONIAN INTERFACE

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ABSTRACT

Normally, some heavy crude oils and several well treatment fluids possess a non-Newtonian nature. This nature makes the fluid behavior to behave differently than a Newtonian one. Therefore, transient pressure analysis, for instance, will have different considerations. Moreover, if for any reason, a non-Newtonian fluid is injected into a reservoir which contains a Newtonian crude oil; two different fluid flow zones will be well established and defined. Therefore, two different diffusivity equations are governing the fluid flow through the porous medium. The simultaneous solution to that problem may be a numerical challenge if not dealt appropriately. In this work we present the numerical solution for a power-law Non-Newtonian diffusivity model coupled with a Newtonian diffusivity model by the finite-difference approximation. Results were successfully compared to those reported in the literature without reporting the detail solution.

Keywords: non-Newtonian/Newtonian, finite difference, viscoplastic fluid, reservoir, consistency, viscosity.

1. INTRODUCTION

Ikoku has been the most outstanding researcher in the field of non-Newtonian power-law fluids modeling, as shown by Ikuko (1979), Ikoku and Ramey (1979a, 1979b, 1979c) and Lund and Ikoku (1981). Most of these references are focused on presenting the flow models and their applications to well test analysis, but little information is concerned to the numerical solutions of the model. Ikoku and Ramey (1979b) solved the problem by the Douglas-Jones predictor/corrector method without detailing on the numerical procedure. Escobar and Civan (1996) presented a numerical solution for the flow of foam in porous media using the Quadrature method. They solved the model proposed by Ikoku and Ramey (1979b) under different boundary conditions and compared to the solution of the diffusivity equation for the Newtonian case. Since, it is quite important to know the detailed procedure for the solution, in this paper the model presented by Ikoku and Ramey (1979b) is detailed solved and successfully compared to the solution of by Lund and Ikoku (1981).

2. MODELING ASPECTS

The diffusivity equation proposed by Ikoku and Ramey (1979b) governing the flow of a power-law Non-Newtonian fluid through an isotropic and homogeneous porous medium is:

$$\frac{\partial^2 p}{\partial r^2}+\frac{n}{r}\frac{\partial p}{\partial r}=c_t\phi n\left(\frac{\mu_{eff}}{k}\right)^{1/n}\left(-\frac{\partial p}{\partial r}\right)^{(n-1)/n}\frac{\partial p}{\partial t};\ r_w\le r\le r_a \quad (1)$$

Equation (1) is strongly non-linear. For achievement of analytical solutions a linearized approximation of Equation (1), Ikoku and Ramey (1979b), has transformed it into:

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial p}{\partial r}\right)=Gr^{1-n}\frac{\partial p}{\partial t} \quad (2)$$

being,

$$G=\frac{n\phi c_t\mu_{eff}}{k_{eff}}\left(\frac{2\pi h}{q}\right)^{1-n} \quad (3)$$

$$\mu_{eff}=\left(\frac{H}{12}\right)\left(9+\frac{3}{n}\right)^n\left(150k\phi\right)^{(1-n)/2} \quad (4)$$

The system under study also considers the flow of a Newtonian fluid which is in contact with the non-Newtonian one. For the Newtonian case, the effective viscosity is considered to be constant and $n = 1$. Therefore, Equation (1) becomes:

$$\frac{\partial^2 P}{\partial r^2}+\frac{1}{r}\frac{\partial P}{\partial r}=\left(\frac{\phi\mu_N c_t}{k_N}\right)\frac{\partial P}{\partial t};\ r_a\le r\le r_e \quad (5)$$

Other assumptions include: radial flow of slightly compressible non-Newtonian/Newtonian fluids, an isotropic and homogeneous reservoir, uniform thickness, a pseudo plastic power-law non-Newtonian fluid is injected in the well, the Newtonian fluid has constant viscosity, piston-like displacement takes place and the reservoir has a finite radius. Figure-1 sketches the composite reservoir under consideration.

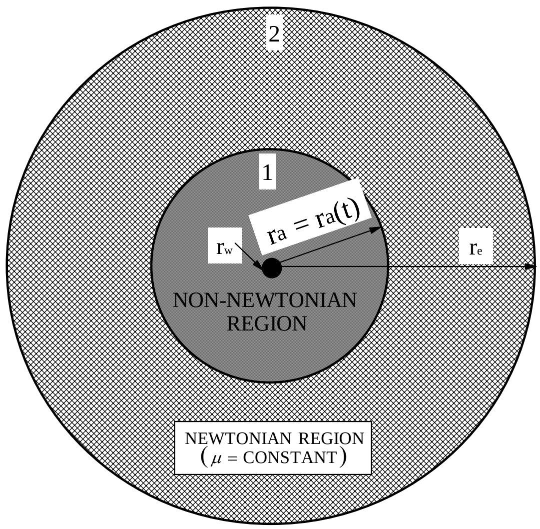

Figure-1. Composite non-Newtonian/Newtonian radial reservoir.

The initial condition is described as:

$$p_1(r,0) = p_2(r,0) = p_i \tag{6}$$

The internal boundary condition indicates a constant rate injection of a non-Newtonian fluid, then:

$$\left(-\frac{\partial p_1}{\partial r}\right)_{r=r_w} = \left(\frac{q}{2\pi r_w}\right)^n \frac{\mu_{eff}}{k}; t>0 \tag{7}$$

The external condition was initially established as no flow;

$$\left(-\frac{\partial p_1}{\partial r}\right)_{r=r_e} = 0,\ \ t>0 \tag{8}$$

But, in this study we treated as an infinite reservoir, then:

$$p_2\left(r_e,t\right) = p_i,\ t>0 \tag{9}$$

Suffixes 1 and 2 refer to the Non-Newtonian and Newtonian regions, respectively. At the fluid interface pressure has to have the same value in the non-Newtonian and Newtonian zones. Then, the continuity condition indicates that:

$$p_1 = p_2\ \ @\ r\ =\ r_a(t) \tag{10}$$

Both fluids have to flow at the same velocity in the interface, then the consistency or Darcy's law condition is given as:

$$\left[\lambda_{eff}\left(-\frac{\partial p_1}{\partial r}\right)\right]^{1/n}_{r=r_a(t)} = \lambda_2\left(-\frac{\partial p_2}{\partial r}\right)_{r=r_a(t)} \tag{11}$$

3. NUMERICAL SOLUTION

The finite-difference discretization for Equation (2) is:

$$\frac{1}{r_i^n \Delta r_i}\left[\left(r_{i+1/2}^n \frac{(p_{i+1}-p_i)^{s+1}}{\Delta r_{i+1/2}} - r_{i-1/2}^n \frac{(p_i-p_{i-1})^{s+1}}{\Delta r_{i-1/2}}\right)\right] = Gr_i^{1-n}\left(\frac{p_i^{s+1}-p_i^s}{\Delta t}\right) \tag{12}$$

For simplification purposes, define:

$$a = \frac{r_{i+1/2}^n}{r_i^n \Delta r_i \Delta r_{i+1/2}} \tag{13}$$

$$b = \frac{r_{i-1/2}^n}{r_i^n \Delta r_i \Delta r_{i-1/2}} \tag{14}$$

$$F = \frac{Gr_i^{1-n}}{\Delta t} \tag{15}$$

$$D = -Fp_i^s \pm \frac{q\mu_{app}}{kV_{ri}} \tag{16}$$

$$c = -(a+b+F) \tag{17}$$

Equation (16) includes the only source/sink term for Equation (12). Once the definitions given by Equations (13) through (17) are replaced, Equation (12) becomes:

$$bp_{i-1}^{s+1} + cp_i^{s+1} + ap_{i+1}^{s+1} = D \tag{18}$$

Discretization of Equation (5) gives,

$$\frac{1}{r_i\ \Delta r_i}\left[\left(r_{i+1/2} \frac{(p_{i+1}-p_i)^{s+1}}{\Delta r_{i+1/2}} - r_{i-1/2} \frac{(p_i-p_{i-1})^{s+1}}{\Delta r_{i-1/2}}\right)\right] = \frac{\phi\mu_N c_t}{k_N}\left(\frac{p_i^{s+1}-p_i^s}{\Delta t}\right) \tag{19}$$

Also, for simplification, define,

$$a' = \frac{r_{i+1/2}}{r_i\ \Delta r_i \Delta r_{i+1/2}} \tag{20}$$

$$b' = \frac{r_{i-1/2}}{r_i\ \Delta r_i \Delta r_{i-1/2}} \tag{21}$$

$$F' = \frac{\phi \mu_N c_t}{k_N \Delta t} \tag{22}$$

$$D' = -F' p_i^s \text{ (No source/sink term)} \tag{23}$$

$$c' = -(a + b + F') \tag{24}$$

Therefore, Equation (19) is rewritten as:

$$b' p_{i-1}^{s+1} + c' p_i^{s+1} + a' p_{i+1}^{s+1} = D' \tag{25}$$

As indicated by Figure-2, the simultaneous solution of Equations (12) and (25) is required. Then, Equation (12) is applied to all points from the well, r_w, until point j-1, just before the interface. Equation (25) is applied from point j+1 until r_e. At the fluids interface, $r = r_a$, the consistency condition given by Equation (11) is applied:

$$\frac{\partial p}{\partial r}\Big|_{NN} = \lambda \frac{\partial p}{\partial r}\Big|_{N} \tag{26}$$

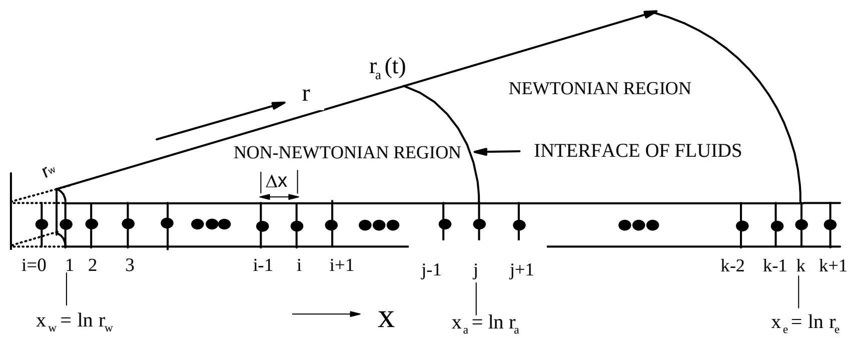

Figure-2. Functions' domain.

Where λ is the relationship between the non-Newtonian fluid apparent viscosity and the viscosity of the Newtonian fluid?

$$\lambda = \frac{\mu_{app}}{\mu_{NN}} \tag{27(a)}$$

$$\mu_{app} = \mu_{eff} \left(\frac{2\pi h r}{q} \right)^{1-n} \tag{27(b)}$$

The finite-difference discretization of the consistency condition, Equation (11), leads to:

$$\frac{p_i^{s+1} - p_{i-1}^{s+1}}{\Delta r_{i-1/2}} = \lambda \frac{p_{i+1}^{s+1} - p_i^{s+1}}{\Delta r_{i+1/2}} \tag{28}$$

For simplification purposes, define:

$$\theta = \frac{\Delta r_{i+1/2}}{\Delta r_{i-1/2}} \tag{29}$$

Then, Equation (28) is rewritten as:

$$\theta p_{i-1}^{s+1} - (\theta + \lambda) p_i^{s+1} + \lambda p_{i+1}^{s+1} = 0 \tag{30}$$

The values of the constant of Equation (18), only for the fluids interface, correspond to:

$$a = \theta \tag{31}$$

$$b = \lambda \tag{32}$$

$$c = -(\theta + \lambda) \tag{33}$$

$$D = 0 \tag{34}$$

The application of Equations (18), (25) and (30) to each grid point leads to the following tridiagonal matrix system which is solved by the Thomas algorithm:

$$\begin{pmatrix} c & a & & & & & & & & \\ b & c & a & & & & & & & \\ & b & c & a & & & & & & \\ & & b & c & a & & & & & \\ & & & b' & c' & a' & & & & \\ & & & & b' & c' & a' & & & \\ & & & & & b' & c' & a' & & \\ & & & & & & . & . & . & \\ & & & & & & & b' & c' & a' \\ & & & & & & & & b' & c' \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_{j-1} \\ p_j \\ p_{j+1} \\ p_{j+2} \\ p_{j+3} \\ . \\ p_{k-1} \\ p_k \end{pmatrix}^{s+1} = \begin{pmatrix} D \\ D \\ D \\ 0 \\ D' \\ D' \\ D' \\ . \\ D' \\ D' \end{pmatrix}^{s} \begin{array}{l} \\ \\ \\ \text{interface} \\ \\ \\ \\ \\ \\ \\ \end{array}$$

The block pressure was converted to well pressure by using Equation (9) of Peaceman (1978). The computer code is reported in the appendix.

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4. RESULTS

The results were tested against the numerical solution presented by Lund and Ikoku (1981). The test example was run with the information given in Table-1 using 500 cells. Comparative results are given in Figure-3 which contains a semilog plot of the results of this study against those from Lund and Ikoku (1981). No deviation error was established since the results are reported at different time values and the solution from Lund and Ikoku (1981) is also numerical which ought to have some small differences with the actual solution. However, the trend of the data looks very good and reasonable. For further verification, the pressure derivative conventionally used in transient pressure analysis was estimated from the simulated data. According to Katime-Meindl and Tiab (2001), who were the first to publish the pressure derivative behavior for pseudoplastic fluids, during radial flow regime the pressure derivative increases as the power-law index decreases? It means, the smaller the *n* value the more pronounced the pressure derivative from the horizontal position. For Newtonian behavior a horizontal line is obtained. This behavior is confirmed by the results shown in Figure-4 which agree with the expectations of the authors.

Table-1. Data used for simulator calibration.

Parameter	Value
r_e, ft	2625
c_t, 1/psi	6.89×10^{-6}
P_R, psi	2500
h, ft	16.4
q, bpd	300
t, day	9
ϕ, %	0.2
Δt, hr	0.01
k, md	100
r_w, ft	0.33
H, cp s^{n-1}	20
B, rb/STB	1
n	0.6

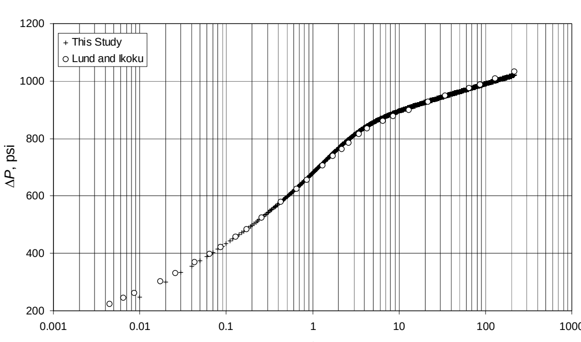

Figure-3. Semilog plot of the results.

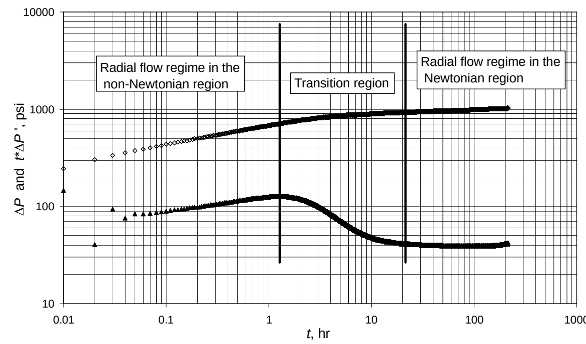

Figure-4. Pressure and pressure derivative of a simulated test.

5. CONCLUSIONS

A numerical solution using the finite-difference method was obtained for a radial composite non-Newtonian/Newtonian reservoir. Results were successfully compared to the numerical solution obtained by Lund and Ikoku (1981). Also, pressure derivative behaves as expected.

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Nomenclature

B	Oil formation factor, rb/STB
c_t	System total compressibility, 1/Pa
h	Formation thickness, m
H	Consistency (Power-law parameter), cp s^{n-1}
k	Permeability, m^2
n	Flow behavior index (power-law parameter)
P	Pressure, psi
P_R	Reservoir pressure, Pa
q	Flow rate, m3/s
r_w	Radius, m
s	Time level
t	Time, s
t*Δp'	Pressure derivative, psi
V_r	Cell volume, m^3

Greek

Δ	Change, drop
Δt	Time step, s
ϕ	Porosity, fraction
μ	Viscosity, Pa s
μ	Effective viscosity for power-law fluids, N
λ	Mobility, m^4/N s
λ_{eff}	Effective mobility for power-law fluids, m^{3+n}/N
ω	Dimensionless storativity (capacity) ratio

Suffices

a	Location of the non-Newtonian fluid front
app	Apparent
e	External
eff	Effective
i	Initial conditions, discretization index
j	Grid point at the non-Newtonian fluid front
N	Newtonian
NN	Non-Newtonian
w	Wellbore
m	Matrix, slope
max	Maximum
min	Minimum

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APPENDIX - visual basic code

```
Dim R () As Double, DR () As Double, R1 () As Double, DR1 () As Double
Dim AAA () As Double, B () As Double, F () As Double, CCC () As Double, DDD () As Double
Dim P () As Double, GG () As Double, QQ () As Double, W () As Double, VM as Double, Ro as Double, Jmodel as Double
Dim Re as Double, Ct as Double, PR as Double, H as Double, HC as Double, Qo as Double, Tiempo as Double
Dim Por as Double, DT as Double, K as Double, rw As Double, BETA As Double, n As Double, UN as Double
Dim Celdas as Double, Ueff as Double, Uapp () As Double, G1 as Double, G2 as Double, Alfa as Double
Dim i As Long, j As Long, v As Long, Temp1 as Double, Temp2 as Double, Ri as Double, X As Long
Const Pi = 3.141592654
Private Sub Command1_Click ()
Re = 2625: Ct = 6.89E-6:    Celdas = 500:    PR = 2500
H = 16.4: Qo = 300: Tiempo = 9: Por = 0.2: DT = 0.01
K = 100: rw = 0.33: HC = 20: n = 0.6: BETA = 1: UN = 3
Ueff = (HC / 12) * (9 + 3 / n) ^ n * (9.869E-16 * 150 * K * Por) ^ ((1 - n) / 2)
G1 = 3792.58489625175 * n * Por * Ct / K
G2 = 3792.58489625175 * Por * Ct * UN / K
Ri = 131.2
ReDim R(1 To Celdas), DR(1 To Celdas), R1(1 To Celdas), DR1(1 To Celdas - 1), Uapp(1 To Celdas)
ReDim AAA(1 To Celdas), B(1 To Celdas), F(1 To Celdas), CCC(1 To Celdas), DDD(1 To Celdas)
ReDim GG (1 to Celdas), QQ (1 to Celdas), W (1 to Celdas)
Alfa = (Re / rw) ^ (1 / (Celdas - 1))
Numbertest = 24 * Tiempo / DT
R (1) = rw
For i = 2 to Celdas
R (i) = Alfa * R (i - 1)
Next i
For i = 1 to Celdas - 1
DR1 (i) = R (i + 1) - R (i)
Next i
For i = 1 to Celdas
If i = Celdas Then
R1 (i) = Re
Else
R1 (i) = ((Alfa - 1) * R (i)) / (Log (Alfa))
If R1 (i) < Ri Then X = i
End If
Next i
For i = 1 to Celdas
If i = 1 Then
DR (i) = R1 (i) - R (i)
Else
DR (i) = R1 (i) - R1 (i - 1)
End If
Next i
For i = 1 to Celdas
Uapp (i) = Ueff * (317196.865977076 * H * R (i) / (Qo * BETA)) ^ (1 - n) 'Viscosidad Aparente
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Next i
VM = Pi * H * (R1 (1) ^ 2 - rw ^ 2) / (5.615 * BETA)
ReDim P (0 to Numbertest, 1 to Celdas)
For i = 1 to Celdas
P (0, i) = PR
Next i
i = 1
Do Until i = X + 1
i = i + 1
B (i) = R1 (i - 1) ^ n / (R (i) ^ n * DR (i) * DR1 (i - 1))
Loop
For i = X + 2 to Celdas
B (i) = R1 (i - 1) / (R (i) * DR (i) * DR1 (i - 1))
Next i
i = 0
If X = 0 Then X = 1
Do Until i = X - 1
i = i + 1
AAA (i) = R1 (i) ^ n / (R (i) ^ n * DR (i) * DR1 (i))
Loop
For i = X To Celdas - 1 'Region Newtoniana "n=1"
AAA (i) = R1 (i) / (R (i) * DR (i) * DR1 (i))
Next i
i = 0
Do Until i = X
i = i + 1
F (i) = G1 * Uapp (i) / DT
Loop
For i = X + 1 to Celdas - 1
F (i) = G2 / DT
Next i
If Ri > 0 And Ri < Re Then
B(X) = DR1(X) / DR1(X - 1)
AAA(X) = Uapp(X) / UN
F(X) = 0
End If
CCC (1) = - (AAA (1) + F (1))
CCC (Celdas) = - (B (Celdas) + F (Celdas))
For i = 2 to Celdas - 1
CCC (i) = - (AAA (i) + B (i) + F (i))
Next i
For v = 1 To Numbertest
For i = 1 to Celdas
If i = 1 Then
DDD (i) = -F (i) * P (v - 1, i) - 158.024370659982 * (Qo /
(K * VM)) * Uapp (i)
Else
DDD (i) = -F (i) * P (v - 1, i)
End If
Next i
QQ (1) = AAA (1) / CCC (1)
GG (1) = DDD (1) / CCC (1)
For j = 2 to Celdas
W (j) = CCC (j) - (B (j) * QQ (j - 1))
GG (j) = (DDD (j) - (B (j) * GG (j - 1))) / W (j)
QQ (j) = AAA (j) / W (j)
Next j
P (v, Celdas) = GG (Celdas)
For j = Celdas - 1 To 1 Step -1
P (v, j) = (GG (j) - (QQ (j) * P (v, j + 1)))
Next j
Next v
Nmax = Numbertest
Temp1 = 1 / (G2 * rw ^ 2)
Temp2 = 141.2 * (UN / K * (Qo * BETA / (H)))
Ro = R1 (1)
Jmodel = 0.00708 * K * H / (Uapp (1) * Log (Ro / rw))
For i = 1 to Nmax
P (i, 1) = P (i, 1) - Qo / Jmodel
Porc = i / Nmax * 100
Next i
End Sub
```