INVESTIGATION OF SFRC CORBEL PERFORMANCE USING A DEVELOPED NINE-NODED LAGRANGIAN ELEMENTS

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ABSTRACT

In present work, novel steel fiber reinforced concrete (SFRC) behavior nonlinearities were proposed depending on various experimental studies outcomes. Nine-noded Lagrangian finite elements procedure was developed using the proposed material behavior nonlinear relationships to investigate the performance of SFRC corbel. The stiffness matrix was determined in many ways dependent on the initial and tangential stiffness values. The results were given in terms of maximum deflections and crack formation of the corbel. Good agreement was observed between present numerical analysis results and that for other related studies. The average ratio between current numerical outcomes and other experimental results is about 0.89655 in terms of maximum displacements and ultimate loads.

Keywords: SFRC corbel, finite element analysis, static behavior.

1. INTRODUCTION

Concrete corbel is a short cantilever member employed for supporting the precast structural elements such as beams and slabs [1, 2]. The corbel is usually cast monolithically with the concrete wall, column and so forth [3]. Appropriate analysis and design detailing for concrete corbels are required due to the high transferred loads from the precast elements which rested on corbels.

Steel fibers are usually introduced in the deep concrete members to improve their shear strength [4]. With respect to the comprehensive literature review, the using of new material nonlinearities in the investigation of SFRC corbels has not been extensively investigated. Two main sorts of nine-noded elements are available for modeling the mechanical behavior of concrete members, namely, heterosis and nine-noded Lagrangian elements. Present endeavor is devoted to study the behavior of SFRC corbels under static loading with considering new material nonlinearities in the plane stress nine-noded Lagrangian finite element procedure. The stiffness matrix was found in different manners through the analysis.

2. METHODOLOGY

The methodology of present research is comprises of two items namely formulation of new and simple material nonlinearities of SFRC material behavior and development of nine-noded plane stress finite elements analytical procedure for SFRC corbels.

2.1 Nonlinear behavior of SFRC material

With introducing of steel fiber in the concrete, non-homogeneous material is regarded in the finite element analysis for fibrous concrete. Thus, different behavior is considered for fibrous concrete behavior under the effect of tensile and compressive forces.

To simulate the performance of SFRC material in proper way, nonlinear relationships are required to be formulated for representing the behavior of this material under both compressive and tensile loads [5]. The compressive behavior of SFRC material was simulated in present plane stress nine-noded finite element procedure with depending on the theories of elasticity and plasticity. This by using the Madrid relationship to represent the growing of the yielding surface of compressive performance as follows [6]:

$$\sigma_{co} = -f_c \frac{\varepsilon_{co.}}{\varepsilon_c}\left[\frac{\varepsilon_{co.}}{\varepsilon_c} - 2\right] \quad (1)$$

where

$\sigma_{co.}$= compression stress
$\varepsilon_{co.}$= compression strain
ε_c = compression strain at concrete compression strength
f_c = concrete compression strength

With considering the derivation of Equation (1), the modulus of elasticity E_c can be found as follows:

$$E_c = 2f_c / \varepsilon_c \quad (2)$$

Thus, Equation (1) can be written as:

$$\sigma_{co} = E_c\varepsilon_{co.} - \frac{E_c\varepsilon_{co.}^2}{2\varepsilon_c} \quad (3)$$

To predict the growing of the compression stress for SFRC materials, a hardening parameter *Ha* should be used which can be calculated by the derivation of Equation (3) as hereunder:

$$Ha = \left(\sqrt{\frac{\varepsilon_c}{\varepsilon_{co.} - \frac{\sigma_{co.}}{E_c}}} - 1\right) \quad (4)$$

To formulate new SFRC material nonlinear relationships in present work, the following parameter was used:

$$FP = \sqrt{fiber\ content\% \times \frac{fiber\ length}{fiber\ equivalent\ diameter}} \tag{5}$$

The strength of SFRC material subjected to two normal forces is more than that in the case of normal uniaxial loading by a magnification magnitude of *Ma* as follows:

$$f_b = Ma.f_c \quad \text{(Where } Ma >= 1.0\text{)} \tag{6}$$

The best formula for *Ma* was proposed in present work with applying the nonlinear regression analysis of various experimental data given in references [7-16] in SPSS 17 program as hereunder:

$$Ma = 2 \times FP \tag{7}$$

A proper mathematical relationship was formulated also for the strain value of compressive strain ε_c with considering regression analysis of experimental outcomes in the aforementioned references as:

$$\varepsilon_c = 0.0046 \times FP \tag{8}$$

The ultimate compression strain $\varepsilon_{ultimate}$ of SFRC material was formulated in the same manner as follows:

$$\varepsilon_{ultimate} = 0.01 \times FP \tag{9}$$

The ultimate strain value is used in the current finite element analysis to predict the crushing of the element at the specified Gauss points.

To simulate the tensile behavior of SFRC material, a formula for tension strain value $\varepsilon_{tension}$ which is accompanied SFRC tensile strength was proposed as hereunder:

$$\varepsilon_{tension} = 0.00027 \times FP \tag{10}$$

The crack pattern was investigated in present finite element work, where the cracks are imposed to form at the integration spots on the elements when the tensile strain is reached the tension strain. After cracking, the SFRC material will lose some mechanical characteristics such as Poisson's ratio and modulus of elasticity. Therefore, a reduced crack modulus must be used to calculate the property matrix instead of un-cracked shear modulus. The reduced crack modulus *Mc* is formulated with depending on the regression analysis procedure for the data given in the references [7-16] as follows:

$$M_c = 5.5 \times FP \tag{11}$$

The formula for the proposed tension strain was employed to modify the SFRC tensile behavior equations which given by [17] as hereunder:

$$\sigma_{t,before} = f_t \left[1 - \left((1 - \frac{\varepsilon_{t,before}}{\varepsilon_{tension}}) \right)^{E_i \cdot \frac{\varepsilon_{mt}}{f_{tu}}} \right] \tag{12}$$

$$\sigma_{t,after} = f_t . e^{-f_t . S . (\frac{\varepsilon_{t,after}}{\varepsilon_{tension}} - 1)^R} \tag{13}$$

where

$\sigma_{t,before}$ = tensile stress before cracking

$\varepsilon_{t,before}$ = tensile strain before cracking

E_i = initial modulus of elasticity

f_t = tensile strength of concrete

$\sigma_{t,after}$ = tensile stress after cracking

$\varepsilon_{t,after}$ = tensile strain after cracking

$$S = 0.66875 - 0.48842\left(fiber\ content \times \frac{Fiber\ length}{Fiber\ diameter} \right) + 0.1125 \left(fiber\ content \times \frac{Fiber\ length}{Fiber\ diameter} \right)^2 \tag{14}$$

$$R = 6.26513 \left(\frac{Fiber\ length}{fiber\ content \times Fiber\ diameter} \right)^{-0.50327} \tag{15}$$

The correlation coefficient of the proposed models is about 95% according to the nonlinear regression analysis in SPSS 17.

2.2 Present finite element formulation

There are two types of nine-noded elements which widely used in finite element analysis namely, heterosis elements and nine-noded lagrangian elements. Nine-noded lagrangian elements [18] were used in present finite element analysis with introducing of embedded elements [19, 20] to model the steel bars in these elements. The perfect bond was assumed between the embedded elements and the surrounded concrete material. In the finite element analysis and design of concrete structures, the nonlinear behavior is predominate due to the influence of concrete cracking and steel bar yielding. In present nonlinear static analysis, the Newton-Raphson approach is used. The stiffness matrix for the elements is found in four cases namely initial stiffness matrix was determined at each load increment, tangential stiffness matrix was used instead of initial one, stiffness matrix is found at the first iteration for each load increment and case IV includes the calculation of stiffness matrix at second iteration for each load increment. The iterations with convergence techniques were used to satisfy the closeness between numerical outcomes and the actual experimental results as in Figure-1. A computer program coded in FORTRAN was developed with using of the proposed models to investigate the behavior of SFRC corbels in terms of load-displacement curves and failure form. The flow chart of present finite element procedure in this program is illustrated in Figure-2.

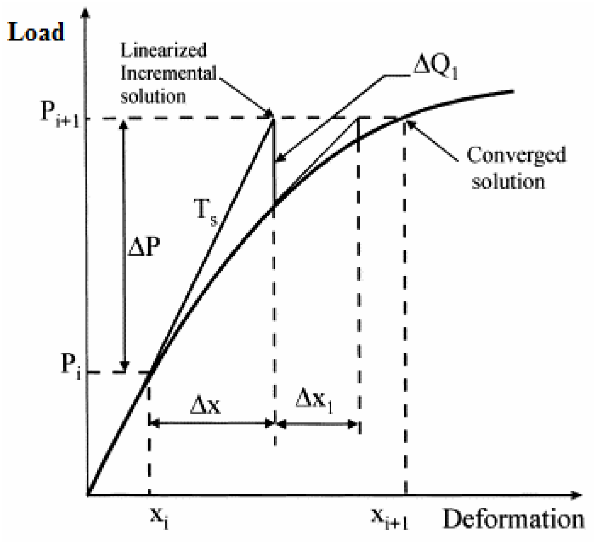

Figure-1. Nonlinear analysis procedure [21].

3. NUMERICAL APPLICATION

Present finite element analytical procedure has been applied for simulation the behavior of SFRC corbel that tested by [22]. The dimensions in millimeters of the corbel are given in Figure-3. Two steel fiber contents were considered in the analysis namely 0.5% and 1.0%. Due to the symmetry of geometry and loading, half of the corbel was taken into account in the analysis. Twenty nine-noded Lagrangian elements (Figure-4) were adopted to model the half of SFRC corbel in the analysis. The results were given in terms of load-maximum deflection curves of the considered corbels as shown in Figures 5-12. Close results are obtained with considering many stiffness determination cases in the analysis. Good agreement was observed between present load-deflection values and that given by [22]. Suitable harmony was appeared also between present crack formation (Figures 13 and 14) and the failure which given by Abdul-Razzak and Ali [2] (Figures 15 and 16), where the cracks are concentrated at the distance between the applied force and the center of the wall. The difference between the experimental and numerical outcomes may be attributed to disregarding of time depending phenomena such as creep in the proposed models. According to Figures 5-12 the ratio between numerical and experimental results was determined in terms of maximum displacement and ultimate load values and for all stiffness determination cases as given in Figure-17.

4. CONCLUSIONS

In present research, new and simple mathematical models for SFRC behavior under compression and tension loadings were proposed depending on many experimental data. Nine-noded Lagrangian elements analytical procedure was developed by using the proposed nonlinearities to perform the static analysis of SFRC corbels. According to present analysis results the following conclusions can be drawn:

a) Present nonlinear models are valid and suitable for simulation the behavior of SFRC materials contain different steel fiber shapes.
b) The developed nine-noded Lagrangian finite element procedure is proper for the analysis of SFRC corbels due to the good agreement between the numerical outcomes and that for other related studies.

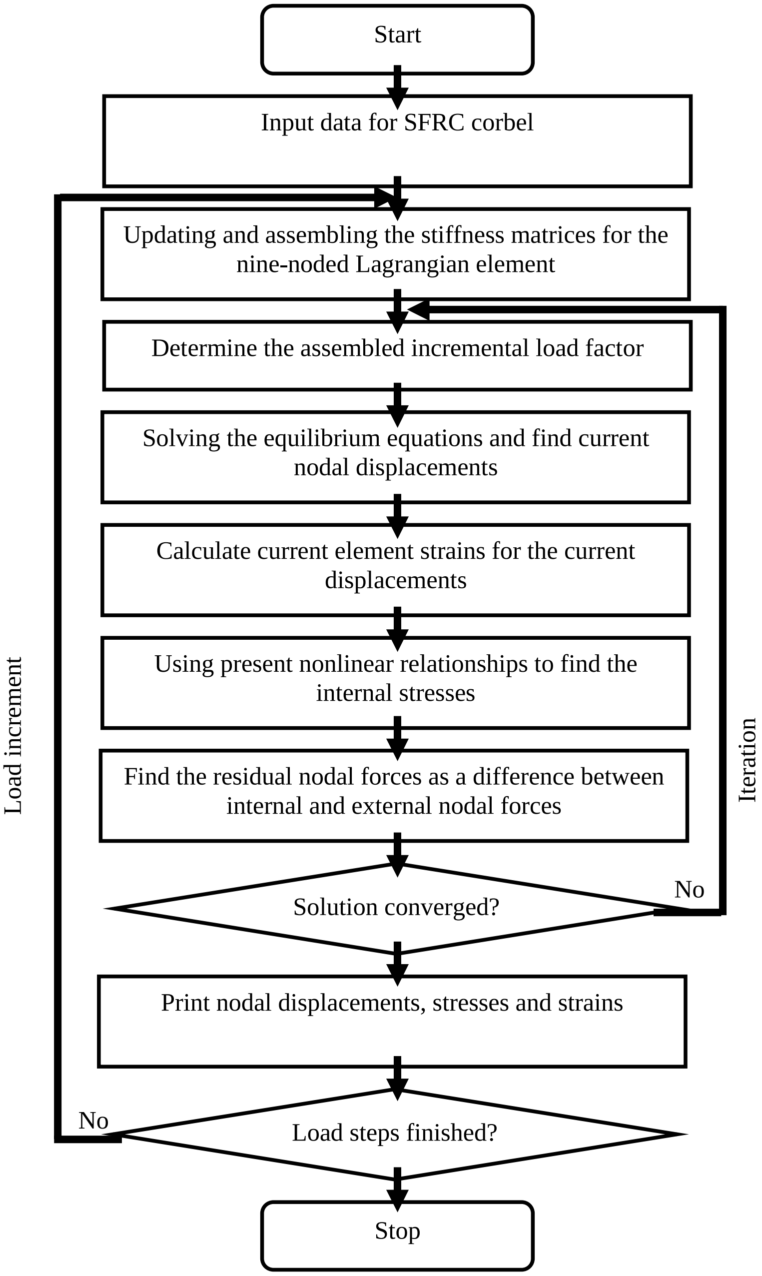

Figure-2. Flow chart of present program for SFRC corbel analysis using nine-noded lagrangian elements.

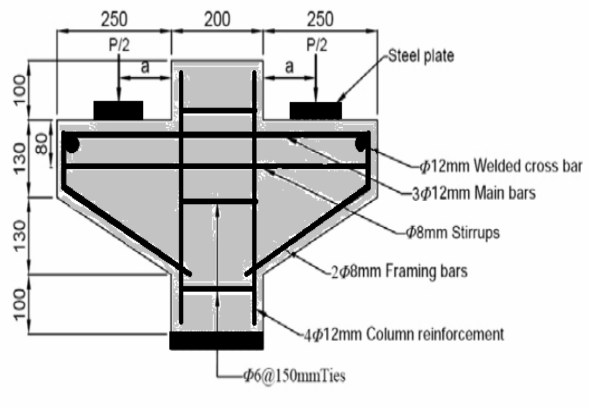

Figure-3. Geometry and loading of SFRC corbel [22].

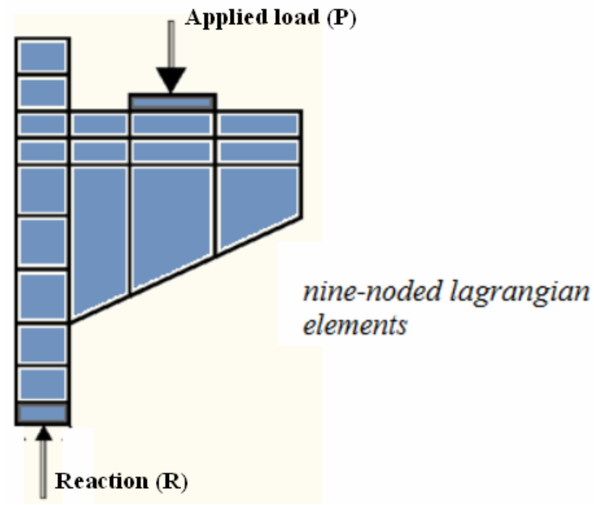

Figure-4. Finite element mesh of SFRC corbel.

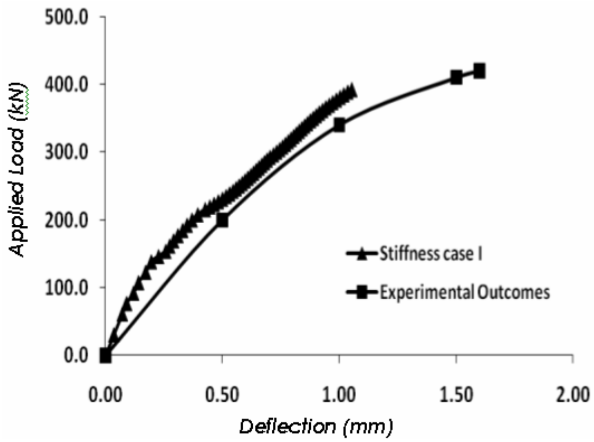

Figure-5. Applied load-deflection curve for SFRC corbel with fiber content of 0.5% - Stiffness case I.

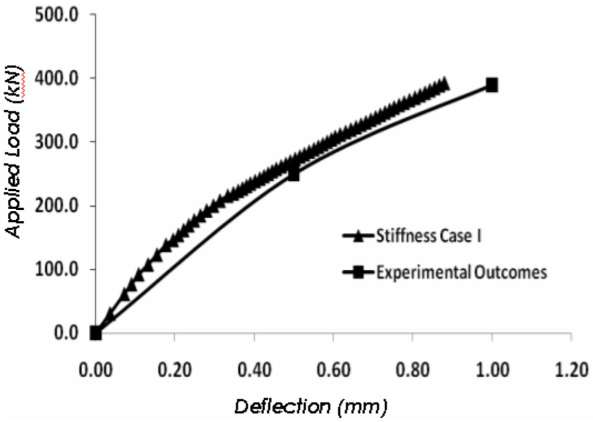

Figure-6. Applied load-deflection curve for SFRC corbel with fiber content of 1.0% - Stiffness case I.

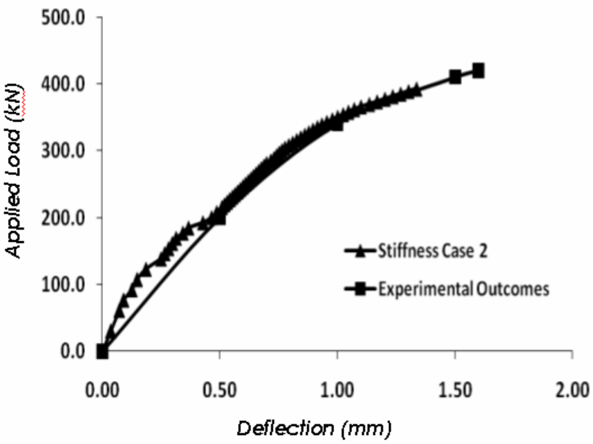

Figure-7. Applied load-deflection curve for SFRC corbel with fiber content of 0.5% - Stiffness case II.

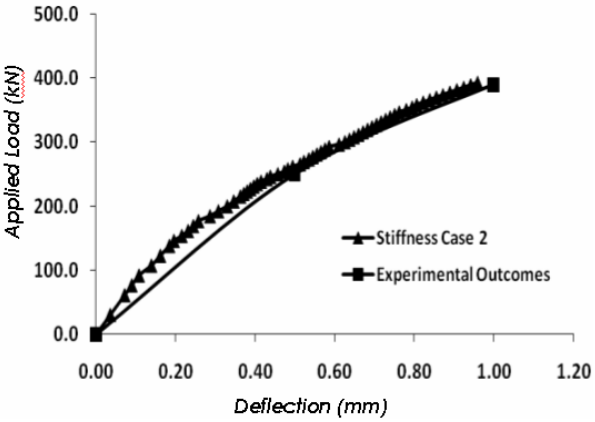

Figure-8. Applied load-deflection curve for SFRC corbel with fiber content of 1.0% - Stiffness case II.

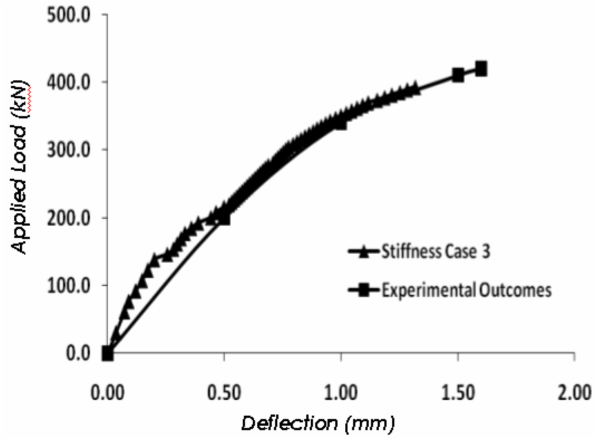

Figure-9. Applied load-deflection curve for SFRC corbel with fiber content of 0.5% - Stiffness case III.

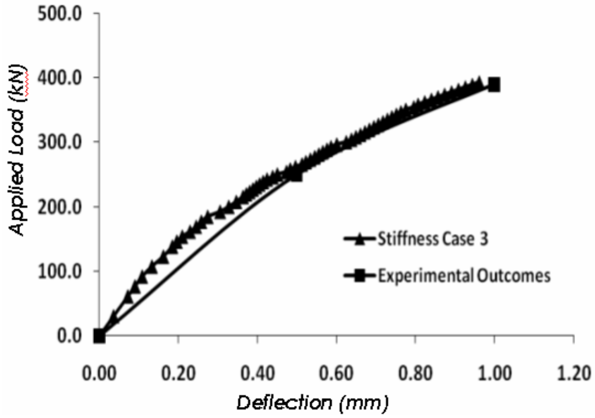

Figure-10. Applied load-deflection curve for SFRC corbel with fiber content of 1.0% - Stiffness case III.

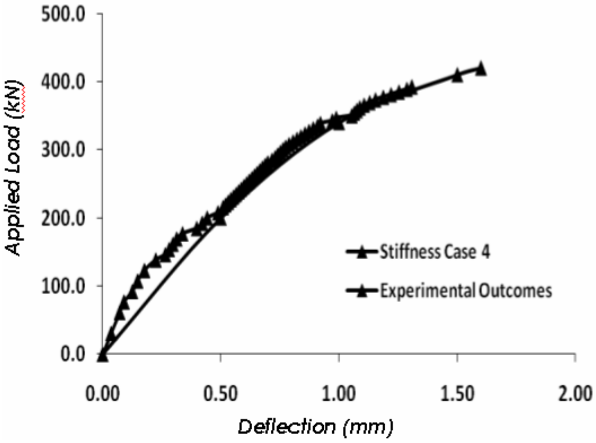

Figure-11. Applied load-deflection curve for SFRC corbel with fiber content of 0.5% - Stiffness case IV.

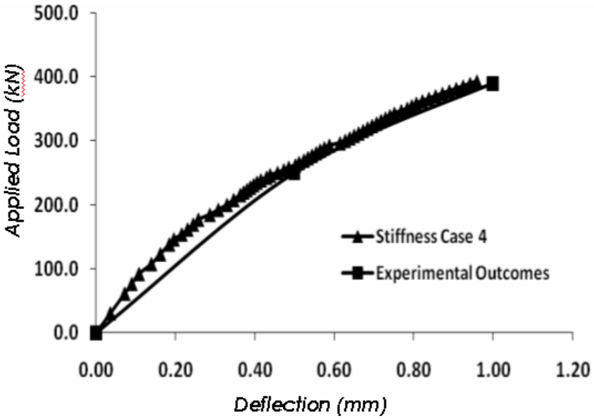

Figure-12. Applied load-deflection curve for SFRC corbel with fiber content of 1.0% - Stiffness case IV.

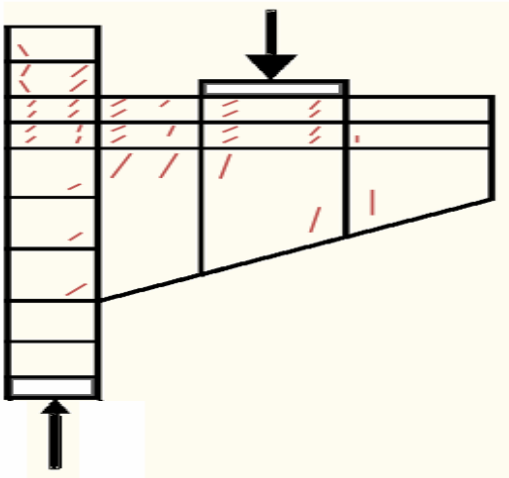

Figure-13. Crack formation of SFRC corbel with fiber content of 0.5% - consider stiffness case I.

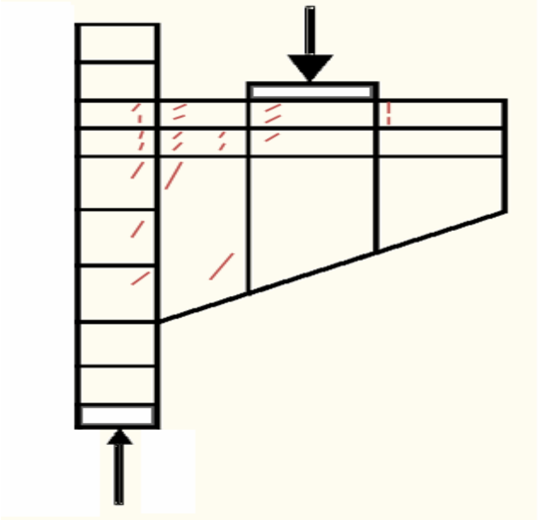

Figure-14. Crack formation of SFRC corbel with fiber content of 1.0% - consider stiffness case I.

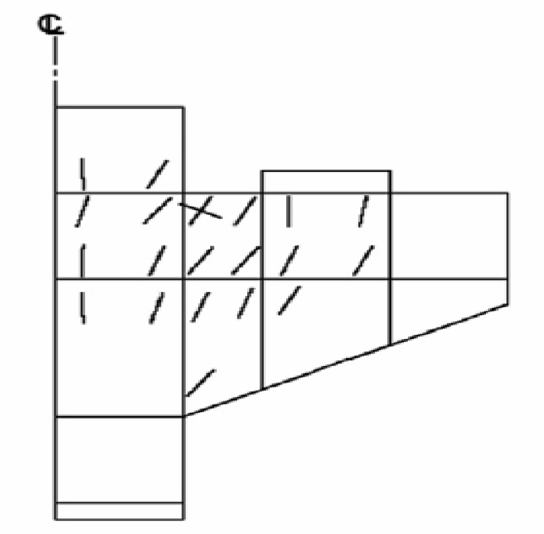

Figure-15. Failure of SFRC with fiber content of 0.5% [2].

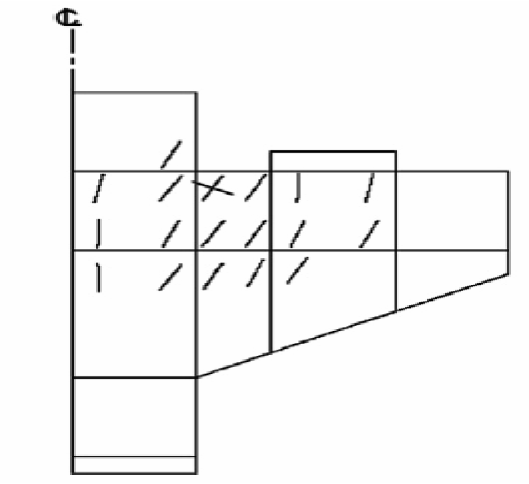

Figure-16. Failure of SFRC with fiber content of 1.0% [2].

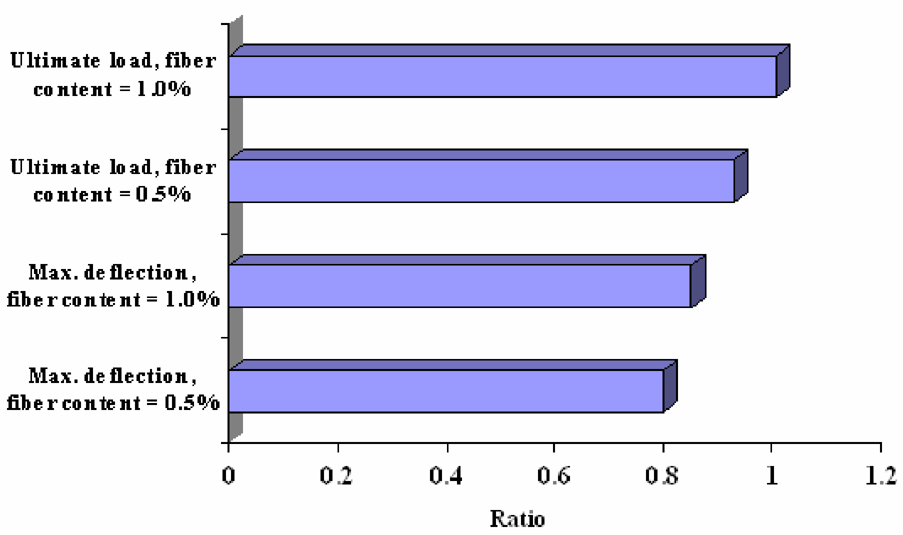

Figure-17. Agreement between numerical and experimental outputs.

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