M L ESTIMATION OF THE RELIABILITY MEASURES OF A TWO UNIT SYSTEM IN THE PRESENCE OF TWO KINDS OF CCS FAILURES

B. R. Sreedhar[1], G. Y. Sagar[2], K. Pushpanjali[3] and Y. R. Reddy[4]
[1]Department of Mathematics, Chaitanya Bharathi Institute of Technology, Hyderabad, India
[2]Department of Mathematics, T R R Engineering College, Hyderabad, India
[3]Department of Statistics, S. K. University, Anantapur, India
[4]Department of OR and SQC, Rayalaseema University, Kurnool, India
E-Mail: gysagar@gmail.com

ABSTRACT

This paper propose the Maximum likelihood estimation (M L estimation) approach for the reliability measures such as Reliability function [$R_s(t)$] and Mean time between failures [MTBF] of two component non-identical system. The system is assumed to be under the influence of Lethal and non-lethal Common Cause Shock failures. The estimates are proposed for the above said reliability indices of the system both for Series and Parallel system. Numerical illustration is also provided to justify the use of M L estimation procedure in this case.

Keywords: maximum likelihood estimation, reliability function, MTBF, lethal and non-lethal common cause shock failures.

1. INTRODUCTION

Quality and reliability play a key role in the present day where competitiveness is the main interest in the globalization process. Engineering system becomes more versatile and powerful than previous days. Sophistication in high-tech industrial process lead to reliability related problems. Thus, reliability problems not only continue to exist but are likely to require more complex solutions. Hence in recent times mathematical and statistical models were developed for evaluating system reliability. Thus system reliability analysis and modeling was one of the interesting areas in reliability theory. In addition to this, life testing and estimation are equally interested in order to assess the performance of the system in the existing conditions.

In recent times, reliability analysts have identified common cause failures (CCFs) which are the event of synchronized failure of components of a system due to external causes instead of outage of components themselves. Two types of common cause shock failures were studied in the reliability analysis. They are known as lethal and non-lethal common cause shock failures (LCCS and NCCS). For the last two decades reliability researchers have significantly accounted for these types of failures in the study of performance of systems. However the methods of estimation accounting CCS failures were not covered very much. Atwood (1986) and Meachunm and Atwood (1984) used BFR model for CCFs in the area of nuclear power plants. The quantification and estimation of CCFs rates were discussed by them. Chari (1988) and Chari *et al*. (1991) have studied the concept of CCFs to arrive the expression of availability indices like availability function and frequency failure function using Markov approach. Y. R. Reddy (2003) discussed and developed reliability measures of non-identical component system in the presence of lethal and non-lethal common cause shock failures. This paper presents the estimation of reliability function and MTBF for series and parallel system in the context of lethal and non-lethal common cause shock failures using maximum likelihood estimation approach.

2. ASSUMPTIONS

a) The system has two statistically independent and non-identical components.
b) The system is affected by both individual failures and lethal as well as non-lethal common cause shock failures.
c) The components fail individually at the rate λ_i and failure probability 'p_1'and also fail simultaneously when lethal common cause shock (LCCS) failures hit the system at a rate 'ω'.
d) The random number of components fail due to non-lethal common cause shock (NCCS) failures, which is occurring at the rate of 'β' and failure probability is 'p_2'.
e) The individual failures, LCCS and NCCS failures occur independently with each other and follow exponential distribution.
f) The failed components are repaired singly or simultaneously.

3. NOTATIONS

$\lambda_{I1}, \lambda_{I2}$ = Individual failure rate of the first and second component respectively
ω = rate of LCCS failures
β = rate of NCCS failures
μ_0, μ_1 = Service rate of the first and second component
$R_{LNS}(t)$ = Reliability function for series system
$R_{LNP}(t)$ = Reliability function for parallel system
$E_{LNS}(T)$ = Mean time between failure for series system
$E_{LNP}(T)$ = Mean time between failure for parallel system
$\hat{R}_{LNS}(t)$ = Maximum likelihood estimate of reliability function for series system
$\hat{R}_{LNP}(t)$ = Maximum likelihood estimate of reliability function for parallel system

ARPN Journal of Engineering and Applied Sciences

$\hat{E}_{LNS}(T)$ = M L Estimate of expected mean time of failure for series system

$\hat{E}_{LNP}(T)$ = M L Estimate of expected mean time of failure for parallel system

$\bar{X}$ = Sample mean of individual failures

$\bar{Y}$ = Sample mean of NCCS failures

$\bar{W}$ = Sample mean of LCCS failures

$\bar{Z}$ = Sample mean of service time of the components

$\hat{\bar{X}}$ = Sample estimate of individual failure rate

$\hat{\bar{Y}}$ = Sample estimate of NCCS failure rate

$\hat{\bar{W}}$ = Sample estimate of LCCS failure rate

$\hat{\bar{Z}}$ = Sample estimate of service time of the components

n = Sample size

N = Number of simulated samples

MSE = Mean square error

4. THE MODEL

The Markovian model was formulated to derive the reliability functions [R_{LNS} (t) and R_{LNP} (t)] under the influence of LCCS and NCCS failures. The Markov graph of the present model is seen in the Figure-4.1. Under the assumptions stated, the quantities λ_0, λ_1, λ_{12} are seen in the Figure-1 are to be defined as:

$\lambda_0 = \lambda_{I1} + \beta\, p_1\,(1 - p_2)$

$\lambda_1 = \lambda_{I2} + \beta\, p_2\,(1 - p_1)$

$\lambda_{12} = \beta\, p_1\, p_2 + \omega$

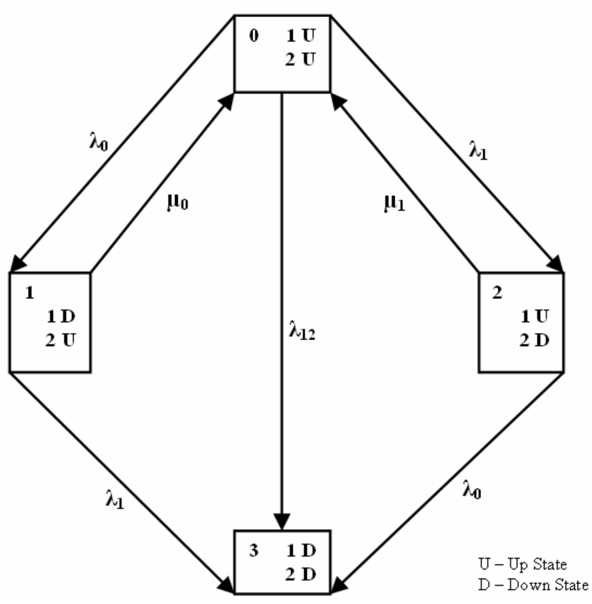

Figure-1. Markov graph for reliability functions of two unit non-identical system with individual, LCCS and NCCS failures.

From the above Markov graph, the expressions of Reliability and MTBF are derived by (Y R Reddy, 2003).

5. RELIABILITY AND MTBF OF TWO-UNIT NON-IDENTICAL SYSTEM

In this section reliability functions and MTBF of the series and parallel system, which are derived and presented (Y R Reddy, 2003). The expressions of reliability functions and MTBF in the case of series and parallel system are as follows:

5.1 Reliability function - series configuration

The expression for reliability function of the two component non-identical series system is given by:

$$R_{LNS}(t) = K1 \exp(\gamma_1 t) - K2 \exp(\gamma_2 t) + K_3 \exp(\gamma_3 t) \quad (1)$$

Where

$K_1 = (\gamma_1^{2} + \gamma_1 G + H) / (\gamma_1 - \gamma_2)(\gamma_1 - \gamma_3)$

$K_2 = (\gamma_2^{2} + \gamma_2 G + H) / (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)$

$K_3 = (\gamma_3^{2} + \gamma_3 G + H) / (\gamma_1 - \gamma_3)(\gamma_2 - \gamma_3)$

$\gamma_1 = -\gamma \sin(\alpha) - A / 3$

$\gamma_2 = \gamma \sin(\pi/3 + \alpha) - A / 3$

$\gamma_3 = \gamma \sin(-\pi/3 + \alpha) - A/ 3$

$\gamma = (2/3)(A^2 - 3B)^{1/2}$

$\alpha = (\sin^{-1}(-4q/\gamma^3))/3$

$q = C - (AB) / 3 + 2A^3/ 27$

$A = 2\,[(\lambda_{I1} + \beta\, p_1 (1- p_2)) + (\lambda_{I2} + \beta\, p_2 (1- p_1))] + \beta\, p_1 p_2 + \omega$

$B = (\lambda_{I1} + \beta\, p_1(1-p_2))[(\lambda_{I1} + \beta\, p_1(1-p_2)) + 3(\lambda_{I2} + \beta\, p_2(1-p_1)) + \beta\, p_1 p_2 + \omega] + (\lambda_{I2} + \beta p_2(1-p_1))[(\lambda_{I2} + \beta\, p_2(1-p_1)) + \beta\, p_1 p_2 + \omega]$

$C = (\lambda_{I1} + \beta\, p_1(1-p_2))(\lambda_{I2} + \beta\, p_2(1-p_1))[(\lambda_{I1} + \beta\, p_1(1-p_2)) + (\lambda_{I2} + \beta\, p_2(1-p_1)) + \beta\, p_1 p_2 + \omega]$

$G = (\lambda_{I1} + \beta\, p_1 (1-p_2) + (\lambda_{I2} + \beta\, p_2 (1-p_1))$

$H = (\lambda_{I1} + \beta\, p_1 (1- p_2)(\lambda_{I2} + \beta\, p_2 (1-p_1))$

γ_1, γ_2 and γ_3 are always negative.

$\forall\ \lambda_{I1}, \lambda_{I2} \geq 0$, p1, $p_2 \in (0, 1)$

5.2 Reliability function – parallel configuration

The expressions for reliability function of the two components non-identical parallel system is given by:

$$R_{LNP}(t) = (K_1+M_1) \exp(\gamma_1 t) - (K_2+M_2) \exp(\gamma_2 t) + (K_3+M_3) \exp(\gamma_3 t) \quad (2)$$

Where

$K_1 = (\gamma_1^{2} + \gamma_1 G_1 + H_1) / (\gamma_1 - \gamma_2)(\gamma_1 - \gamma_3)$

$K_2 = (\gamma_2^{2} + \gamma_2 G_1 + H_1) / (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)$

$K_3 = (\gamma_3^{2} + \gamma_3 G_1 + H_1) / (\gamma_1 - \gamma_3)(\gamma_2 - \gamma_3)$

$M_1 = (\gamma_1 G_2 + H_2) / (\gamma_1 - \gamma_2)(\gamma_1 - \gamma_3)$

$M_2 = (\gamma_2 G_2 + H_2) / (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)$

$M_3 = (\gamma_3 G_2 + H_2) / (\gamma_1 - \gamma_3)(\gamma_2 - \gamma_3)$

$\gamma_1 = -\gamma \sin(\alpha) - A / 3$

$\gamma_2 = \gamma \sin(\pi/3 + \alpha) - A / 3$

$\gamma_3 = \gamma \sin(-\pi/3 + \alpha) - A/ 3$

$\gamma = (2/3)(A^2 - 3B)^{1/2}$

ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

$\alpha = (\sin^{-1}(- 4q/\gamma^3))/3$
$q = C - (AB) / 3 + 2A^3/ 27$
$A = 2 [(\lambda_{I1} + \beta\ p_1 (1- p_2)) + (\lambda_{I2} + \beta\ p_2 (1- p_1))] + \beta\ p_1p_2 + \omega + \mu_0 + \mu_1$

$B = [(\lambda_{I1} + \beta\ p_1(1-p_2))\ [(\lambda_{I1} + \beta\ p_1(1-p_2)) + 3(\lambda_{I2} + \beta\ p_2(1-p_1)) + \beta\ p_1p_2 + \omega + \mu_0 + \mu_1] + (\lambda_{I2} + \beta\ p_2(1-p_1))\ [(\lambda_{I2} + \beta\ p_2 (1-p_1)) + \beta\ p_1p_2 + \omega + \mu_0 + \mu_1] + (\beta\ p_1p_2 + \omega)\ (\mu_0 + \mu_1) + \mu_0\ \mu_1]$

$C = [(\lambda_{I1} + \beta\ p_1(1-p_2))\ (\lambda_{I2} + \beta\ p_2(1-p_1))\ [\ (\lambda_{I1} + \beta\ p_1(1-p_2)) + (\lambda_{I2} + \beta\ p_2(1-p_1)) + \beta\ p_1p_2 + \omega + \mu_0 + \mu_1] + (\beta\ p_1p_2 + \omega)\ (\mu_0)\ [(\lambda_{I1} + \beta\ p_1(1-p_2)) + \mu_1] + (\lambda_{I2} + \beta\ p_2(1-p_1))\ (\beta\ p_1p_2 + \omega)\ \mu_1]$

$G_1 = [(\lambda_{I1} + \beta\ p_1\ (1-p_2)) + (\lambda_{I2} + \beta\ p_2\ (1-p_1)) + \mu_0 + \mu_1]$
$H_1 = [(\lambda_{I1} + \beta\ p_1\ (1- p_2)) + \mu_1]\ [(\lambda_{I2} + \beta\ p_2\ (1-p_1)) + \mu_0]$
$G_2 = [(\lambda_{I1} + \beta\ p_1\ (1-p_2)) + (\lambda_{I2} + \beta\ p_2\ (1-p_1))]$
$H_2 = [(\lambda_{I1} + \beta\ p_1(1-p_2))^2 + (\lambda_{I2} + \beta\ p_2(1-p_1))^2 + (\lambda_{I1} + \beta\ p_1(1-p_2))\ \mu_0 + (\lambda_{I2} + \beta\ p_2(1-p_1))\ \mu_1]$

5.3 MTBF - series configuration

Thus, the mean time between failure (MTBF) of series system is given by:

$$E_{LNS}(T) = \int_0^\infty R_{LNS}\ (t).\ dt$$

$$= (K_2 / \gamma_2) - (K_1 / \gamma_1) - (K_3 / \gamma_3) \qquad (3)$$

Where K_1, K_2, K_3 and γ_1, γ_2, γ_3 are seen in "section 5.1".

5.4 MTBF - parallel configuration

The expression for mean time between failure of the two component non-identical parallel system is given by:

$$E_{LNP}(T) = \int_0^\infty R_{LNP}\ (t).\ dt$$

$$= (K_2+M_2) / \gamma_2 - (K_1+M_1) / \gamma_1 - (K_3+M_3) / \gamma_3 \qquad (4)$$

Where K_1, K_2, K_3 and M_1, M_2, M_3 and γ_1, γ_2, γ_3 are defined in "section 5.2".

6. ESTIMATION OF RELIABILITY AND MTBF OF THE SYSTEM UNDER THE INFLUENCE OF LETHAL AND NON-LETHAL CCS FAILURES - M L ESTIMATION APPROACH

This section discusses the maximum likelihood estimation of reliability function and MTBF of two component non-identical system in the case of series and parallel configurations.

Let X_{11}, X_{12}, X_{13}… X_{1n} and X_{21}, X_{22}, X_{23}, ….. X_{2n} be samples of size 'n` representing time between individual failures components 1 and 2 respectively, which will obey exponential law.

Let Y_1, Y_2, Y_3 …Y_n; be sample of size 'n' representing time between NCCS failures which follow exponential as well.

Let W_1, W_2, W_3 ….W_n be sample of size 'n' representing times between LCCS failures which will obey exponential population.

Let Z_{11}, Z_{12}, Z_{13} ….Z_{1n} and Z_{21}, Z_{22}, Z_{23} ….Z_{2n} be 'n' number of times between repair of the components with exponential population.

Where, $\hat{\bar{X}}_1$, $\hat{\bar{X}}_2$, $\hat{\bar{Y}}$, $\hat{\bar{W}}$, $\hat{\bar{Z}}_1$ and $\hat{\bar{Z}}_2$ are the maximum likelihood estimates of individual non-identical failure rates of λ_{I1}, λ_{I2}, non-lethal common cause shock failure rate 'β', lethal common cause shock failure rate 'ω' and repair rates of μ_0, μ_1 of the system, respectively.

Where
$\hat{\bar{X}}_1 = 1 / \bar{X}_1$, $\hat{\bar{X}}_2 = 1 / \bar{X}_2$, $\hat{\bar{Y}} = 1 / \bar{Y}$, $\hat{\bar{W}} = 1 / \bar{W}$, $\hat{\bar{Z}}_1 = 1 / \bar{Z}_1$, $\hat{\bar{Z}}_2 = 1 / \bar{Z}_2$ and $\bar{X}_1 = \sum X_{1i} / n$, $\bar{X}_2 = \sum X_{2i} /n$, $\bar{Y} = \sum Y_i /n$, $\bar{W} = \sum W_i /n$, $\bar{Z}_1 = \sum Z_{1i} /n$, $\bar{Z}_2 = \sum Z_{2i} /n$ are sample estimates of the individual failure rates (λ_{I1}, λ_{I2}), non-lethal common cause shock failure rate (β), lethal common cause shock failure rate (ω) and repair rates of the components (μ_0, μ_1), respectively.

6.1 Estimation of reliability function- series system

The maximum likelihood estimate of reliability function for series system is given by:

$$\hat{R}_{LNS}(t) = K_1' \exp(D_1 t) - K_2' \exp(D_2 t) + K_3' \exp(D_3 t) \qquad (5)$$

Where

$$K_1' = (D_1^2 + D_1\ G' + H') / (D_1 - D_2)\ (D_1 - D_3)$$
$$K_2' = (D_2^2 + D_2\ G' + H') / (D_1 - D_2)\ (D_2 - D_3)$$
$$K_3' = (D_3^2 + D_3\ G' + H') / (D1 - D_3)\ (D_2 - D_3)$$

$D_1 = - D \sin(\alpha') - A' / 3$

$D_2 = D \sin(\pi/3 + \alpha') - A' / 3$

$D_3 = D \sin(-\pi/3 + \alpha') - A' / 3$

$D = (2/3)\ ((A')^2 - 3B')^{1/2}$

$\alpha' = (\sin^{-1}(- 4q' / D^3)) / 3$

$q' = C' - (A'\ B') / 3 + 2 (A')^3 / 27$

$A' = 2\ [(\hat{\bar{X}}_1 + \hat{\bar{Y}}\ p_1\ (1- p_2)) + (\hat{\bar{X}}_2 + \hat{\bar{Y}}\ p_2\ (1- p_1))] + \hat{\bar{Y}}\ p_1p_2 + \hat{\bar{W}}$

$B' = (\hat{\bar{X}}_1 + \hat{\bar{Y}}\ p_1\ (1-p_2))\ [(\hat{\bar{X}}_1 + \hat{\bar{Y}}\ p_1\ (1-p_2)) + 3(\hat{\bar{X}}_2 + \hat{\bar{Y}}\ p_2\ (1-p_1)) + \hat{\bar{Y}}\ p_1p_2 + \hat{\bar{W}}\ ] + (\hat{\bar{X}}_2 + \hat{\bar{Y}}\ p_2\ (1-p_1))\ [(\hat{\bar{X}}_2 + \hat{\bar{Y}}\ p_2\ (1-p_1)) + \hat{\bar{Y}}\ p_1p_2 + \hat{\bar{W}}\ ]$

$C' = (\hat{\bar{X}}_1 + \hat{\bar{Y}}\ p_1\ (1-p_2))\ (\hat{\bar{X}}_2 + \hat{\bar{Y}}\ p_2\ (1-p_1))\ [(\hat{\bar{X}}_1 + \hat{\bar{Y}}\ p_1\ (1-p_2)) + (\hat{\bar{X}}_2 + \hat{\bar{Y}}\ p_2\ (1-p_1)) + \hat{\bar{Y}}\ p_1p_2 + \hat{\bar{W}}\ ]$

$$G' = (\hat{X}_1 + \hat{Y} p_1 (1-p_2)) + (\hat{X}_2 + \hat{Y} p_2 (1-p_1))$$

$$H' = (\hat{X}_1 + \hat{Y} p_1 (1-p_2)) (\hat{X}_2 + \hat{Y} p_2 (1-p_1))$$

Where, $\hat{X}_1$, $\hat{X}_2$, $\hat{Y}$ and $\hat{W}$ are the maximum likelihood estimates of individual non-identical failure rates of first and second components (λ_{I1} and λ_{I2}), NCCS failure rate 'β' and LCCS failure rate 'ω' of system, respectively.

6.2 Estimation of reliability function - parallel system

The maximum likelihood estimate of reliability function for parallel system is given by:

$$\hat{R}_{LNP}(t) = (K_1' + M_1') \exp(D_1 t) - (K_2' + M_2') \exp(D_2 t) + (K_3' + M_3') \exp(D_3 t) \quad (6)$$

Where

$$K_1' = (D_1^2 + D_1 G_1' + H_1') / (D_1 - D_2)(D_1 - D_3)$$

$$K_2' = (D_2^2 + D_2 G_1' + H_1') / (D_1 - D_2)(D_2 - D_3)$$

$$K_3' = (D_3^2 + D_3 G_1' + H_1') / (D1 - D_3)(D_2 - D_3)$$

$$M_1' = (D_1 G_2' + H_2') / (D_1 - D_2)(D_1 - D_3)$$

$$M_2' = (D_2 G_2' + H_2') / (D_1 - D_2)(D_2 - D_3)$$

$$M_3' = (D_3 G_2' + H_2') / (D_1 - D_3)(D_2 - D_3)$$

$$D_1 = -D \sin(\alpha') - A'/3$$

$$D_2 = D \sin(\pi/3 + \alpha') - A'/3$$

$$D_3 = D \sin(-\pi/3 + \alpha') - A'/3$$

$$D = (2/3)((A')^2 - 3B')^{1/2}$$

$$\alpha' = (\sin^{-1}(-4q'/D^3))/3$$

$$q' = C' - (A'B')/3 + 2(A')^3/27$$

$$A' = 2[(\hat{X}_1 + \hat{Y} p_1 (1-p_2)) + (\hat{X}_2 + \hat{Y} p_2 (1-p_1))] + \hat{Y} p_1 p_2 + \hat{W} + \hat{Z}_1 + \hat{Z}_2$$

$$B' = (\hat{X}_1 + \hat{Y} p_1 (1-p_2)) [(\hat{X}_1 + \hat{Y} p_1 (1-p_2)) + 3(\hat{X}_2 + \hat{Y} p_2 (1-p_1)) + \hat{Y} p_1 p_2 + \hat{W} + \hat{Z}_1 + \hat{Z}_2] + (\hat{X}_2 + \hat{Y} p_2 (1-p_1)) [(\hat{X}_2 + \hat{Y} p_2 (1-p_1)) + \hat{Y} p_1 p_2 + \hat{W} + \hat{Z}_1 + \hat{Z}_2] + (\hat{Y} p_1 p_2 + \hat{W})(\hat{Z}_1 + \hat{Z}_2) + \hat{Z}_1 \hat{Z}_2]$$

$$C' = [(\hat{X}_1 + \hat{Y} p_1 (1-p_2))(\hat{X}_2 + \hat{Y} p_2 (1-p_1)) [(\hat{X}_1 + \hat{Y} p_1 (1-p_2)) + (\hat{X}_2 + \hat{Y} p_2 (1-p_1)) + \hat{Y} p_1 p_2 + \hat{W} + \hat{Z}_1 + \hat{Z}_2] + (\hat{Y} p_1 p_2 + \hat{W})(\hat{Z}_1)[(\hat{X}_1 + \hat{Y} p_1 (1-p_2)) + \hat{Z}_2] + (\hat{X}_2 + \hat{Y} p_2 (1-p_1))(\hat{Y} p_1 p_2 + \hat{W})\hat{Z}_2]$$

$$G_1' = [(\hat{X}_1 + \hat{Y} p_1 (1-p_2)) + (\hat{X}_2 + \hat{Y} p_2 (1-p_1)) + \hat{Z}_1 + \hat{Z}_2]$$

$$H_1' = [(\hat{X}_1 + \hat{Y} p_1 (1-p_2)) + \hat{Z}_2][(\hat{X}_2 + \hat{Y} p_2 (1-p_1)) + \hat{Z}_1]$$

$$G_2' = [(\hat{X}_1 + \hat{Y} p_1 (1-p_2)) + (\hat{X}_2 + \hat{Y} p_2 (1-p_1))]$$

$$H_2' = [(\hat{X}_1 + \hat{Y} p_1 (1-p_2))^2 + (\hat{X}_2 + \hat{Y} p_2 (1-p_1))^2 + (\hat{X}_1 + \hat{Y} p_1 (1-p_2))\hat{Z}_1 + (\hat{X}_2 + \hat{Y} p_2 (1-p_1))\hat{Z}_2]$$

Where, $\hat{X}_1$, $\hat{X}_2$, $\hat{Y}$, $\hat{W}$, $\hat{Z}_1$ and $\hat{Z}_2$ are the maximum likelihood estimates of individual non-identical failure rates of first and second components (λ_{I1} and λ_{I2}), NCCS failure rate 'β' and LCCS failure rate 'ω' and repair rates of the components μ_0, μ_1, respectively.

6.3 Estimation of mtbf function - series system

Thus, the expression of maximum likelihood estimate of mean time between failure function for series system is given by:

$$\hat{E}_{LNS}(T) = (K_2'/D_2) - (K_1'/D_1) - (K_3'/D_3) \quad (7)$$

Where K_1', K_2', K_3' and D_1, D_2, D_3 are seen in "section 6.1"

6.4 Estimation of mtbf function- parallel system

Thus the expression of maximum likelihood estimate of mean time between failure function for parallel system is given by:

$$\hat{E}_{LNP}(T) = [(K_2' + M_2')/D_2 - (K_1' + M_1')/D_1 - (K_3' + M_3')/D_3] \quad (8)$$

Where K_1', K_2', K_3' and M_1', M_2', M_3' and D_1, D_2, D_3 are seen in "section 6.2"

7. SIMULATION AND VALIDITY

The proposed estimates of reliability function and MTBF by maximum likelihood estimation approach do not find analytical form of density and it is not possible to attempt or develop analytical verification of properties of proposed M L estimates. Hence in this paper empirical approach is considered and Monte Carlo simulation procedure is used for validity of results.

For a range of specified values of the rates of individual (λ_{I1}, λ_{I2}), LCCS failures (ω), NCCS failures (β) and service rates (μ_0, μ_1) and for the samples of sizes n = 5 (5) 30 are simulated using computer package in this paper and the sample estimates are computed for N = 10000 (20000) 90000 and mean square error (MSE) of the estimates for R_{LNS} (t), R_{LNP} (t), E_{LNS} (T), E_{LNP} (T) were obtained and given in numerical illustration. For large samples M L estimates are undisputedly better since they are CAN estimators. However it is interesting to note that for a sample size as low as five i.e., (n=5) M L estimate is still reasonably good giving near accurate estimate in this case.

7.1 Numerical illustration

Table-1.

Two unit non-identical Reliability function - Series Configuration with $\lambda_{I1} = 0.5$; $\lambda_{I2} = 0.6$; $\beta = 0.1$; $\omega = 0.2$; $p_1 = 0.5$; $p_2 = 0.5$; t = 1.

Sample size n = 5			
N	$R_{LNS}(t)$	$\hat{R}_{LNS}(t)$	M S E
10000	0.253388	0.177137	0.013622
30000	0.253388	0.176391	0.013603
50000	0.253388	0.176434	0.013552
70000	0.253388	0.177673	0.013510
90000	0.253388	0.176922	0.013571

Sample size n = 10			
N	$R_{LNS}(t)$	$\hat{R}_{LNS}(t)$	M S E
10000	0.253388	0.201044	0.006946
30000	0.253388	0.200229	0.007057
50000	0.253388	0.201018	0.006991
70000	0.253388	0.200649	0.007039
90000	0.253388	0.200374	0.007062

Sample size n = 15			
N	$R_{LNS}(t)$	$\hat{R}_{LNS}(t)$	M S E
10000	0.253388	0.209512	0.004791
30000	0.253388	0.209703	0.004787
50000	0.253388	0.209052	0.004845
70000	0.253388	0.209251	0.004836
90000	0.253388	0.209254	0.004836

Sample size n = 20			
N	$R_{LNS}(t)$	$\hat{R}_{LNS}(t)$	M S E
10000	0.253388	0.213712	0.003733
30000	0.253388	0.214647	0.003683
50000	0.253388	0.214205	0.003724
70000	0.253388	0.213985	0.003710
90000	0.253388	0.213791	0.003757

Sample size n = 25			
N	$R_{LNS}(t)$	$\hat{R}_{LNS}(t)$	M S E
10000	0.253388	0.216783	0.003112
30000	0.253388	0.216958	0.003083
50000	0.253388	0.216739	0.003103
70000	0.253388	0.216571	0.003108
90000	0.253388	0.216591	0.003104

Sample size n = 30			
N	$R_{LNS}(t)$	$\hat{R}_{LNS}(t)$	M S E
10000	0.253388	0.218981	0.002617
30000	0.253388	0.218587	0.002669
50000	0.253388	0.218520	0.002661
70000	0.253388	0.218368	0.002692
90000	0.253388	0.218394	0.002668

Table-2.

Two unit non-identical Reliability function - Parallel Configuration with $\lambda_{I1} = 0.2$; $\lambda_{I2} = 0.3$; $\beta = 0.02$; $\omega = 0.03$; $\mu = 0.05$; $\mu = 0.06$; $p_1 = 0.5$; $p_2 = 0.5$; t = 1.

Sample size n = 5			
N	$R_{LNP}(t)$	$\hat{R}_{LNP}(t)$	M S E
10000	0.585360	0.697889	0.188680
30000	0.585360	0.697769	0.154186
50000	0.585360	0.696715	0.225490
70000	0.585360	0.695914	0.209205
90000	0.585360	0.696791	0.204763

Sample size n = 10			
N	$R_{LNP}(t)$	$\hat{R}_{LNP}(t)$	M S E
10000	0.585360	0.630603	0.006568
30000	0.585360	0.631440	0.010734
50000	0.585360	0.631280	0.007021
70000	0.585360	0.631242	0.008062
90000	0.585360	0.631209	0.007769

Sample size n = 15			
N	$R_{LNP}(t)$	$\hat{R}_{LNP}(t)$	M S E
10000	0.585360	0.624279	0.002953
30000	0.585360	0.624266	0.002942
50000	0.585360	0.623939	0.002949
70000	0.585360	0.624107	0.002927
90000	0.585360	0.624091	0.002940

ARPN Journal of Engineering and Applied Sciences

Sample size n = 20			
N	$R_{LNP}(t)$	$\hat{R}_{LNP}(t)$	M S E
10000	0.585360	0.622717	0.002432
30000	0.585360	0.622174	0.002350
50000	0.585360	0.622905	0.002392
70000	0.585360	0.622563	0.002386
90000	0.585360	0.622368	0.002368

Sample size n = 25			
N	$R_{LNP}(t)$	$\hat{R}_{LNP}(t)$	M S E
10000	0.585360	0.622462	0.002177
30000	0.585360	0.622218	0.002148
50000	0.585360	0.622041	0.002145
70000	0.585360	0.622143	0.002143
90000	0.585360	0.621987	0.002139

Sample size n = 30			
N	$R_{LNP}(t)$	$\hat{R}_{LNP}(t)$	M S E
10000	0.585360	0.621610	0.001983
30000	0.585360	0.621753	0.001987
50000	0.585360	0.621876	0.001997
70000	0.585360	0.621941	0.002002
90000	0.585360	0.621832	0.001990

Table-3.

Simulation results for Mean Time between Failures function - Series Configuration with $\lambda_{I1} = 0.2$; $\lambda_{I2} = 0.3$; $\beta = 0.02$; $\omega = 0.03$; $p_1 = 0.5$; $p_2 = 0.5$.

	Sample size n = 5		
N	$E_{LNS}(T)$	$\hat{E}_{LNS}(T)$	M S E
10000	1.843660	1.507307	0.384593
30000	1.843660	1.502634	0.381706
50000	1.843660	1.503561	0.380992
70000	1.843660	1.510876	0.381490
90000	1.843660	1.506544	0.382465

Sample size n = 10			
N	$E_{LNS}(T)$	$\hat{E}_{LNS}(T)$	M S E
10000	1.843660	1.608480	0.195839
30000	1.843660	1.603750	0.197966
50000	1.843660	1.608191	.197106
70000	1.843660	1.606037	.198584
90000	1.843660	1.604597	.198782

Sample size n = 15			
N	$E_{LNS}(T)$	$\hat{E}_{LNS}(T)$	M S E
10000	1.843660	1.641517	0.135792
30000	1.843660	1.642902	0.135640
50000	1.843660	1.639277	0.137177
70000	1.843660	1.640475	0.137315
90000	1.843660	1.640735	0.137167

Sample size n = 20			
N	$E_{LNS}(T)$	$\hat{E}_{LNS}(T)$	M S E
10000	1.843660	1.657060	0.106342
30000	1.843660	1.662879	0.105028
50000	1.843660	1.660965	0.106286
70000	1.843660	1.659527	0.105605
90000	1.843660	1.658251	0.107077

Sample size n = 25			
N	$E_{LNS}(T)$	$\hat{E}_{LNS}(T)$	M S E
10000	1.843660	1.669831	0.089679
30000	1.843660	1.671068	0.088325
50000	1.843660	1.670444	0.088690
70000	1.843660	1.669435	0.088811
90000	1.843660	1.669131	0.088752

Sample size n = 30			
N	$E_{LNS}(T)$	$\hat{E}_{LNS}(T)$	M S E
10000	1.843660	1.679359	0.074938
30000	1.843660	1.676950	0.076391
50000	1.843660	1.676595	0.076162
70000	1.843660	1.675720	0.077014
90000	1.843660	1.675966	0.076293

Table-4.

Simulation results for Mean Time between Failures function - Parallel Configuration with $\lambda_{I1} = 0.2$; $\lambda_{I2} = 0.3$; $\beta = 0.02$; $\omega = 0.03$; $p_1 = 0.5$; p2 = 0.5; $\mu_0 = 0.01$; $\mu_1 = 0.02$.

Sample Size n = 5			
N	$E_{LNP}(T)$	$\hat{E}_{LNP}(T)$	M S E
10000	2.299145	2.806930	6.031752
30000	2.299145	2.813680	4.742666
50000	2.299145	2.800186	5.543337
70000	2.299145	2.788599	5.659295
90000	2.299145	2.801859	5.459275

Sample size n = 10			
N	**E_{LNP} (T)**	**$\hat{E}_{LNP}$ (T)**	**M S E**
10000	2.299145	2.403138	0.369342
30000	2.299145	2.402448	0.451706
50000	2.299145	2.402406	0.341820
70000	2.299145	2.400231	0.396070
90000	2.299145	2.401181	0.391577

Sample size n = 15			
N	**E_{LNP} (T)**	**$\hat{E}_{LNP}$ (T)**	**M S E**
10000	2.299145	2.333296	0.138836
30000	2.299145	2.330864	0.137853
50000	2.299145	2.331183	0.140487
70000	2.299145	2.330628	0.137249
90000	2.299145	2.330697	0.138414

Sample size n = 20			
N	**E_{LNP} (T)**	**$\hat{E}_{LNP}$ (T)**	**M S E**
10000	2.299145	2.304328	0.092116
30000	2.299145	2.300806	0.087171
50000	2.299145	2.306305	0.088652
70000	2.299145	2.303950	0.089073
90000	2.299145	2.302454	0.088497

Sample size n = 25			
N	**E_{LNP} (T)**	**$\hat{E}_{LNP}$ (T)**	**M S E**
10000	2.299145	2.295125	0.067497
30000	2.299145	2.291784	0.066676
50000	2.299145	2.290966	0.067142
70000	2.299145	2.291181	0.066722
90000	2.299145	2.289789	0.066789

Sample size n = 30			
N	**E_{LNP} (T)**	**$\hat{E}_{LNP}$ (T)**	**M S E**
10000	2.299145	2.282674	0.055078
30000	2.299145	2.280037	0.054193
50000	2.299145	2.281962	0.054084
70000	2.299145	2.282627	0.054227
90000	2.299145	2.282307	0.053688

8. CONCLUSSIONS

This paper is aimed to establish the estimates of reliability measures [R_{LNS} (t), R_{LNP} (t), E_{LNS} (T), E_{LNP} (T)] of two component non-identical system in the presence of LCCS and NCCS failures as well as individual failures for both series and parallel configurations. The M L method proposed here is giving almost accurate estimation in the case of sample size 20 and above (n ≥ 20) which is verified by simulation process in the absence of analytical approach. Thus empirical evidence was developed which indicate that MSE is found very small and satisfactory for the estimation process. For all the reliability indices even for very small sample of size n = 5 are found reasonable. This shows that M L method of estimator is quite useful in estimating reliability indices like R_{LNS} (t), R_{LNP} (t), E_{LNS} (T), E_{LNP} (T).

REFERENCES

Atwood C. L. 1986. The binomial failure rate common cause model. In Techno metrics. 28: 139-147.

Atwood C. L. 1984. Approximate tolerance intervals, based on Maximum Likelihood Estimates. Journal of the American Statistical Association. 79: 459-465.

Billinton R and Allan R. N. 1983. Reliability Evaluation of Engineering Systems; Concepts and Techniques. Plenum Press, New York, USA.

Chari A. A. 1988. A Markovian approach to system and availability measures with Common Cause Failure. Ph.D., Thesis, Sri Venkateswara University, Tirupati, India.

Chari A. A., Sastry M. P and Madhusudhana Verma S. 1991. Reliability analysis in the presence of common cause shock failures. Micro-Electronics and reliability. 31: 15-19.

Reddy Y. R. 2003. Reliability analysis for two unit non-identical system with CCS failures. Ph.D thesis, S.K. University, Anantapur, India.

Sagar G. Y. 2007. Markovian approach to system reliability measures with common cause shock failures and human errors. Ph. D thesis, S. K. University, Anantapur, India.

Verma S. M and Reddy E.M.K. 2003. Reliability Indices for 2-Component Non-identical system. The Institution of Engineers (India). 84: 30-33.