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RENORMALIZATION OF QCD USING MODIFIED PRE-REGULARIZATION METHOD AND β-FUNCTION

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ABSTRACT

In an earlier paper we have prescribed a modified form of pre-regularization and used this prescription to study the renormalization of QED and Yang-Mills theory. We have seen that this is one of the best prescriptions in studying the quantum field theory problems. In this paper we have applied this prescription in studying the renormalization of QCD and also found the corresponding β-function. Here also we obtained the same result as found by other regularization prescriptions.

Keywords: renormalization, QCD, modified pre-regularization method, dimensional regularization method, Loop integrals, β-function.

1. INTRODUCTION

Quantum electrodynamics (QED) is a gauge theory to study electromagnetic interactions, which is well established through rigorous study of many well known scientists and researchers. In the same way Gross and wilezek [1], Politzer [2] and Weinberg [3] showed that quantum chromodynamics (QCD) is a gauge theory to study strong interactions phenomena. QCD stemps from incorporating various remarkable ideas of hadronic physics, such as quarks, partons, colour and current algebra etc. Asymptotic freedom i.e., coupling strength decreases at short distances is the key point in establishing the theory of QCD. That means if the theory is asymptotically free then the quarks can interact weakly at short distances.

The problem of renormalization in QED can be studied perturbatively through the use of regularization method. That means when we consider radiative corrections in QED Lagrangian then we have to evaluate loop diagrams which are not always finite. Then the problem of divergencies can be consistently studied by the use of proper regularization method. Although there are many regularization methods but not all regularization methods [4, 5, 6] are suitable for all problems even in QED and many of them can not be used in studying the problems of QCD. However, like dimensional regularization [4], pre-regularization [7] is one of the best prescriptions, which can be applied in studying the problems both in QED and QCD.

In a recent paper [8] we have prescribed a modified form of pre-regularization and explained clearly why one needs to modify the original pre-regularization [7] method. In that paper we have demonstrated how the modified form of pre-regularization can be applied to study the renormalization problem and the calculation of β-function in QED and also in Yang-Mills theory. There we have explained the advantage of using modified form of pre-regularization than that of others. The main advantage of this prescription is that one can study the problem in physical dimension and the calculations are more simpler than that of other methods.

Because of the simplicity and straight forward calculation of the combination of pre-regularization and modified form of pre-regularization it is plausible to check whether this modified method is applicable in the renormalization of QCD and other interested problems. In this view we are interested to apply this new prescription to study renormalization of QCD.

In section 2, QCD Lagrangian is described and for renormalization of the theory at one-loop level appropriate Feynman diagrams are depicted. Since pre-regularization is described in reference [7] and modified pre-regularization is describe in our earlier paper [8] that is why instead of describing the prescriptions we are using the prescription and relevant results to evaluate the loop diagrams in section 3. In section 4 we have found different renormalization constants from the calculation of section 3, which are needed for renormalization of QCD. Then we have evaluated the β-function for QCD.

2. QCD LAGRANGIAN AND ONE-LOOP FEYNMAN DIAGRAMS

Following [6, 7] we can write the QCD Lagrangian in the following form:

$$L_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_k^m \hat{q}_k (i\gamma^\mu D_\mu - m_k) q_k \tag{1}$$

Where

$$G^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g\, f^{abc} B^b_\mu B^c_\nu \tag{2}$$

$$D_\mu q_k = (\partial_\mu - igB_\mu) q_k \tag{3}$$

$$B_\mu = \sum_{a=1}^{\infty} B^a_\mu t^a = \sum_{a=1}^{8} B^a_\mu \frac{\lambda^a}{2} \tag{4}$$

Here B_μ^a is the colour gauge field similar to the iso-spin gauge field in the original Yang-Mills theory and g is the strong interaction coupling constant, k is the flavour index $k = 1, 2, 3, ..., n_f$ (number of quark flavour). That is, $q_k : u, d, s, c, b, ...$.

The λ's are the Gell-Mann matrices that satisfy the $SU(3)_c$ commutation relations

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = if^{abc}\frac{\lambda_c}{2} \qquad (5)$$

And the normalization condition

$$tr(\lambda^a \lambda^b) = 2\delta^{ab} \qquad (6)$$

QED describes the interaction between gauge field i.e., photon and matter field i.e., electron. On the other hand QCD describes the interaction between colour gauge field i.e., gluon and quarks. In QED there is no self interaction between photon, but in QCD due to the last term in (2) there is self interaction between gluons. Hence we get some more Feynman diagrams than QED. The Lagrangian (1) satisfies $SU(3)_c$ symmetries, so it has eight generators.

To find the Feynman rules in a consistent way we have to add gauge fixing term and Faddeev-Popov ghost term to the Lagrangian (1). Then the complete Lagrangian can now be written as:

$$L_{QCD} = -\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} + \sum_k^m \hat{q}_k (i\gamma^\mu D_\mu - m_k) q_k$$
$$+ g\, f^{abc} B_\mu^b B_\nu^c - \frac{1}{2\alpha}(\partial^\mu B_\mu^a)^2 + (\partial^\mu \bar{c}^a)(D_\mu^{ab} c^b) \qquad (7)$$

where α is a parameter in covariant gauge and c^a is the Faddeev-Popov ghost field.

From this Lagrangian now we are able to draw all loop diagrams. Since we are interested only to one-loop diagrams, so let us draw only the appropriate one-loop Feynman diagrams, which contributes to the problem of one-loop renormalization. The one-loop diagrams are:

One-loop quark self-energy diagram

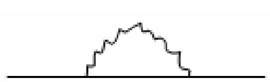

Figure-1.

One-loop gluon self-energy diagrams

Figure-2(a)

Figure-2(b)

Figure-2(c)

One-loop ghost self-energy diagram

Figure-3.

One-loop quark-gluon vertex

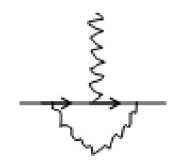

Figure-4(a)

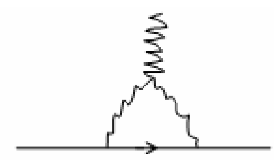

Figure-4(b)

If we compare these diagrams with QED one-loop diagrams [8], we can see that due to gluon and the presence of ghost field we got four extra diagrams which are depicted in Figure-2(b), Figure-2(c), Figure-3 and Figure-4(b). This is because photon can not interact with itself where as gluon can for which we got the Figure-2(b) and Figure-4(b). Again ghost field can interact with gluon and quark which is represented in Figure-2(c) and Figure-3. Again Figure-2(b) and Figure-2(c) also one can get in Yang-Mills theory which is also evaluated in [8]. Since Figure-1, Figure-2(a) and Figure-4(a) are almost same as in QED diagram and Figure-2(b) and Figure-2(c) are same as Yang-Mills loops that we have demonstrated in [8] using modified form of pre-regularization, so in this paper we will only use these results except some minor replacement of the factors such as coupling constant g instead of e and appropriate colour factor C_F which was absent in QED. However, we will evaluate the other diagrams using modified form of pre-regularization and taking the result of all loop- diagrams we will try to find the renormalization of QCD. Also we will evaluate the β-function with this modified form of pre-regularization. These calculations demonstrate the advantage of this method in evaluating loop-diagrams.

3. EVALUATION OF ONE-LOOP QCD DIAGRAMS WITH MODIFIED PRE-REGULARIZATION

3.1. One-loop quark self-energy

The one-loop contribution to the quark self-energy is found from Figure-1. Using pre-regularization we can write the amplitude of quark self-energy as:

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$$\sum(p) = -ig^2 t^a t^b \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_\mu(\frac{\not p}{2} - \not k - \not s_1 + m)\gamma^\mu}{\{(\frac{p}{2} - k - s_1)^2 - m^2\}(\frac{p}{2} + k + s_1)^2} \quad (8)$$

This is the same as equation (11) of [8]; except the factor g for coupling constant and $t^a t^b$ for quark-gluon vertex in a colour diagram.

After doing the γ-algebra in 4-dimension and performing the 4-dimensional momentum integral we get (like equation (15) of [8]).

$$\sum(p) = -\frac{g^2}{16\pi^2} t^a t^b \Gamma(0)(\not p - 4m) + \frac{g^2}{32\pi^2} t^a t^b \not s_1 \quad (9)$$

Using modified pre-regularization, that is replacing $\Gamma(0)$ by $\Gamma(\varepsilon/2)$ where $\varepsilon = 4 - d$; d is the dimension of the integral, the equation (9) can now be written as:

$$\sum(p) = -\frac{g^2}{8\pi^2 \varepsilon} C_F (\not p - 4m) + \text{ finite term} \quad (10)$$

Here C_F is the colour factors which come from $t^a t^b$.

We have used the divergent parameter ε for renormalization, which was absent in the original form of pre-regularization.

3.2 One-loop gluon self-energy

Figure-2(a) to Figure-2(c) contributes to the gluon self-energy in QCD; where as only Figure-2(a) arises in QED, which is not for gluon but for electron. The amplitude of Figure-2(a) is evaluated in [8] for QED. In QCD problem, we can use the same result for Figure-2(a) with appropriate corrections for quark-gluon vertex and colour factor. Hence we can write the amplitude of Figure-2(a) as:

$$\prod_{\mu\nu}^{(a)}(p) = -\frac{g^2}{6\pi^2}\frac{1}{\varepsilon} T_F n_f (p^2 g_{\mu\nu} - p_\mu p_\nu) + \text{finiteterms} \quad (11)$$

Similarly, writing the amplitudes of Figure-2(b) and Figure-2(c) following equation (56) and (59) of [8] we get,

$$\prod_{\mu\nu}^{(b)}(p) = \frac{g^2}{16\pi^2}\frac{1}{\varepsilon} C_A (\frac{19}{6} p^2 g_{\mu\nu} - \frac{11}{3} p_\mu p_\nu) + \text{finiteterms} \quad (12)$$

$$\prod_{\mu\nu}^{(c)}(p) = \frac{g^2}{16\pi^2}\frac{1}{\varepsilon} C_A (\frac{1}{6} p^2 g_{\mu\nu} + \frac{1}{3} p_\mu p_\nu) + \text{finiteterms} \quad (13)$$

Form these calculation it is transparent that the amplitude of Figure-2(a) is transverse but amplitude of Figure-2(b) and Figure-2(c) are not individually transverse. But if we add the result of Figure-2(b) and Figure-2(c) then the result is transverse. Hence the total amplitude for one loop gluon self-energy is transverse which can be written in the following form:

$$\prod_{\mu\nu} = -\frac{g^2}{6\pi^2}\frac{1}{\varepsilon} T_F n_f (p^2 g_{\mu\nu} - p_\mu p_\nu)$$
$$+ \frac{5g^2}{24\pi^2}\frac{1}{\varepsilon} C_A (p^2 g_{\mu\nu} - p_\mu p_\nu) + \text{ finite terms}$$
$$= -\frac{g^2}{24\pi^2}\frac{1}{\varepsilon}(p^2 g_{\mu\nu} - p_\mu p_\nu)(4T_F n_f - 5C_A) + \text{ finite terms} \quad (14)$$

3.3. One-loop ghost self-energy

In the ghost propagator only Figure-3 contributes to the ghost self-energy at the one-loop level. Using pre-regularization method and substituting the proper Feynman rules we can write the amplitude of ghost self-energy as:

$$\sum(p^2) = -iC_A g^2 \int \frac{d^4k}{(2\pi)^4} p_\mu \frac{1}{(k+p+s_5)^2}(k+p+s_5)_\nu \frac{g_{\mu\nu}}{(k+s_5)^2} \quad (15)$$

Where C_A the colour factor is arises from the product of the structure constants.

After simplification and combining the denominators using Feynman identity we get:

$$\sum(p^2) = -iC_A g^2 \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \times \frac{p.(k+p+s_5)}{[(k+s_5+px)^2 + p^2 x(1-x)]^2} \quad (16)$$

Shifting the variable of integration keeping track of the surface terms with the help of pre-regularization this becomes:

$$\sum(p^2) = -iC_A g^2 \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \frac{p.(k+p(1-x))}{[k^2 + p^2 x(1-x)]^2}$$
$$-\frac{C_A g^2}{32\pi^2} \int_0^1 dx\, p.(s_5 + px) \quad (17)$$

Performing the k-integral and x-integral using modified pre-regularization method, we obtain:

$$\sum(p^2) = -C_A g^2 \frac{1}{16\pi^2 \varepsilon} p^2 + \text{finiteterms} \quad (18)$$

Which is the amplitude of one-loop ghost energy.

3.4. One-loop quark-gluon vertex

In the quark-gluon vertex only the two diagrams Figure-4(a) and Figure-4(b) contributes at the one-loop level. The amplitude of Figure-4(a) is almost same as QED except the proper colour factor. Using the appropriate QCD Feynman rules and following the method of pre-regularization, the amplitude of Figure-4(a) is:

$$\Lambda^{(1)}_\mu(p,p') = -g^2 t^a t^b t^c \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\alpha(\frac{\not p}{2} - \not k - \not s_6 + m)\gamma_\mu(\not p' - \frac{\not p}{2} - \not k - \not s_6 + m)\gamma_\alpha}{(\frac{p}{2}+k+s_6)^2\{(\frac{p}{2}-k-s_6)^2 - m^2\}\{p' - (\frac{p}{2}-k-s_6)^2 - m^2\}} \quad (19)$$

Doing the γ-algebra in 4-dimensions and after going through all algebraic and other technical manipulations and following equation (22) and equation (26) of [8] we can write the amplitude of Figure-4(b) in the following form:

$$\Lambda^1_\mu(p,p') = \frac{g^2}{8\pi^2\varepsilon}(C_F - C_A/2)\gamma_\mu + \textit{finite terms} \quad (20)$$

where $(C_F - C_A/2)$ came from the colour factor of $t^a t^b t^c$.

Since the calculation of Figure-4(b) is complicated so let us give a bit detail of it in this paper. Following QCD Feynman rules the amplitude of Figure-4(b) can be written as:

$$\Lambda^{(2)}_\mu = ig^2 t^b t^c f^{abc} \int \frac{d^4q}{(2\pi)^4}\frac{1}{D}[\gamma_\alpha(\not p + \not q + \not s_8)\gamma_\beta \times \{(2q+k+2s_8)^\mu g^{\alpha\beta} - (q+2k+2s_8)^\alpha g^{\beta\mu} - (q-k+s_8)^\beta g^{\mu\alpha}\}] \quad (21)$$

where

$$D = (q+p+s_8)^2(q+k+s_8)^2(q+s_8)^2$$

Let

$$N_\mu = \gamma_\alpha(\not p + \not q + \not s_8)\gamma_\beta[(2q+k+2s_8)_\mu g^{\alpha\beta} - (q+2k+2s_8)^\alpha g^\beta_\mu - (q-k+s_8)^\beta g^\alpha_\mu]$$

$$\Lambda^{(2)}_\mu = ig^2 t^b t^c f^{abc} \int \frac{d^4q}{(2\pi)^4}\frac{N_\mu}{D}$$

$$\frac{1}{D} = \frac{1}{(q+p+s_8)^2(q+k+s_8)^2(q+s_8)^2}$$

Combining the denominators using Feynman identity we get,

$$= 2\int_0^1 dx \int_0^{1-x} dy \frac{1}{[(q+p+s_8)^2 x + (q+k+s_8)^2 y + (q+s_8)^2(1-x-y)]^3}$$

$$= 2\int_0^1 dx \int_0^{1-x} dy \frac{1}{[(q+s_8+px+ky)^2 - X]^3}$$

$$X = -p^2 x(1-x) - k^2 y(1-y) + 2p.k\, xy$$

$$= p^2 x(x-1) + k^2 y(y-1) + 2p.k\, xy$$

Using the above results in (21), we get:

$$\Lambda^{(2)}_\mu = 2ig^2 t^b t^c f^{abc} \int \frac{d^4q}{(2\pi)^4}\int_0^1 dx \int_0^{1-x} dy \frac{1}{[q^2 - X]^3}$$

$$\times[(p+q-s_8-px-ky+s_8) - 2(2q-2s_8-2px - 2ky+k+2s_8) + (q-s_8-px-ky+2k+s_8)\gamma_\mu + 2(q-s_8-px-ky-k+s_8)_\mu + (q-s_8-px-ky+k+s_8)\gamma_\mu + 2(p+q-s_8-px-ky+s_8)_\mu \times(q-s_8-px-ky-k+s_8)]$$

Since the odd integrals of q vanishes, so after simplification the terms that contributes to the amplitudes are:

$$\Lambda^{(2)}_\mu = 2ig^2 t^b t^c f^{abc} \int \frac{d^4q}{(2\pi)^4}\int_0^1 dx \int_0^{1-x} dy \frac{1}{[q^2 - X]^3} \times[-4\not q q_\mu - 2\{\not p(1-x) - \not k y\}\{2p_\mu x - k_\mu(1-2y)\} + \not q \not q \gamma_\mu - \{\not p(1-x) - \not k y\}\{\not p x + \not k(y-2)\}\gamma_\mu + 2\not q q_\mu - 2\{\not p(1-x) - \not k y\}\{p_\mu x + k_\mu(1+y)\} - \not q \not q \gamma_\mu + \{\not p(1-x) - \not k y\}\{\not p x + \not k(1+y)\}\gamma_\mu - 2\not q q_\mu + 2\{p_\mu(1-x) - k_\mu y\}\{\not p x + \not k(1+y)\} \quad (22)$$

$$= 2ig^2 t^\alpha t^\beta f^{\mu\alpha\beta} \int \frac{d^4q}{(2\pi)^4}\int_0^1 dx \int_0^{1-x} dy [I_1 + I_2] \quad (23)$$

I_1 and I_2 are of the form

$$I_1 = \int \frac{d^4q}{(2\pi)^4}\frac{4\not q q_\mu}{[q^2 - X]^3} \quad I_2 = \int \frac{d^4q}{(2\pi)^4}\frac{N}{[q^2 - X]^3}$$

N are the factors without q.

Performing the q-integral we get the one-loop gluon vertex function in the form:

$$\Lambda^{(2)}_\mu = -2g^2 t^b t^c f^{abc} \int_0^1 dx \int_0^{1-x} dy \frac{1}{16\pi^2}\gamma_\mu \Gamma(0) - \frac{1}{32\pi^2}\frac{N}{X}$$

$$\Lambda^{(2)}_\mu = -\frac{1}{32\pi^2} g^2 C_A \gamma_\mu \Gamma(0) + \textit{finite terms}$$

$$= \frac{-1}{32\pi^2} g^2 C_A \gamma_\mu \Gamma(\frac{\varepsilon}{2}) + \textit{finite terms}$$

$$= \frac{-1}{16\pi^2} g^2 C_A \gamma_\mu \frac{1}{\varepsilon} + \textit{finite terms} \quad (24)$$

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Adding (20) and (24) we get the full contribution to one-loop quark-gluon vertex:

$$\Lambda_\mu = \Lambda_\mu^{(1)} + \Lambda_\mu^{(2)}$$
$$= \frac{1}{8\pi^2\varepsilon} g^2 (C_F - C_A/2)\gamma_\mu - \frac{1}{16\pi^2} g^2 C_A \gamma_\mu \frac{1}{\varepsilon} + \textit{finite terms}$$

$$= \frac{1}{8\pi^2\varepsilon} g^2 (C_F - C_A)\gamma_\mu + \textit{finite terms} \qquad (25)$$

4. RENORMALIZATION AND EVALUATION OF β–FUNCTION IN QCD WITH MODIFIED PRE-REGULARIZATION METHOD

In section-3 using modified pre-regularization method, we have evaluated all one-loop graphs arise for renormalization of QCD model. From our result let us write all field renormalization constants in a convenient form.

4.1. Renormalization constants

The quark field renormalization constant z_q can be found from equation (10) which is:

$$z_q = 1 + \frac{g^2}{8\pi^2\varepsilon} C_F$$

Putting $\alpha = 1 + \frac{g^2}{4\pi}$ (26)

We can write z_q as:

$$z_q = 1 + \frac{\alpha}{2\pi}\frac{1}{\varepsilon} C_F \qquad (27)$$

Similarly, the gluon field renormalization constant z_A can be obtained from equation (14) which is:

$$z_A = 1 - \frac{g^2}{24\pi^2\varepsilon}(4T_f n_f - 5C_A) \qquad (28)$$

$$z_A = 1 - \frac{\alpha}{2\pi}\frac{1}{\varepsilon}(\frac{4}{3}T_f n_f - \frac{5}{3}C_A) \qquad (29)$$

Where we have used relation (26)

The ghost field renormalization constant z_C can be obtained from equation (18), which is:

$$z_C = 1 + \frac{g^2}{16\pi^2}\frac{1}{\varepsilon} C_A$$

$$z_C = 1 + \frac{\alpha}{4\pi}\frac{1}{\varepsilon} C_A \qquad (30)$$

The quark-gluon vertex renormalization constant z_Γ can be obtained from equation (25), which is:

$$z_\Gamma = 1 - \frac{g^2}{8\pi^2}\frac{1}{\varepsilon}(C_F - C_A)$$
$$z_\Gamma = 1 - \frac{\alpha}{2\pi}\frac{1}{\varepsilon}(C_F - C_A) \qquad (31)$$

4.2. The β-function in QCD

The beta function of any model or theory is the most important results to evaluate. Because it assures the correctness of the theory and also the validity of the renormalization prescription used in evaluating the diagrams. The relation between bare g and renormalization coupling constant g_R is given by:

$$g = z_\alpha^{\frac{1}{2}} g_R \mu^{\varepsilon/2} \qquad (32)$$

Where

$$z_\alpha = (z_\Gamma z_q)^{-2} z_A^{-1} \qquad (33)$$

Using equations (27), (29) and (31) we can calculate z_α in (33).

$$z_\alpha = \left\{\left[1 - \frac{\alpha}{2\pi}\frac{1}{\varepsilon}(C_F - C_A)\right].\left[1 + \frac{\alpha}{2\pi}\frac{1}{\varepsilon}C_F\right]\right\}^{-2} \times \left[1 - \frac{\alpha}{2\pi}\frac{1}{\varepsilon}(\frac{4}{3}T_f n_f - \frac{5}{3}C_A\right]^{-1}$$

$$z_\alpha = \left[1 - \frac{\alpha}{2\pi}\frac{1}{\varepsilon}(\frac{11}{3}C_A - \frac{4}{3}T_f n_f) + - - - - - -\right] \qquad (34)$$

This gives us the coupling renormalization constant with one-loop accuracy. This is a gauge invariant result, which is one of the strong check of our regularization method.

Now, let us define renormalized α_R as:

$$\alpha_R = \frac{g_R^2}{4\pi} \qquad (35)$$

Then using (32), we can write:

$$\frac{g^2}{4\pi} = z_\alpha \alpha_R \mu^\varepsilon \qquad (36)$$

$$\alpha = z_\alpha \alpha_R \mu^\varepsilon \qquad (37)$$

This gives the relation between bare α and renormalized α_R. Taking ln on both sides of (37) we get

$$\ln\alpha = \ln z_\alpha + \ln\alpha_R + \varepsilon \ln\mu \qquad (38)$$

Let us define,

$$\ln z_\alpha = \sum_{n=1}^{\infty} \frac{F_u(\alpha)}{\varepsilon^n} \quad (39)$$

Then equation (38) becomes

$$\ln \alpha = \sum_{n=1}^{\infty} \frac{F_u(\alpha)}{\varepsilon^n} + \ln\alpha_R + \varepsilon \ln \mu \quad (40)$$

Again taking $\ln$ on both sides of (34), we get:

$$\ln z_\alpha = \ln\left[1 - \frac{\alpha}{2\pi}\frac{1}{\varepsilon}(\frac{11}{3}C_A - \frac{4}{3}T_f n_f) + - - - - - -\right]$$

$$= -\frac{\alpha}{2\pi}(\frac{11}{3}C_A - \frac{4}{3}T_f n_f)\frac{1}{\varepsilon} + O(\alpha^2) \quad (41)$$

Comparing (41) with (39) for $n = 1$, we get the coefficient of one-loop divergent term:

$$F_1(\alpha) = -\frac{\alpha}{2\pi}(\frac{11}{3}C_A - \frac{4}{3}T_f n_f)\frac{1}{\varepsilon} + O(\alpha^2) \quad (42)$$

Then the β -function in QCD is:

$$\beta(\alpha) = \alpha^2 F_1'(\alpha) = -\frac{\alpha^2}{2\pi}(\frac{11}{3}C_A - \frac{4}{3}T_f n_f) + O(\alpha^3) \quad (43)$$

In QCD the gauge group is $SU(3)$ and the quarks are in the fundamental representation. Hence if we consider $C_A = 3$ and $T_f = 1/2$ then equation (43) becomes:

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}(11 - \frac{2}{3}n_f) + O(\alpha^2) \quad (44)$$

Equation (44) shows that if $n_f \le 16$, then the β-function is negative. This implies that the gauge coupling in QCD becomes weaker at high energies and stronger at low energies. This behavior is called the asymptotic freedom. This is another strong check of our modified pre-regularization method.

5. CONCLUSIONS

The result in section-3 and 4 shows that modified pre-regularization is one of the best methods to apply in studying the problem in quantum field theory. Here we reproduced the same result with other regularization methods, such as dimensional regularization method for renormalization constants and β -function in QCD. The advantage of this method is that the calculation is straight forward and easy to handle. More rigorously we can say that the γ -algebra can be done more conveniently with this prescription. Because one can perform the algebra exactly in 4-dimensions or dimension that we are seeking to do. That means in this method we can study the problem in exact dimension and after calculation the divergent part is separated from the finite part. Then in the divergent part we can use some parameter to study other features of the problem. This sorts of clear cut separation and demonstration is absent is other regularization methods.

REFERENCES

[1] Gross D and Wilezek F. 1973a. Phys. Rev. Lett. 30, 1343; Phys. Rev. D8: 3633(1973b).

[2] Politzer H.D. 1973. Phys. Rev. Lett. 30, 1346; Phys. Rev. 14c: 129(1974).

[3] Weinberg S. 1973b. Phys. Rev. Lett. 31: 494.

[4] 't Hooft G and Veltman M. 1972. Nuel. Phys. B44: 189.

[5] Pauli P and Villars F. 1949. Rev. Mod. Phys. 21: 434.

[6] Lee B. W. and Zinn-Justin. 1973. Phys. Rev. D5, 3121, 3137 and 3155 (1972) and Phys. Rev. D7: 1049.

[7] Chowdhury M. A *et al*. 1985. Can. J. Phys. 63: 1453.

[8] Hafez G., Tarafder S. A. 2012. Chowdhury M. A, Can J. Sc. and Eng., Math. 3: 70.

[9] Cheng T. P and Li L. F. 1984. Gauge Theory of Elementary Particle Physics. Clarerdon Press, London, U.K.

[10] Grozin A. 2007. Lectures on QED and QCD. World Scientific.