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ON PARTLY CENSORED DATA WITH THE WEIBULL DISTRIBUTION

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ABSTRACT

Having been observed as one of the most useful distributions for modelling and analyzing lifetime data in medical, biological, engineering and others, the Weibull distribution has been studied enthusiastically in the literature to determine the best method in estimating its parameters. The objective of our study is to determine the reliable estimator among three methods for estimating the parameters of Weibull distribution and how bias the estimators' estimates of the parameters are to the true values. The methods being examined here are maximum likelihood estimator, least square estimators of Y on X and X on Y. The methods are compared using MSE and Bias base on simulation study. From the study it is observed that least square on X is more reliable for estimating the shape parameter with relatively small samples but with larger samples LSY will be the preferred method whiles maximum likelihood is reliable for estimating the scale parameter.

Keywords: partly censored data, maximum likelihood estimator, least square estimators, Weibull distribution, and simulation study.

INTRODUCTION

Weibull distribution is one of the most widely used life-time distributions in biological and reliability studies. It has shown to be satisfactory in modelling the phenomenon of fatigue and the life of many devices such as electric bulbs, capacitors and others, according to Zhang *et al*. (2008).

Many methods have been proposed for the estimation of Weibull distribution parameters among which are: Maximum Likelihood Estimation (MLE), Methods of Moment, Weighted Least Square Estimation (WLSE), and Least Square Estimation of Y on X and X on Y and others. According to Zhang *et al*. (2008), researchers prefer to use MLE in estimating the Weibull parameters because of its good statistical properties, while practitioners prefer that of LSE which seem to be more convenient.

The objective of this study is to determine a reliable method for estimating the parameters of the Weibull distribution. The methods being considered here are the Maximum Likelihood Estimation (MLE), which has been used by many researchers to estimate the Weibull parameters. Among them are: Guure and Ibrahim (2012b), Al Omari and Ibrahim (2011), Stefano *et al.* (2007), Scallan (1999), Flygare *et al*. (1985), Sirvanci (1984) and others, and the Least Square Estimation of X on Y (LSY) and Y on X (LSX) which have not been given much attention. Those who have considered these methods are, Zhang (2008, 2007). MLE is used to estimate the parameters by maximizing the likelihood function to make the values of the parameters consistent with the data. LS methods are used to linearize the Weibull cumulative distribution so that a straight line can be fitted on the Weibull Probability Paper (WPP) by least square regression technique. Guure *et al*. (2012a) studied Bayesian estimation of two-parameter Weibull distribution using extension of Jeffreys' prior information with three loss functions.

Different types of data can be applied in lifetime analysis such as right censored, left censored, time censored, failure censored, interval censored, multiply censored data and others.

In this study our focus is on partly censored data. This type of censoring occurs when units which are put to test begin at different times and the test is terminated before all units are said to have failed, and where it is observed that there is some intertwine among censoring times and that of failure times. What has been considered under partly censoring here are failure, right censoring and interval censoring observations.

Failure is said to have occurred when an item stops functioning before or at the end of the termination of the study and the failure is in connection to the purpose for which it is being tested or investigated. Right-Censoring occurs when an item has not failed by the last inspection. Interval-Censoring occurs when an item's failure time is only known to be in a range. Left and Right censoring are a special case of interval censoring.

The rest of the paper is arranged as follows: The second section is based on the derivation of maximum likelihood estimators for the parameters followed by least square regression procedure which is divided into three, with the first part obtaining the estimators for the parameters using Least Square on Y, next is Least Square on X and the last part deals with determination of Median Ranks with censored data. Simulation study is given followed by Results/Discussions and then Conclusions.

MAXIMUM LIKELIHOOD ESTIMATION

In this section, maximum likelihood techniques are used to develop estimators for the parameters of the Weibull distribution.

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Let the probability density function (pdf) and the cumulative distribution function (cdf) be represented by:

$$f(t,\alpha,\beta)=\frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}\exp\left\{-\left(\frac{t}{\alpha}\right)^{\beta}\right\} \tag{1}$$

and

$$F(t,\alpha,\beta)=1-\exp\left\{-\left(\frac{t}{\alpha}\right)^{\beta}\right\} \tag{2}$$

With data that involve failure, right censored and interval-censored, the likelihood function is given as:

$$L(t_i,u_i,v_i,\alpha,\beta)=\prod_{i=1}^{k}f(t_i)\prod_{j=k+1}^{r}(1-F(t_i))\prod_{j=r+1}^{n}\left(F(v_i,\alpha,\beta)-F(u_i,\alpha,\beta)\right) \tag{3}$$

This implies that,

$$L(t_i,u_j,v_j,\alpha,\beta)=\prod_{i=1}^{k}\left[\frac{\beta}{\alpha}\left(\frac{t_i}{\alpha}\right)^{\beta-1}\exp\left\{-\left(\frac{t_i}{\alpha}\right)^{\beta}\right\}\right]\times\prod_{j=k+1}^{r}\left[\exp\left\{-\left(\frac{t_k}{\alpha}\right)^{\beta}\right\}\right]\times$$

$$\prod_{j=r+1}^{r}\left[\exp\left\{-\left(\frac{u_j}{\alpha}\right)^{\beta}\right\}-\exp\left\{-\left(\frac{v_j}{\alpha}\right)^{\beta}\right\}\right] \tag{4}$$

Let $L(t_i,u_j,v_j,\alpha,\beta)=A$

then

$$A=\left(\left(\frac{\beta}{\alpha}\right)^{k}\prod_{i=1}^{k}\left[\left(\frac{t_k}{\alpha}\right)^{\beta-1}\right]\exp\left\{-\left(\frac{t_i}{\alpha}\right)^{\beta}\right\}\prod_{j=k+1}^{r}\exp\left\{-\left(\frac{t_k}{\alpha}\right)^{\beta}\right\}\times\right.$$

$$\left.\prod_{j=r+1}^{r}\left[\exp\left\{-\left(\frac{u_j}{\alpha}\right)^{\beta}\right\}-\exp\left\{-\left(\frac{v_j}{\alpha}\right)^{\beta}\right\}\right]\right) \tag{5}$$

Implying

$$A=\left(\left(\frac{\beta}{\alpha}\right)^{k}\prod_{i=1}^{k}\left[\left(\frac{t_k}{\alpha}\right)^{\beta-1}\right]\exp\left\{\prod_{j=k+1}^{r}\left\{-\left(\frac{t_i}{\alpha}\right)^{\beta}\right\}\prod_{j=k+1}^{r}\left[-\left(\frac{t_k}{\alpha}\right)^{\beta}\right]^{r-k}\right\}\times\right.$$

$$\left.\prod_{j=r+1}^{r}\left[\exp\left\{-\left(\frac{u_j}{\alpha}\right)^{\beta}\right\}-\exp\left\{-\left(\frac{v_j}{\alpha}\right)^{\beta}\right\}\right]\right) \tag{6}$$

The log-likelihood of (6) becomes:

$$\ln(A)=\left(k\left[\ln\beta-\beta\ln\alpha\right]+(\beta-1)\sum_{i=1}^{k}\ln(t_i)-\frac{1}{\alpha^{\beta}}\left[\sum_{i=1}^{k}(t_i)^{\beta}+(r-k)(t_k)^{\beta}\right]+\right.$$

$$\left.\sum_{j=r+1}^{r}\ln\left[\exp\left\{-\left(\frac{u_j}{\alpha}\right)^{\beta}\right\}-\exp\left\{-\left(\frac{v_j}{\alpha}\right)^{\beta}\right\}\right]\right) \tag{7}$$

Differentiating (7) with respect to α and β give the following:

$$\frac{\partial\ln(A)}{\partial\alpha}=\left(-\frac{k\beta}{\alpha}+\frac{\beta}{\alpha}\left[\sum_{i=1}^{k}\left(\frac{t_i}{\alpha}\right)^{\beta}+(r-k)\left(\frac{t_k}{\alpha}\right)^{\beta}\right]+\right.$$

$$\left.\sum_{i=r+1}^{n}\left[\frac{\left(\frac{u_j}{\alpha}\right)^{\beta}\left(\frac{u_j}{\alpha}\right)\exp\left\{-\left(\frac{u_j}{\alpha}\right)^{\beta}\right\}-\left(\frac{v_j}{\alpha}\right)^{\beta}\left(\frac{v_j}{\alpha}\right)\exp\left\{-\left(\frac{v_j}{\alpha}\right)^{\beta}\right\}}{\exp\left\{-\left(\frac{u_j}{\alpha}\right)^{\beta}\right\}-\exp\left\{-\left(\frac{v_j}{\alpha}\right)^{\beta}\right\}}\right]\right) \tag{8}$$

and

$$\frac{\partial\ln(A)}{\partial\beta}=\left(\frac{k}{\beta}-k\ln\alpha+\sum_{i=1}^{k}\ln(t_i)-\frac{1}{\alpha^{\beta}}\left[\sum_{i=1}^{k}(t_i)^{\beta}\ln\left(\frac{t_i}{\alpha}\right)+(r-k)(t_k)^{\beta}\ln\left(\frac{t_k}{\alpha}\right)\right]+\right.$$

$$\left.\sum_{i=r+1}^{n}\left[\frac{\left(\frac{u_j}{\alpha}\right)^{\beta}\ln\left(\frac{u_j}{\alpha}\right)\exp\left\{-\left(\frac{u_j}{\alpha}\right)^{\beta}\right\}-\left(\frac{v_j}{\alpha}\right)^{\beta}\ln\left(\frac{v_j}{\alpha}\right)\exp\left\{-\left(\frac{v_j}{\alpha}\right)^{\beta}\right\}}{\exp\left\{-\left(\frac{u_j}{\alpha}\right)^{\beta}\right\}-\exp\left\{-\left(\frac{v_j}{\alpha}\right)^{\beta}\right\}}\right]\right) \tag{9}$$

LEAST SQUARE REGRESSION PROCEDURES

Least square estimator on Y

According to Gibbons and Vance (1981), if available data are plotted on Weibull probability paper (WPP), and the order of observations which are $t_1 \le t_2 \le ... \le t_n$ are represented on the abscissa against some plotting rule which estimates the CDF $F(t_i)$, then the failure time that has been observed will be converted by the probability paper to $\ln(t_i)$ and the plotting rule to $\ln\left[-\ln\left(1-F(t_i)\right)\right]$. Since $\ln\left[-\ln\left(1-F(t_i)\right)\right]$ is random, then the sum of squares in the vertical direction can be minimized to obtain the estimates of the parameters by employing the following:

By taking the natural log twice to equation (2) we have;

$$\ln\left[-\ln\left(1-F(t_i)\right)\right]=\beta\ln(t_i)-\beta\ln(\alpha) \tag{10}$$

with $i=1,2,...,n$

To minimize $\hat{\alpha}$ and $\hat{\beta}$ we have

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$$\sum_{i=1}^{n}\left[\ln\left[-\ln\left(1-F(t_i)\right)\right]-\left(\hat{\beta}\ln(t_i)-\hat{\beta}\ln(\hat{\alpha})\right)\right]^2 \tag{11}$$

According to Zhang *et al* (2007),

$$\hat{\beta}_{xy}=\left\{\frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right\} \tag{12}$$

$$\hat{\alpha}_{xy}=\left\{\exp\left[-\left(\frac{\overline{y}}{\hat{\beta}_{xy}}-\overline{x}\right)\right]\right\} \tag{13}$$

From (12) and (13), the following equations are obtained:

$$\hat{\beta}_{xy}=\left\{\frac{n\sum_{i=1}^{n}\ln(t_i)\ln\left[-\ln\left(1-F(t_i)\right)\right]-\sum_{i=1}^{n}\ln(t_i)\sum_{i=1}^{n}\ln\left[-\ln\left(1-F(t_i)\right)\right]}{n\sum_{i=1}^{n}\left(\ln(t_i)\right)^2-\left[\sum_{i=1}^{n}\ln(t_i)\right]^2}\right\} \tag{14}$$

and

$$\hat{\alpha}_{xy}=\exp\left\{\frac{\sum_{i=1}^{n}}{n}-\frac{1}{n\hat{\beta}}\sum_{i=1}^{n}\ln\left[-\ln\left(1-F(t_i)\right)\right]\right\} \tag{15}$$

Least square estimator on X

Applying the same methodology as above but taking note of the fact that the horizontal deviations of the points to the line rather than the vertical are minimized.

$$\sum_{i=1}^{n}\left[\ln(t_i)-\left[\frac{1}{\beta}\ln\left(-\ln\left(1-F(t_i)\right)\right)+\ln(\alpha)\right]\right]^2 \tag{16}$$

$$\hat{\beta}_{yx}=\left\{\frac{\sum_{i=1}^{n}(y_i-\overline{y})^2}{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}\right\} \tag{17}$$

$$\hat{\alpha}_{yx}=\left\{-\frac{(\overline{x}-\overline{y})}{\hat{\beta}_{yx}}\right\} \tag{18}$$

Making use of (10) we obtain the following,

$$\hat{\beta}_{yx}=\left\{\frac{n\sum_{i=1}^{n}\ln\left[-\ln\left(1-F(t_i)\right)\right]^2-\left[\sum_{i=1}^{n}\ln\left[-\ln\left(1-F(t_i)\right)\right]\right]^2}{n\sum_{i=1}^{n}\ln(t_i)\ln\left[-\ln\left(1-F(t_i)\right)\right]-\sum_{i=1}^{n}\ln(t_i)\sum_{i=1}^{n}\ln\left[-\ln\left(1-F(t_i)\right)\right]}\right\} \tag{19}$$

$$\hat{\alpha}_{yx}=\left\{\frac{\left[\frac{1}{n}\sum_{i=1}^{n}\ln\left[-\ln\left(1-F(t_i)\right)\right]-\frac{1}{n}\sum_{i=1}^{n}\ln(t_i)\right]}{\hat{\beta}_{yx}}\right\} \tag{20}$$

These are applicable to both failure and censored data.

For censored data, $n = r$

Determination of median rank with censored data

Table A.

Item number (I)	State, F or C
1	F_1
2	C_1
3	F_2
4	C_2
5	F_3

where F is failure and C is censored

In reference to the above table, it can be observed that the first item is a failure, therefore, it can be assigned a Failure Order Number $i = 1$, but the actual Failure Order Number of the second failure is in limbo. It cannot be said to be $i = 2$ because $i = 2$ is likely to have failed before F_2 if it was not withdrawn from the test. One is likely to say that then F_2 is in the position of $i = 3$, again it cannot be given $i = 3$ due to the reason that C_1 might have ran more than $i = 3$, if it had stayed in the test. This makes it tricky to determine the actual position for F_2. It can only be said that the Failure Order Number for F_2 lies between $i = 2$ and $i = 3$, we therefore have to make use of the Mean Order Number (MON) to obtain the ranks.

According to Kececioglu (1993), Mean Order Number of a failure in suspended-items test may be obtained by calculating an increment, represented by I ,

where, $I = \frac{N+1-Q}{1+M}$

with N = sample size,

Q = previous Mean Order Number

M = number of items beyond present suspended set

MON is the most likely position of a failure in the sample under consideration.

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This procedure for determining the Median Rank is applicable to suspensions from the left, middle or right. When suspension is observed first then Q is zero (0).

SIMULATION STUDY

In this study, the sample size chosen was n = 25, 50, 75 and 100. The percentage of censoring for both right and interval were taken to be 20% and 10% respectively. The actual parameters values for α were 0.5, 1.0 and 1.5, and that of β were 0.8, 1.0 and 1.2. 1000 iterations (R) were ran on all the data with the actual values of the parameters and the estimates calculated. Mean Squared Errors and Biases were calculated and used for the comparison of the estimators.

$$MSE(\hat{\theta}) = \frac{\sum_{r=1}^{R}(\hat{\theta}^r - \theta)^2}{R-1} \text{ and } Bias(\hat{\theta}) = \frac{\sum_{r=1}^{R}(\hat{\theta}^r - \theta)}{R-1}$$

RESULTS AND DISCUSSIONS

From Tables 1-3, it is observed that the methods are somehow reliable in the estimation of the scale parameter but MLE is seen to be more reliable than any of the two other methods since from Tables 1-3 it gives the smallest values of biasness followed by LSX.

For the shape parameter, it is observed that LSX gives a very minimal bias among the others. MLE and LSX overestimate the shape parameter with an increase in sample size except at the point where the scale parameter is 1.0 with a decreasing and constant shape parameter and also with constant shape parameter at α =1.5 that LSX underestimates the shape parameter. LSY underestimates the shape parameter throughout even as the sample size increases with a decreasing, constant and increasing shape parameter except at α =1.5 with β =0.8 and 1.0 where it overestimates the shape parameter as indicated in Tables 1-3. It is also clear from Table-6 that as the sample size increases to n = 100 LSY became the preferred estimator for the shape parameter since it gives a very small MSE among the three methods.

In Tables 4-6 are the MSE values of the estimates from the true parameter values. It indicates that as the sample size increases all the estimators correspondingly give a decreasing MSE for the scale parameter and the shape parameter, which indicates how good the estimators are.

Table-1. Biasness of $\hat{\alpha}$ and $\hat{\beta}$ with n = 25.

		MLE		LSY		LSX	
α	β	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
0.5	0.8	7.19x10^{-3}	5.08x10^{-3}	1.20x10^{-2}	-3.41x10^{-3}	9.01x10^{-3}	-3.30x10^{-4}
	1.0	4.51x10^{-3}	7.96x10^{-3}	7.78x10^{-3}	-2.26x10^{-3}	5.87x10^{-3}	1.92x10^{-3}
	1.2	3.88x10^{-3}	8.59x10^{-3}	6.75x10^{-3}	-4.68x10^{-3}	4.79x10^{-3}	3.50x10^{-4}
1.0	0.8	1.32x10^{-2}	6.21x10^{-3}	2.20x10^{-2}	-1.78x10^{-3}	1.67x10^{-2}	1.43x10^{-3}
	1.0	1.04x10^{-2}	7.18x10^{-3}	1.77x10^{-2}	-4.15x10^{-3}	1.29x10^{-2}	6.00x10^{-5}
	1.2	5.49x10^{-3}	8.66x10^{-3}	1.07x10^{-2}	-3.91x10^{-3}	7.34x10^{-3}	6.10x10^{-4}
1.5	0.8	1.97x10^{-2}	6.22x10^{-3}	3.30x10^{-2}	-1.78x10^{-3}	2.49x10^{-2}	1.43x10^{-5}
	1.0	1.57x10^{-2}	7.18x10^{-3}	2.66x10^{-2}	-4.16x10^{-3}	1.93x10^{-2}	6.00x10^{-5}
	1.2	8.25x10^{-3}	8.66x10^{-3}	1.60x10^{-2}	-3.91x10^{-3}	1.10x10^{-2}	6.10x10^{-4}

Table-2. Biasness of $\hat{\alpha}$ and $\hat{\beta}$ with n = 50.

		MLE		LSY		LSX	
α	β	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
0.5	0.8	6.84x10^{-4}	2.95x10^{-4}	1.02x10^{-3}	-1.79x10^{-4}	8.07x10^{-4}	3.20x10^{-5}
	1.0	4.79x10^{-4}	3.60x10^{-4}	7.50x10^{-4}	-2.36x10^{-4}	5.80x10^{-4}	4.10x10^{-5}
	1.2	3.82x10^{-4}	5.45x10^{-4}	5.78x10^{-4}	-2.42x10^{-4}	4.59x10^{-4}	1.00x10^{-4}
1.0	0.8	1.37x10^{-3}	2.95x10^{-4}	2.05x10^{-3}	-1.79x10^{-4}	1.63x10^{-3}	3.10x10^{-5}
	1.0	1.03x10^{-3}	4.63x10^{-4}	1.55x10^{-3}	-2.17x10^{-4}	1.25x10^{-3}	7.90x10^{-5}
	1.2	7.64x10^{-4}	5.45x10^{-4}	1.16x10^{-3}	-2.42x10^{-4}	9.18x10^{-4}	1.00x10^{-4}
1.5	0.8	2.22x10^{-3}	3.69x10^{-4}	2.98x10^{-3}	-2.00x10^{-4}	2.31x10^{-3}	1.90x10^{-5}
	1.0	1.42x10^{-3}	4.60x10^{-4}	2.09x10^{-3}	-6.40x10^{-5}	1.65x10^{-3}	2.22x10^{-4}
	1.2	1.08x10^{-3}	5.06x10^{-4}	1.75x10^{-3}	-2.64x10^{-4}	1.44x10^{-3}	2.80x10^{-5}

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Table-3. Biasness of $\hat{\alpha}$ and $\hat{\beta}$ with n = 100.

α	β	MLE $\hat{\alpha}$	MLE $\hat{\beta}$	LSY $\hat{\alpha}$	LSY $\hat{\beta}$	LSX $\hat{\alpha}$	LSX $\hat{\beta}$
0.5	0.8	$6.34x10^{-5}$	$2.74x10^{-5}$	$8.89x10^{-5}$	$-6.70x10^{-6}$	$7.77x10^{-5}$	$6.50x10^{-6}$
	1.0	$5.02x10^{-5}$	$3.18x10^{-5}$	$6.93x10^{-5}$	$-1.12x10^{-5}$	$6.01x10^{-5}$	$1.05x10^{-5}$
	1.2	$3.38x10^{-5}$	$4.21x10^{-5}$	$4.62x10^{-5}$	$-8.20x10^{-6}$	$3.89x10^{-5}$	$1.19x10^{-5}$
1.0	0.8	$1.39x10^{-4}$	$2.67x10^{-5}$	$1.82x10^{-4}$	$-7.90x10^{-6}$	$1.58x10^{-4}$	$8.10x10^{-6}$
	1.0	$9.76x10^{-5}$	$3.69x10^{-5}$	$1.39x10^{-4}$	$-1.12x10^{-5}$	$1.19x10^{-4}$	$1.00x10^{-5}$
	1.2	$6.78x10^{-5}$	$3.93x10^{-5}$	$9.79x10^{-5}$	$-9.30x10^{-6}$	$8.48x10^{-5}$	$9.10x10^{-6}$
1.5	0.8	$2.25x10^{-4}$	$3.41x10^{-5}$	$2.97x10^{-4}$	$1.30x10^{-6}$	$2.58x10^{-4}$	$1.44x10^{-5}$
	1.0	$1.46x10^{-4}$	$4.03x10^{-5}$	$1.87x10^{-4}$	$1.90x10^{-6}$	$1.64x10^{-4}$	$1.92x10^{-5}$
	1.2	$1.12x10^{-4}$	$4.01x10^{-5}$	$1.57x10^{-4}$	$-1.00x10^{-6}$	$1.34x10^{-4}$	$1.84x10^{-5}$

Table-4. MSE of $\hat{\alpha}$ and $\hat{\beta}$ with n = 25.

α	β	MLE $\hat{\alpha}$	MLE $\hat{\beta}$	LSY $\hat{\alpha}$	LSY $\hat{\beta}$	LSX $\hat{\alpha}$	LSX $\hat{\beta}$
0.5	0.8	$5.17x10^{-4}$	$2.58x10^{-4}$	$1.45x10^{-3}$	$1.16x10^{-6}$	$8.12x10^{-4}$	$1.09x10^{-6}$
	1.0	$2.03x10^{-4}$	$6.34x10^{-4}$	$6.05x10^{-6}$	$5.11x10^{-5}$	$3.45x10^{-4}$	$3.69x10^{-5}$
	1.2	$1.51x10^{-4}$	$7.38x10^{-4}$	$4.56x10^{-6}$	$2.19x10^{-4}$	$1.29x10^{-4}$	$1.23x10^{-6}$
1.0	0.8	$1.73x10^{-3}$	$3.86x10^{-4}$	$4.85x10^{-5}$	$3.17x10^{-5}$	$2.78x10^{-3}$	$2.04x10^{-5}$
	1.0	$1.09x10^{-3}$	$5.16x10^{-4}$	$3.14x10^{-5}$	$1.72x10^{-4}$	$1.65x10^{-3}$	$3.60x10^{-6}$
	1.2	$3.01x10^{-4}$	$7.49x10^{-4}$	$1.14x10^{-5}$	$1.53x10^{-4}$	$5.39x10^{-4}$	$3.72x10^{-6}$
1.5	0.8	$3.89x10^{-3}$	$3.87x10^{-4}$	$1.09x10^{-4}$	$3.17x10^{-5}$	$6.25x10^{-3}$	$2.04x10^{-5}$
	1.0	$2.45x10^{-3}$	$5.16x10^{-4}$	$7.05x10^{-5}$	$1.73x10^{-4}$	$3.72x10^{-3}$	$3.62x10^{-6}$
	1.2	$6.81x10^{-4}$	$7.49x10^{-4}$	$2.56x10^{-5}$	$1.53x10^{-4}$	$1.21x10^{-3}$	$3.73x10^{-6}$

Table-5. MSE of $\hat{\alpha}$ and $\hat{\beta}$ with n = 50.

α	β	MLE $\hat{\alpha}$	MLE $\hat{\beta}$	LSY $\hat{\alpha}$	LSY $\hat{\beta}$	LSX $\hat{\alpha}$	LSX $\hat{\beta}$
0.5	0.8	$4.68x10^{-5}$	$8.70x10^{-6}$	$1.05x10^{-5}$	$3.20x10^{-6}$	$6.51x10^{-5}$	$1.02x10^{-7}$
	1.0	$2.29x10^{-5}$	$1.29x10^{-5}$	$5.63x10^{-6}$	$5.57x10^{-6}$	$3.36x10^{-5}$	$1.68x10^{-7}$
	1.2	$1.46x10^{-5}$	$2.97x10^{-5}$	$3.34x10^{-6}$	$5.86x10^{-6}$	$2.11x10^{-5}$	$1.00x10^{-6}$
1.0	0.8	$1.87x10^{-4}$	$8.70x10^{-6}$	$4.19x10^{-5}$	$3.20x10^{-6}$	$2.64x10^{-4}$	$9.61x10^{-7}$
	1.0	$1.05x10^{-4}$	$2.14x10^{-5}$	$2.41x10^{-5}$	$4.71x10^{-6}$	$1.56x10^{-4}$	$6.24x10^{-7}$
	1.2	$5.84x10^{-5}$	$2.97x10^{-5}$	$1.35x10^{-5}$	$5.86x10^{-6}$	$8.43x10^{-5}$	$1.00x10^{-6}$
1.5	0.8	$4.94x10^{-4}$	$1.36x10^{-5}$	$8.85x10^{-5}$	$4.00x10^{-6}$	$5.35x10^{-4}$	$3.61x10^{-7}$
	1.0	$2.00x10^{-4}$	$2.12x10^{-5}$	$4.39x10^{-5}$	$4.09x10^{-7}$	$2.72x10^{-4}$	$4.93x10^{-6}$
	1.2	$1.59x10^{-4}$	$2.56x10^{-5}$	$3.07x10^{-5}$	$6.97x10^{-6}$	$2.09x10^{-4}$	$7.84x10^{-7}$

Table-6. MSE of $\hat{\alpha}$ and $\hat{\beta}$ with n = 100.

α	β	MLE $\hat{\alpha}$	MLE $\hat{\beta}$	LSY $\hat{\alpha}$	LSY $\hat{\beta}$	LSX $\hat{\alpha}$	LSX $\hat{\beta}$
0.5	0.8	$4.02x10^{-6}$	$7.78x10^{-7}$	$7.90x10^{-7}$	$4.49x10^{-8}$	$6.04x10^{-6}$	$4.23x10^{-8}$
	1.0	$2.52x10^{-6}$	$1.01x10^{-6}$	$4.80x10^{-6}$	$9.80x10^{-8}$	$3.61x10^{-6}$	$1.10x10^{-7}$
	1.2	$1.14x10^{-6}$	$1.77x10^{-6}$	$2.13x10^{-6}$	$6.72x10^{-8}$	$1.51x10^{-6}$	$1.42x10^{-7}$
1.0	0.8	$1.95x10^{-5}$	$7.13x10^{-7}$	$3.29x10^{-5}$	$6.24x10^{-8}$	$2.49x10^{-5}$	$6.56x10^{-8}$
	1.0	$9.53x10^{-6}$	$1.36x10^{-6}$	$1.94x10^{-5}$	$1.24x10^{-7}$	$1.43x10^{-5}$	$1.00x10^{-7}$
	1.2	$4.59x10^{-6}$	$1.54x10^{-6}$	$9.58x10^{-6}$	$8.65x10^{-8}$	$7.19x10^{-6}$	$8.28x10^{-8}$
1.5	0.8	$5.06x10^{-5}$	$1.16x10^{-6}$	$8.84x10^{-5}$	$1.69x10^{-9}$	$6.67x10^{-5}$	$2.07x10^{-7}$
	1.0	$2.13x10^{-5}$	$1.62x10^{-6}$	$3.50x10^{-5}$	$3.61x10^{-9}$	$2.68x10^{-5}$	$3.69x10^{-7}$
	1.2	$1.26x10^{-5}$	$1.61x10^{-6}$	$2.45x10^{-5}$	$1.00x10^{-9}$	$1.78x10^{-5}$	$3.39x10^{-7}$

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CONCLUSIONS

From the results/discussions, it is clear that, LSX is more reliable in estimating the shape parameter when overestimation is preferred to underestimation but if vice versa and with large sample size then LSY will be more appropriate. MLE is more reliable for estimating the scale parameter than any of the other methods. All the estimators have their mean squared error decreasing as the sample size increases.

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