VOL. 7, NO. 11, NOVEMBER 2012 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

MICROPOLAR FLUID FLOW THROUGH A DUCT CONTAINING HIGHLY POROUS MEDIUM

V. Narasimha Charyulu[1] and M. Sunder Ram[2]
[1]Sumathi Reddy Institute of Technology, Warangal, A.P., India
[2]Department of Mathematics, Chaitanya Degree College (Autonomous), Kishanpura, Hanamkonda, Warangal, A.P., India
E-Mail: vn_charyulu@yahoo.com

ABSTRACT

The flow of the incompressible Micropolar fluid through a duct containing high porous medium is examined under the influence of magnetic field. The induced magnetic effect is taken into consideration in study of flow. The duct is bounded by two horizontal infinite and non conducting plates. The duct is subjected to the rotation with an angular velocity. The non-Darcian equation of the flow is applied to examine the flow in porous medium. The solution for velocity field, shear stress and induced magnetic field are obtained in closed form. The effect of permeability parameter magnetic parameter and angular velocity on the flow of the fluid is investigated.

Keywords: micropolar fluid, permeability, rotation, MHD flow.

1. INTRODUCTION

The study of flow of micropolar fluid has attracted the interest of many researchers in view of its application in a number of engineering problems such as oil exploration, chemical catalystic reactors, thermal insulation and geo thermal energy extraction. The lung alveolar is an example that finds application in the animal body. The classical Darcy's law [1, 2] states that the pressure gradient pushes the fluid against the body force exerted by the medium which can be expressed as $\bar{V} = \left(\frac{k}{\mu}\right)\nabla P$ with usual notation.

A generalized Darcy's law is proposed by Brinkman [3].

$$\rho \frac{d\bar{v}}{dt} = Div\, s_{ij} - \left(\frac{\mu}{k}\right)\vec{v}$$

Where

s_{ij} = stress tensor

ρ = density

$\bar{v}$ = velocity of fluid

k = permeability coefficient

Several investigators adapted the Brinkman generalized law for the flow through porous medium. Raptis [4], Narasimha Charyulu [5, 6] Yamamoto *et al.*, [7], Eringen [8] formulated the constitutive equations of micro polar fluid which contain particles under certain conditions have not only linear velocity but also intrinsic rotation velocity different from those of the medium element in which they are contained. The micropolar fluids include colloidal fluids, ferroliquids, polymers with suspensions etc.

The problem of micropolar fluid n the presence of transverse magnetic field finds application in nuclear engineering and other fields. The rotatory flow of the fluid has special applications in various engineering fields such as Mechanical, petroleum and chemical. In addition to the geophysical fluid dynamic which helps in explaning the phenomena give oceanic circulation? The flow through porous medium in the presence of transverse magnetic field under the assumption shows that the induced magnetic effect is negligible on the flow of the fluid [9-12].

In this paper the flow of incompressible micropolar fluid through a duct containing high porous medium is examined under the influence of magnetic field. The induced magnetic effect is taken into consideration in studying the flow. The solutions for velocity field shear stress and induced magnetic field are obtained in closed form. The effect of permeability parameter magnetic parameter and angular velocity is studied at length.

2. FORMULATION AND SOLUTION

The flow of in compressible micropolar fluid which is electrically conducting is considered through a porous medium contained between two infinite parallel plates, which are non permeable and non conducting in nature. The plates are kept at $z = \pm h$ horizontally and a uniform magnetic field H_o is applied parallel to Z axis. A constant pressure gradient G is applied in the direction of x axis. The whole system is subjected to rotation about z axis with angular velocityΩ. The flow is assumed to be steady. All the physical quantities other than pressure are independent of x, y and time variables.

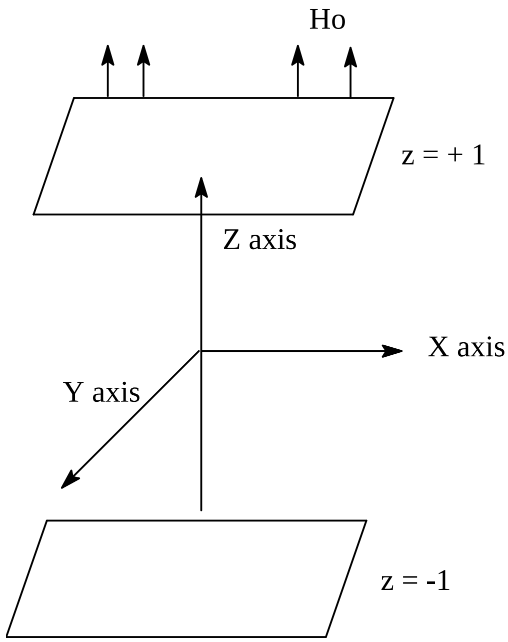

Figure-1. Non Darcian micropolar flow through porous duct.

The governing equations of motion of the fluid through porous medium in the presence of transverse magnetic field while the frame of reference is subjected to the rotation with angular velocity are given by Ludford [9], Brikman [3], Ramachandran *et al*. [12].

Equation of continuity is satisfied by choice of velocity field and magnetic field.

$$\overline{V} = [u(Z), v(Z), 0],\ \overline{H} = [H_x(Z), H_y(Z), 0] \quad (1)$$

$$-2\Omega v = -G + \left(\mu_e \frac{H_o}{\rho}\right)\frac{dH_x}{dz} + (\nu + \nu_c)\frac{d^2u}{dz^2} - \frac{\nu}{k}u + 2\nu_c \frac{dN}{dz} \quad (2)$$

$$-2\Omega u = \left(\mu_e \frac{H_o}{\rho}\right)\frac{dH_y}{dz} + (\nu + \nu_c)\frac{d^2v}{dz^2} - \frac{\nu}{k}.v + 2\nu_c \frac{dN}{dz} \quad (3)$$

$$\frac{-d^2H_x}{dz^2} = \sigma\mu_e H_o \frac{du}{dz} \quad (4)$$

$$\frac{-d^2H_y}{dz^2} = \sigma\mu_e H_o \frac{dv}{dz} \quad (5)$$

With the boundary conditions

$$u = v = 0;\ H_x = H_y = 0 \quad \text{at } z = \pm h \quad (6)$$

To make the system of equations non dimensionalized we use the following non dimensional quantities.

$$\left.\begin{aligned} & z^* = \frac{z}{h} \quad u^* = \frac{uh}{\nu},\ v^* = \frac{vh}{\nu} \\ & H_x^* = \frac{H_x}{\rho\,\mu_e \nu H_0},\ H_y^* = \frac{H_y}{\sigma\,\mu_e \nu H_0} \\ & G^* = \frac{Gh^3}{\nu} \quad \overline{\Omega} = \frac{\overline{\Omega}h^3}{\nu},\ \beta = \frac{h}{\sqrt{k}} \end{aligned}\right\} \quad (7)$$

Applying these non dimensional quantities to the equations (2) to (5) and adding equations (2) with (3) and (4) with (5), the following equations are obtained after removing *

$$\frac{d^2U}{dz^2} + \frac{M^2}{(1+s)}\frac{dH}{dz} - \left(\frac{2\Omega i}{1+s} + \frac{\beta^2}{1+s}\right)U = \frac{G}{1+s} \quad (8)$$

$$\frac{d^2H}{dz^2} + \frac{dU}{dz} = 0 \quad (9)$$

Where

$$M = \frac{\mu H_0 h.\sqrt{\sigma}}{\rho\nu},\ \beta = \frac{h}{\sqrt{k}},\ U = u + iv,\ H = H_x + iH_y$$

By writing $M_1 = \dfrac{\mu^2}{1+s}$

$$\Omega_1 = \frac{\Omega}{1+s}$$

$$\beta_1^2 = \frac{\beta^2}{1+s}$$

$$G_1 = \frac{G}{1+s}$$

The equation (8) reduces to

$$\frac{d^2U}{dz^2} + m_1^2\frac{dH}{dz} - \left(2\Omega_1 i + \beta_1^2\right)U = G_1 \quad (10)$$

$$\frac{d^2H}{dz^2} + \frac{dU}{dz} = 0 \quad (11)$$

Solving (10) and (11), we get

$$U(z)=\frac{R[\cosh\alpha z-\cosh\alpha]}{\cosh\alpha} \tag{12}$$

$$H(z)=\frac{R[z\sinh\alpha-\sinh\alpha z]}{\alpha.\cosh\alpha} \tag{13}$$

where $\alpha = p+iq$

$$\left.\begin{aligned}
R &= \frac{\alpha.G_1}{\alpha^3+M_1^2(Tan\ h\alpha-\alpha)}\\
P &= r\left[1+\left[\left(\frac{\Omega_1}{r}\right)^4+1\right]^{\frac{1}{2}}\right]^{\frac{1}{2}}\\
q &= r\left[\left\{\left(\frac{\Omega_1}{r}\right)^4+1\right\}^{\frac{1}{2}}-1\right]^{\frac{1}{2}}\\
r^2 &= \frac{\left(M_1^2+\beta_1^2\right)}{2}
\end{aligned}\right\} \tag{14}$$

The velocity component in x, y directions are obtained from (12) as:

$$u(z)=c_1[\cosh p(1+z)\cos q(1-z)+\cosh p(1-z)\cos q(1+z)]\times \left[(\cosh 2p+\cos 2q)^{-1}-1\right] \tag{15}$$

$$v(z)=c_2[\sinh p(1+z)\sin q(1-z)+\sinh p(1-z)\sin q(1+z)]\times \left[(\cosh 2p+\cos 2q)^{-1}\right] \tag{16}$$

with

$$\left.\begin{aligned}
c_1 &= \frac{G_1(PR_1+qR_2)}{R_1^2+R_2^2}\\
c_2 &= \frac{G_1(qR_1+pR_2)}{R_1^2+R_2^2}\\
R_1 &= \frac{p^3-3pq^2=pM_1^2+M_1^2\sinh 2p}{\cosh 2p+\cos 2q}\\
R_2 &= \frac{3p^2q-q^3-qM^2+M_1^2\sinh 2q}{\cosh 2p+\cos 2q}
\end{aligned}\right\} \tag{17}$$

The induced magnetic field components are given by equations (13)

$$\left.\begin{aligned}
H_x &= R_1z-R_4[\sinh p(1+z)\cos q(1-z)-\sinh p(1-z)\cos q(1+z)]+\\
&\quad R_5[\cosh p(1+z)\sin q(1-z)-\cosh p(1-z)\sin q(1+z)]\\
H_y &= R_6z-R_4[\cosh p(1+z)\sin q(1-z)-\cosh p(1-z)\sin q(1+z)]+\\
&\quad R_5[\sinh p(1+z)\cos q(1-z)-\sinh p(1-z)\cos q(1+z)]
\end{aligned}\right\} \tag{18}$$

where

$$\left.\begin{aligned}
R_3 &= c_1c_2-c_2c_4\\
R_4 &= c_1c_5+c_2c_6\\
R_5 &= c_1c_6-c_2c_5\\
R_6 &= c_2c_3+c_1c_4
\end{aligned}\right\} \tag{19}$$

with

$$\left.\begin{aligned}
c_3 &= c_5\sinh 2p+c_6\sin 2q\\
c_4 &= c_5\sinh 2q-c_6\sinh 2q\\
c_5 &= \frac{(pc_1+qc_2)}{(p^2+q^2)(\cosh 2p+\cos 2q)}\\
c_6 &= \frac{(pc_1-qc_2)}{(p^2+q^2)(\cosh 2p+\cos 2q)}
\end{aligned}\right\} \tag{20}$$

Shear stress acting on the plates at z = $\pm$ 1 due to the velocity component u, v in x, y directions are

$$T_x=-\frac{du}{dz} \quad \text{at} \quad z=\pm 1$$

$$T_x=\frac{\pm[(c_1p-c_2q)\sinh 2p-(c_1q+c_2p)\sin 2q]}{(\cosh 2p+\cos 2q)} \tag{21}$$

$$T_y=\frac{dv}{dz} \quad \text{at} \quad z=\pm 1$$

$$=\frac{\pm[(c_1p-c_2q)\sinh 2p+(c_2p-c_1q)\sin 2q]}{(\cosh 2p+\cos 2q)} \tag{22}$$

3. SPECIAL CASES

Case-1: The Newtonian fluid flow with in duct is obtained by substituting s = 0 in the solution [13].

$$U(z)=\frac{R[\cosh\alpha z-\cosh\alpha]}{\cosh\alpha} \tag{23}$$

$$H(z)=\frac{R[z\sinh\alpha-\sinh\alpha z]}{\alpha\cosh\alpha} \tag{24}$$

$$\left.\begin{aligned}
\alpha &= p+iq\\
R &= \frac{\alpha.G}{\alpha^3+M_1^2(Tan\,h\alpha-\alpha)}\\
P &= r\left[1+\left[\left(\frac{\Omega_1}{r}\right)^4+1\right]^{\frac{1}{2}}\right]^{\frac{1}{2}}\\
q &= r\left[\left\{\left(\frac{\Omega_1}{r}\right)^4+1\right\}^{\frac{1}{2}}-1\right]^{\frac{1}{2}}\\
r^2 &= \frac{\left(M^2+\beta_1^2\right)}{2}
\end{aligned}\right\} \tag{25}$$

VOL. 7, NO. 11, NOVEMBER 2012 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

Case-2: The flow of micro polar fluids through clear medium is given by taking $k \to \infty$ i.e. $\beta = 0$

$$U(z) = \frac{R[\cosh \alpha z - \cosh \alpha]}{\cosh \alpha} \tag{26}$$

$$H(z) = \frac{R[z \sinh \alpha - \sinh \alpha z]}{\alpha \cosh \alpha} \tag{27}$$

where

$$\left.\begin{aligned} &\alpha = p + iq \\ &R = \frac{\alpha . G_1}{\alpha^3 + M_1^2 (Tan\,h\alpha - \alpha)} \\ &P = r\left[1 + \left[\left(\frac{\Omega_1}{r}\right)^4 + 1\right]^{\frac{1}{2}}\right]^{\frac{1}{2}} \\ &q = r\left[\left\{\left(\frac{\Omega_1}{r}\right)^4 + 1\right\}^{\frac{1}{2}} - 1\right]^{\frac{1}{2}} \\ &r^2 = \frac{M_1^2}{2} \end{aligned}\right\} \tag{28}$$

Case 3: Steady flow of micro polar fluid with out any rotation of the system is given by substituting $\Omega_1 = 0$

$$U(z) = \frac{R[\cosh \alpha z - \cosh \alpha]}{\cosh \alpha} \tag{29}$$

$$H(z) = \frac{R[z \sinh \alpha - \sinh \alpha z]}{\alpha \cosh \alpha} \tag{30}$$

where

$$\left.\begin{aligned} &\alpha = p + iq \\ &R = \frac{\alpha . G_1}{\alpha^3 + M_1^2 (Tan\,h\alpha - \alpha)} \\ &p = r.\sqrt{2} \\ &q = 0 \\ &r^2 = \left(\frac{M_1^2 + \beta_1^2}{2}\right) \end{aligned}\right\} \tag{31}$$

4. CONCLUSIONS

The velocity profile is drawn for different values of Ω fixing the values $M_1 = 0$, $\beta_1 = 0.5$, $S = 3$, $G_1 = 1$. The profile is parabolic as rotation is increasing the profile parabolo is decreasing. The effect of rotation is seen to be decreasing the velocity profile.

As rotation is constant β = 0 and magnetic parameter M_1 is increasing the velocity profiles are decreasing as seen in Figure-3.

From Figure-4 as the porous medium permeability β_1 is increasing i.e., the permeability k is decreasing; the velocity of the fluid is increasing.

As the Newtonian parameter S is increasing, the velocity of the fluid is decreasing as seen in Figure-5.

Pictorial representation of u (t), v (t) is shown in Figure-6 to Figure-9 for different values of Ω_1 and M_1 which self explanatory.

Figure-10 to Figure-13 represents the induced magnetic field for different values of Ω_1, M_1, β_1, S.

The line representing magnetic field is straight line passing through origin for Ω = 0.33 for Ω = 0.6 and 1.1 the lines are close to Z axis (Figure-10). For different values of M_1 the magnetic field is shown in Figure-11. For different values of permeability of the porous medium the magnetic field is represented as straight lines showing more oblique nature as β is increasing (Figure-12).

The induced magnetic field is represented for different values of the non Newtonian parameters S (Figure-13). As S increases, the deviation from Z axis becoming more.

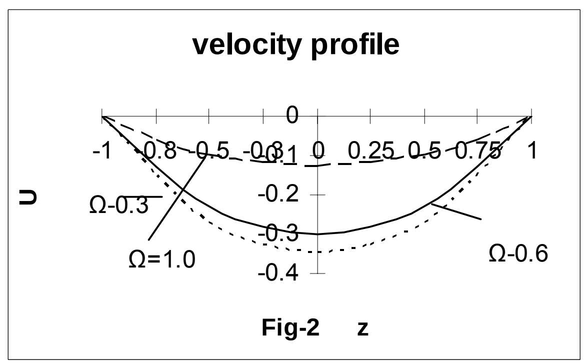

Fig-2

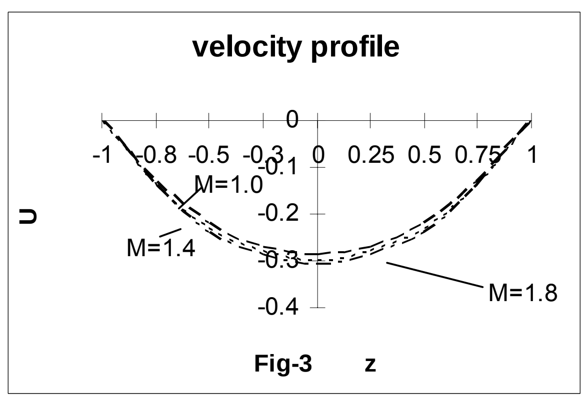

Fig-3

ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

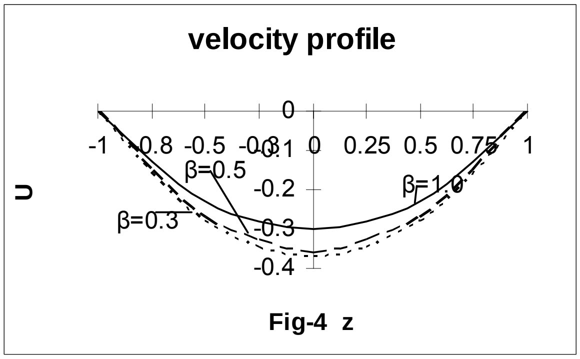

Fig-4

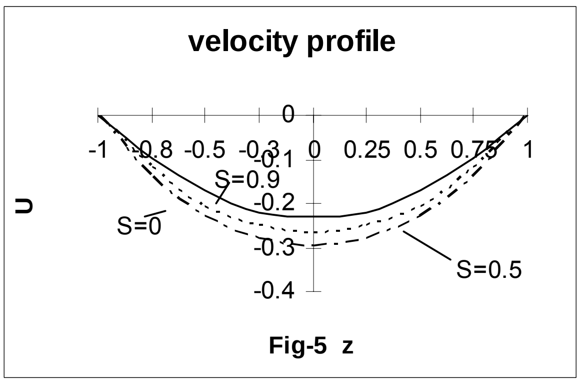

Fig-5

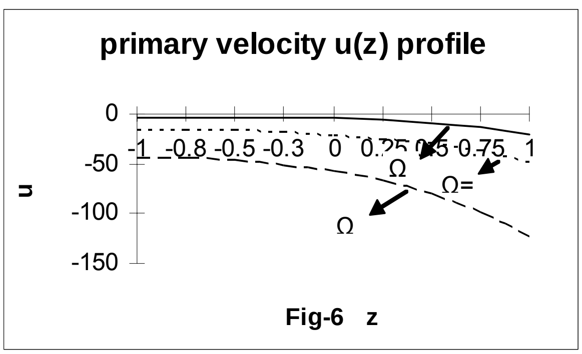

Fig-6

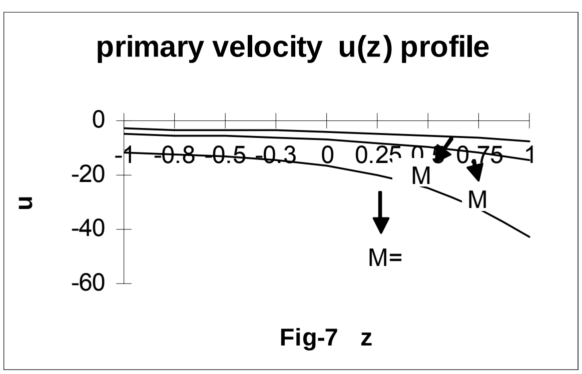

Fig-7

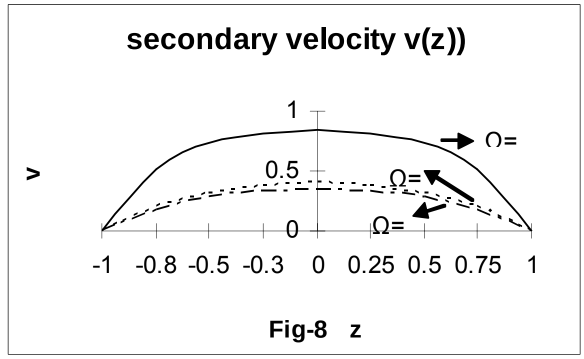

Fig-8

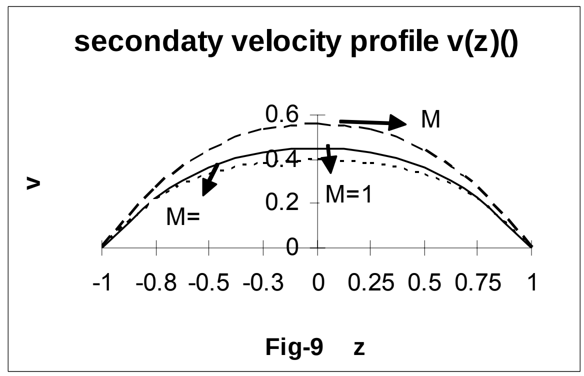

Fig-9

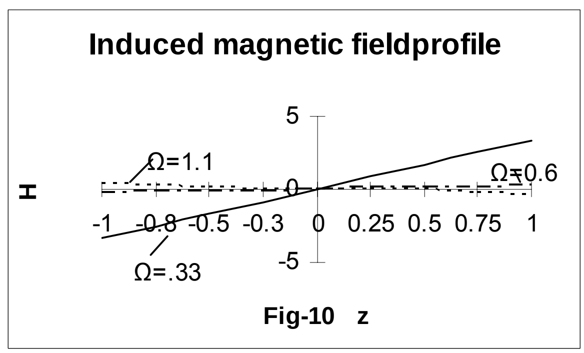

Fig-10

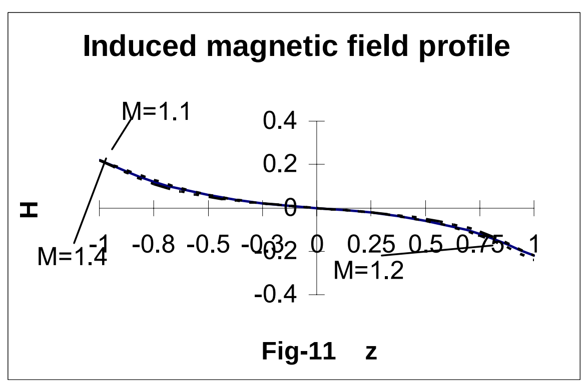

Fig-11

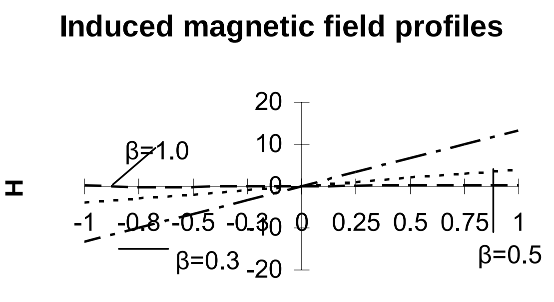

Fig-12

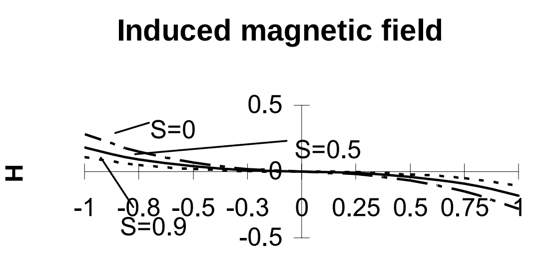

Fig-13

REFERENCES

[1] M. Musakat. 1937. Flow of Homogeneous fluid through porous medium McGraw Hill Inc. New York.

[2] M. Musakat. 1949. Physical principles of oil production. McGraw Hill Inc. New York.

[3] Brinkman J. of Applied Science Research. 27(A_1) 27.

[4] A. Raptis. 1986. Int. J. Eng. Research. 10: 97.

[5] Narasimha Charyulu. 1997. Indian J. of Mathematics. 39: 267.

[6] Narasimha Charyulu. 1997. International e Journal of Engineering Mathematics Theory and Applications.

[7] K. Yamamoto. J. of Engineering Mathematics. 10: 1.

[8] A.C. Eringen. 1966. J. Maths Mech. 16.

[9] G.S. Ludford. 1959. Arch. Rat. Mech. Anal. 3: 14.

[10] H.P. Greenspan. 1969. Theory of rotating fluids. Cambridge University Press.

[11] R. Herbert. 1975. ZAAM. 55: 443.

[12] P.S. Ramachandran, M.N. Mathur and S.K. Ojha. 1979. Int. J. Eng. Sci. 17: 625.

[13] V. Narasimha Charyulu and P. Venkat Ranam. 2003. Far East Appl. Math. 10(3): 221.