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EXCITATION CONTROLLER DESIGN OF A SYNCHRONOUS MACHINE USING DISCRETE OPTIMAL MULTIRATE TECHNIQUES

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ABSTRACT

An optimal control strategy based on Two-Point-Multirate Controllers (TPMRCs) is used to design a desirable excitation controller of a hydro generator system, in order to enhance its dynamic stability characteristics. In the TPMRCs based scheme, the control is constrained to a certain piecewise constant signal, while each of the controlled plant outputs is detected many times over a fundamental sampling period T_0. On the basis of this strategy, the original problem is reduced to an associate discrete-time linear quadratic (LQ) regulation problem for the performance index with cross product terms, for which a fictitious static state feedback controller is needed to be computed. Simulation results for the actual 160 MVA synchronous generator with conventional exciter supplying line to an infinite grid show the effectiveness of the proposed method which has a quite satisfactory performance.

Keywords: multirate controllers, discrete system representation, power systems, turbo generators.

INTRODUCTION

The typical control problem has always been to start with a suitable linear (or linearized) open-loop mathematical model of a physical plant (in continuous or discrete form) and attempt to design a proper controller for it, i.e., to obtain an associated closed-loop system with enhanced dynamic stability characteristics. [1, 2, 3, 4, 5-7]. The digital controller applied for the discrete linear systems may be obtained by using new TPMRCs [8-11].

It is pointed out that the used TPMRCs technique reduced the original LQ regulation problem to an associated discrete-time LQ regulation problem for the performance index with crossed product terms, for which is computer a fictitious static state feedback controller [12-17]. In addition thus technique offers more flexibility in choosing the sampling rates and provides a power design computed method.

In the present work the discrete linear open-loop system model under consideration systematically derived from the associated continuous 8th order MIMO linearized open-loop model of a practical power system, hawing on 160 MVA synchronous generator supplying power to an infinite grid through a step-up transformer and a transmission line [18]. The sought digital controller for the enhancement of the dynamic characteristics of the above 6th order discrete model is accomplished by the proper application of the new TPMRCs to it.

OVERVIEW OF RELEVANT MATHEMATICAL CONSIDERATIONS

The general description of the controllable and observable continuous, linear, time-invariant, multivariable MIMO dynamical open-loop system expressed in state-space form is:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \qquad (1)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$$

where

$\mathbf{x}(t) \in \mathbf{R}^n$, $\mathbf{u}(t) \in \mathbf{R}^m$, $\mathbf{y}(t) \in \mathbf{R}^p$ are state, input and output vectors respectively; and A, B and C are real constant system matrices with proper dimensions.

The associated general discrete description of the system of equation (1) is as follows:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \qquad (2)$$
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)$$

where

$\mathbf{x}(k) \in \mathbf{R}^n$, $\mathbf{u}(k) \in \mathbf{R}^m$, $\mathbf{y}(k) \in \mathbf{R}^p$ are state, input and output vectors respectively; and A, B and C are real constant system matrices with proper dimensions.

OVERVIEW OF NEW OMCM FOR LINEAR DISCRETE SYSTEMS

This method with H_o and H_N being zero-order holds and with holding times T_o and T_N, respectively (see Figure-1) is presented here in a concise manner, whereas the details are found [8].

Starting with the general linear state space system description in continuous form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \qquad (3)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$$

where

$\mathbf{x}(t) \in R_n$, $\mathbf{u}(t) \in R^m$ and $\mathbf{y}(t) \in R^p$ are the state, input and output vectors, respectively.

The associated discrete system description is obtained by letting n_i, $i \in J_p = \{1, 2, ..., p\}$, be used of

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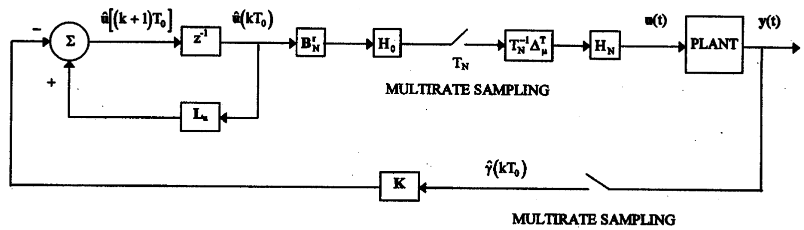

Figure-1. Simplified representation of power system under investigation in discrete form.

the observability indices of the pair **(A,C),** and $T_o \in R^+$ be a sampling period. Also, by letting

$$\Phi = \exp(\mathbf{A}T_0) \tag{4}$$

and $\mathbf{B}_N \in R^{nxp_N}$ be the full rank matrix defined by

$$\mathbf{B}_N\mathbf{B}_N^T = \mathbf{W}_N(T_0,0) \geq 0$$

with the generalized reachability Grammian of ord N in the interval $[0, T_0]$ being

$$\mathbf{W}_N(T_0,0) = T^{*-1}\sum_{\mu=0}^{N_0-1}\Delta_\mu\Delta_\mu^T,\ p_N = rank\mathbf{W}_N(T_0,0)$$

and

$$T^* = T_0 / N_0,\ \Delta_\mu = \hat{A}_N^{N_0-\mu-1}\hat{B}_{T^*}$$

$$\hat{\mathbf{A}}_N = \exp(\mathbf{A}T^*),\ \hat{\mathbf{B}}_{T^*} = \int_0^{T^8}\exp(\mathbf{A}\lambda)\mathbf{B}d\lambda \tag{5}$$

Next follows the application of the OMCM technique to the above descriptions. The input of the plant are constrauned to the following piecewise constant control.

$$\mathbf{u}(kT_0 + \mu T^* + \varsigma) = T^{*-1}\Delta_\mu^T\mathbf{B}_N^\tau\hat{u}(kT_0),\quad \hat{\mathbf{u}}(kT_0) \in R^{p_N} \tag{6}$$

for

$$t = kT_0 + \mu T^*,\ \mu = 0,...,N_0 - 1,\ k >> 0$$

and $J \in [0, T^*)$, where $B_N^\tau = B_N(B_N^TB_N)^{-1}$.

The ith plant output $\mathbf{y_i(t)}$ is detected at every $T_i = T_0 / M_i$, such that

$$y_i(kT_0 + \rho T_i) = c_i^T x(kT_0 + \rho T_i),\quad \rho = 0,1,...,M_i - 1 \tag{7}$$

where $M_i \in Z^+, i \in J_p$ are the output multiplicities of the sampling. In general $M_i \neq N$. the sampled values of the plant outputs obtained over $[kT_0, (k+1)T_0)$ are stored in the M^*-dimentional column vector $\hat{\gamma}(kT_0)$ of the form:

$$\hat{\gamma}(kT_0) = [y_1(kT_0)...y_1(kT_0 + (M_1 - 1)T_1)...\ y_p(kT_0)...y_p(kT_0 + (M_p - 1)T_p)]^T$$

where

$M^* = \sum_{i=1}^{p} M_i$. The vector $\hat{\gamma}(kT_0)$ is used in the control law of the form

$$\hat{\mathbf{u}}[(k+1)T_0] = \mathbf{L}_u\hat{\mathbf{u}}(kT_0) - \mathbf{K}\hat{\gamma}(kT_0) \tag{8}$$

where $\mathbf{L}_u \in \mathbf{R}^{p_N x p_N}$, $\mathbf{K} \in \mathbf{R}^{p_N x M^*}$.

Finally one searches a controller in the form of (5) and (7) which, when applied to system (1), minimizes the following performance index

$$J = \frac{1}{2}\int_0^\infty[\mathbf{y}^T(t)\mathbf{Q}\mathbf{y}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)]dt \tag{9}$$

where $\mathbf{Q} \in R^{pxp}$ *and* $\mathbf{R} \in R^{mxm}$ are symmetric matrices with $\mathbf{Q} \geq 0$, $\mathbf{R} > 0$ while $(\mathbf{AC^TQC})$ is an observable pair.

The above problem is equivalent to the problem of designing a control law of the form of equation (9), in order to minimize the following index:

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$$J = \frac{1}{2}\sum_{k=0}^{\infty}\left[\mathbf{x}^{T}(kT_0) \quad \hat{\mathbf{u}}^{T}(kT_0)\right]\begin{bmatrix}\tilde{\mathbf{Q}}_N & \tilde{\mathbf{G}}_N \\ \tilde{\mathbf{G}}_N^{T} & \tilde{\Gamma}_N\end{bmatrix}\begin{bmatrix}\mathbf{x}(kT_0) \\ \hat{\mathbf{u}}(kT_0)\end{bmatrix}$$

for the system

$$\mathbf{x}\left[(k+1)T_0\right] = \Phi\,\mathbf{x}(kT_0) + B_N\,\hat{u}(kT_0)$$

Where $\tilde{\mathbf{Q}}_N$, $\tilde{\mathbf{G}}_N$, $\tilde{\Gamma}_N$ are giver explicitly (Al - Rahmani and Franklin, 1990).

Theorem: The following basic formula of the multirate sampling mechanism holds

$$\mathbf{Hx}\left[(k+1)T_0\right] = \hat{\gamma}(kT_0) - \mathbf{D}\hat{\mathbf{u}}(kT_0)\ ,\ k \geq 0$$

where, matrices $\mathbf{x}(kT_0 + \rho T_i) = \hat{\mathbf{A}}_i^{\rho - M_i}\mathbf{x}\left[(k+1)T_0\right] + \hat{\mathbf{B}}_{i,\rho}\hat{\mathbf{u}}(kT_0)$ are defined as follows:

$$\mathbf{H} = \begin{bmatrix} \mathbf{c}_1^{T}\left(\hat{\mathbf{A}}_1^{M_1}\right)^{-1} \\ \vdots \\ \mathbf{c}_1^{T}\hat{\mathbf{A}}_1^{-1} \\ \vdots \\ \mathbf{c}_p^{T}\left(\hat{\mathbf{A}}_p^{M_p}\right)^{-1} \\ \vdots \\ \mathbf{c}_p^{T}\hat{\mathbf{A}}_p^{-1} \end{bmatrix},\quad \mathbf{D} = \begin{bmatrix} \mathbf{c}_1^{T}\hat{\mathbf{B}}_{1,0} \\ \vdots \\ \mathbf{c}_1^{T}\hat{\mathbf{B}}_{1,M_1-1} \\ \vdots \\ \mathbf{c}_p^{T}\hat{\mathbf{B}}_{p,0} \\ \vdots \\ \mathbf{c}_p^{T}\hat{\mathbf{B}}_{p,M_p-1} \end{bmatrix} \qquad (10)$$

and where,

$$y_i(kT_0 + \rho T_i) = \mathbf{c}_i^{T}\hat{\mathbf{A}}_i^{\rho - M_i}\mathbf{x}\left[(k+1)T_0\right] + \mathbf{c}_i^{T}\hat{\mathbf{B}}_{i,\rho}\hat{\mathbf{u}}(kT_0)$$

The ultimate expressions for the control law optimal gain matrices L_u and K are as follows:

$$\mathbf{L}_u = (\tilde{\mathbf{R}}_N + \mathbf{B}_N^{T}\mathbf{P}\mathbf{B}_N)^{-1}(\tilde{\mathbf{G}}_N + \mathbf{B}_N^{T}\mathbf{P}\mathbf{\Phi})\mathbf{H}^{-1}\mathbf{D} \qquad (11)$$

$$\mathbf{K} = (\tilde{\mathbf{R}}_N + \mathbf{B}_N^{T}\mathbf{P}\mathbf{B}_N)^{-1}(\tilde{\mathbf{G}}_N + \mathbf{B}_N^{T}\mathbf{P}\mathbf{\Phi})\mathbf{H}^{-1} \qquad (12)$$

where $\tilde{\mathbf{R}}_N, \tilde{\mathbf{G}}_N$ *and* $\mathbf{H}$ are defined in [2, 3]. The resulting discrete closed-loop system matrix $\left(\mathbf{A}_{cl/d}\right)$ takes the following:

$$\mathbf{A}_{cl/d} = \mathbf{A}_{ol/d} - \mathbf{B}_N\mathbf{K}\mathbf{H} \qquad (13)$$

where cl = closed-loop, ol = open-loop and d = discrete.

TPMRCs DESIGN AND SIMULATIONS OF RESULTING DISCRETE CLOSED-LOOP POWER SYSTEM MODEL

The power system under study is taken from [18] and is shown here in Figure-2. It consists of a 160 MVA synchronous generator with conventional exciter supplying power through a transformer and a transmission line to an infinite grid. System parameters are given in Table 1.

Table-1. Numerical values of the system parameters and the operating point (p.u. values on generator ratings).

Turbogenerator
160 MVA, 2-pole, pf = 0.894, x_q = 1.7, x_q = 1.6, x'_d = 0.245p.u. τ'_{d0} = 5.9, H = 5.5s; ω_R = 377rad/s D=2, 0p.u.
External system
R_e = 0.02, X_e = 0.40p.u. (on a 160 MVA base).
Operating point
P_0 = 1.0, Q_0 = 0.5, E_{FDo} = 2.5128, E_{qo} = 0.9986, V_{t0} = 1.0, T_{mo}= 1.0p.u; δ_0 = 1,1966rad; K_1 = 1, 1330, K_2 = 1.3295, K_3 = 0.3072, K_4 = 1.8235, K_5 = -0, 0433, K_6 = 0.4777.

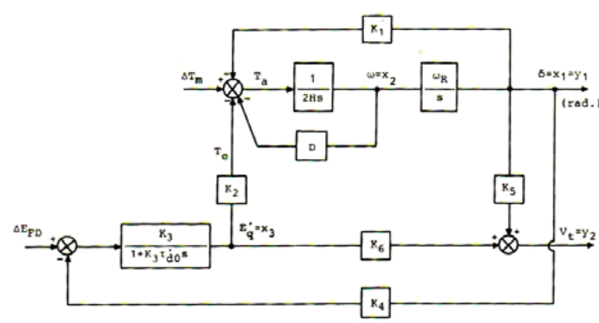

Figure-2. Simplified representation of investigated practical power system.

The continuous open-loop model describing this power system (taken from) [18] in the form of equation (1) (with p_1 = 4 and p_2 = 8) is as follows:

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3\end{bmatrix} = \begin{bmatrix} 0 & 377 & 0 \\ -0.1030 & -0.1818 & -0.1209 \\ -0.3091 & 0 & -5.5517 \end{bmatrix}\begin{bmatrix}x_1 \\ x_2 \\ x_3\end{bmatrix} +$$

$$+\begin{bmatrix} 0 & 0 \\ 0.0909 & 0 \\ 0 & 0.1695 \end{bmatrix}\begin{bmatrix}\Delta T_m \\ \Delta E_{FD}\end{bmatrix}$$

$$\begin{bmatrix}y_1 \\ y_2\end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -0.0433 & 0 & 0.4777 \end{bmatrix}\begin{bmatrix}x_1 \\ x_2 \\ x_3\end{bmatrix}$$

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where

$$x = \left[\delta \;\; \omega \;\; E_q^{'}\right]^T, \quad u = \left[\Delta T_m \;\; \Delta E_{FD}\right]^T, \quad y = \left[\delta \;\; v_t\right]^T$$

The the simulated responses of the output variables (δ, v_t, ω) and the eigenvalues of the original open-loop power system models, are shown in Figure 3 and Table 2, repectively.

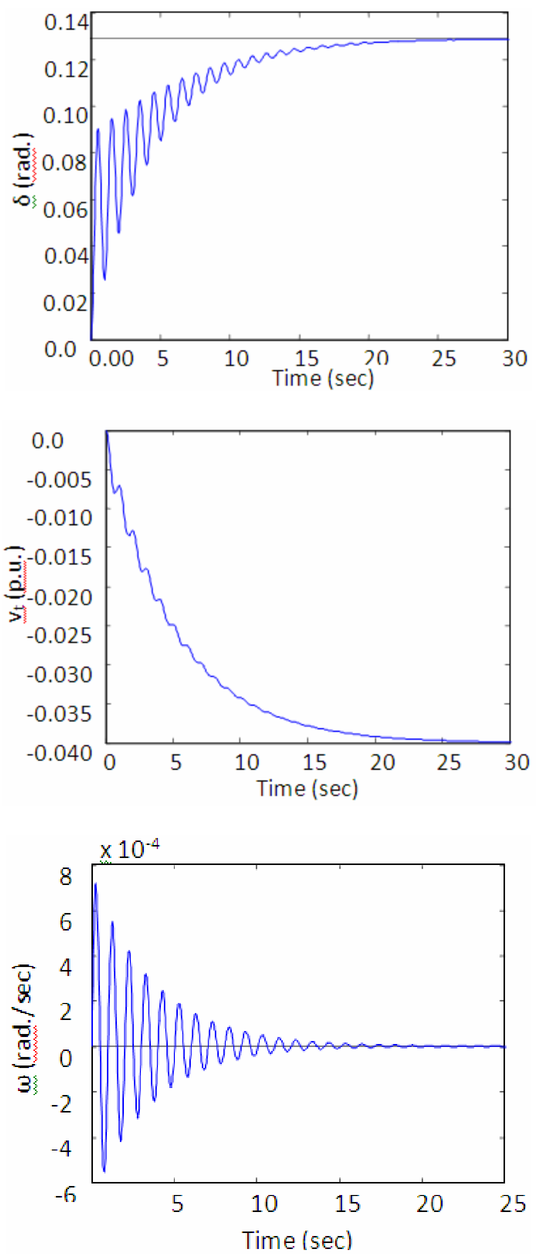

Figure-3. δ, v_t, ω, responses of the output variables of the original open-loop power system models to step input change: $\Delta T_m = 0.05$ and $\Delta E_{FD} = 0.0$p.u.

Table-2. Eigenvalues of original open-loop power system model.

λ	-0.2723 + 6.2253i -0.2723 - 6.2253i -0.1889

Based on the transformed continuous open-loop power system model, the associated discrete one, in relation to equation (2), is given as follow:

The properly selected sampling period, $T_0 = 1.2$sec

$$\mathbf{A}_{ol/d} = \begin{bmatrix} 0.2992 & 42.3326 & -0.6094 \\ -0.0111 & 0.2788 & -0.0127 \\ -0.0332 & -1.5581 & 0.7592 \end{bmatrix}$$

$$\mathbf{B}_{ol/d} = \begin{bmatrix} 0.9264 & -0.1913 \\ 0.0102 & -0.0003 \\ -0.2623 & 0.1812 \end{bmatrix}$$

$$\mathbf{C}_{ol/d} = \begin{bmatrix} 1 & 0 & 0 \\ -0.0433 & 0 & 0.4777 \end{bmatrix}$$

Due to space limitations the numerical description of the resulting discrete close loop system model is not presented here, but it depends on the following derive weight matrices.

$$\mathbf{Q} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

and the chosen output multiplicities of the sampling M=[4 5], N= [8].

Evaluation relations (11) and (12) we obtain the admissible TPMRC gains.

The computed values of **K**, $\mathbf{L_u}$ and feedback gain matrices are:

$$\mathbf{K} = \left[\begin{matrix} -0.0164 & 0.1552 & 0.0674 & -0.0386 \\ -0.0743 & -0.0701 & 0.1352 & 0.0191 \\ -0.01048 & -0.1023 & -0.2513 & -0.1513 \end{matrix}\right. \ldots$$

$$\ldots \left.\begin{matrix} -0.0219 & -0.0277 & -0.0301 & -0.0233 & -0.0184 \\ -0.0058 & 0.0006 & -0.0062 & -0.0123 & -0.0067 \\ 0.0782 & 0.0701 & 0.0722 & 0.0739 & 0.0667 \end{matrix}\right]$$

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$$\mathbf{L}_u = \begin{bmatrix} -0.4582 & -0.5101 & -0.1141 \\ 0.5575 & -0.3912 & -0.1122 \\ -0.0532 & -0.0708 & 0.3103 \end{bmatrix}$$

The magnitudes of the eigenvalues of the discrete original open-loop and of the designed closed-loop power system model are shown in Table-3.

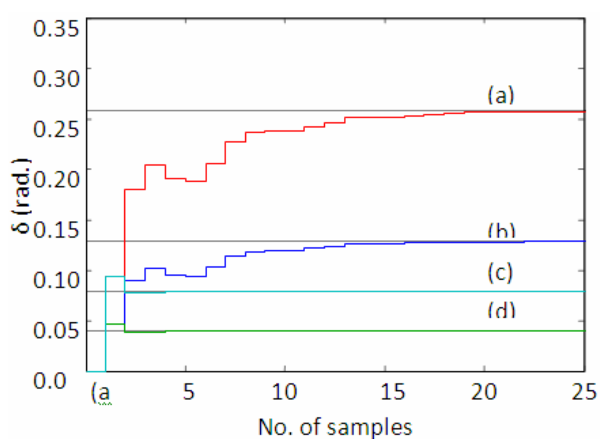
Table-3. Magnitude of eigenvalues of discrete original open-loop and designed closed-loop power system models.

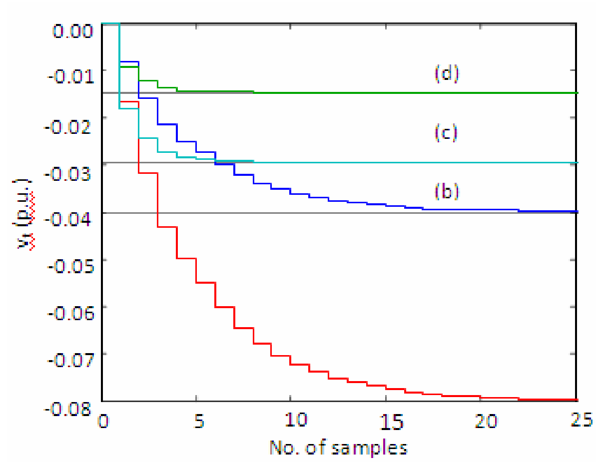
Original open-loop power system model	$\lvert\lambda\rvert$	0.7213 0.7213 0.7972
Designed closed-loop power system model	$\lvert\hat{\lambda}\rvert$	0.1984 0.1984 0.4137

The simulated responses of the output variables (δ, ω, v_t) of the discrete original open-loop and designed closed-loop power system models, for zero initial conditions and unit step input disturbance, are shown in Figure-3 and Figure-4.

By comparing the computed eigenvalues of the simulated responses of the discrete original open-loop power system model and the associated designed discrete closed-loop models, it is clear that the resulting enhancement in the dynamic system stability of the closed-loop system model is remarkable.

It is to be noted that the solution results of the discrete system models (i.e., eigenvalues, eigenvectors, responses of system variables etc.) for zero initial conditions were obtained using a special software program (which is based on the theory of § 3 and runs on MATLAB program environment).





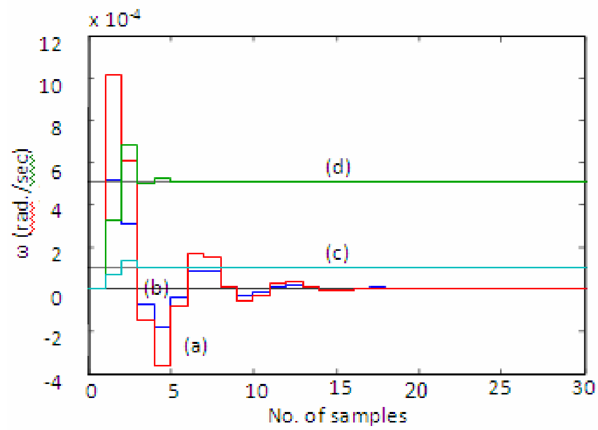


Figure-4. Responses of δ, v_t and ω outputs subject to unit step input change where: (a) and (b) refer to discrete open-loop and closed-loop to step input change ΔT_m= 0.05 and ΔE_{FD} = 0.0p.u., (c) and (d) refer to discrete open-loop and closed-loop to step input change ΔT_m = 0.10 and ΔE_{FD} = 0.0p.u.).

CONCLUSIONS

An optimal digital control strategy based on Two-Point-Multirate Controllers has been used in this paper in order to design a desirable excitation controller of an unstable hydro generator system, for the purpose of enhancing its dynamic stability characteristics. The proposed method offers acceptable closed loop response as well as more design flexibility (particularly in cases where the system states are not measurable), and its performance is at least comparable to known LQ optimal regulation methods.

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