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OPTIMIZATION OF WATER ADDUCTION MAINS PATH

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ABSTRACT

The efficient design of water adduction main involves several optimization processes among which an important place is held by their path optimization. In this paper are developed two deterministic mathematical models for optimization of water adduction main path, based on techniques of sequential operational calculus, implemented in a computer program. Using these optimization models could be obtained an optimal solution for selection of source location and of water adduction main path based on graph theory and dynamic programming. Numerical examples will be presented to demonstrate the accuracy and efficiency of the proposed optimization models. These show a good performance of the new models.

Keywords: water adduction main, optimal path, dynamic programming, graph theory, optimization models, computer program.

1. INTRODUCTION

The water adduction mains have a significant importance in water supply systems due to their large investment amount and important energy consumption [1]. As a consequence, the efficient design of water adduction main involves several optimization processes among which an important place is held by their path optimization.

In current design practice, the choice of the optimal solution is made usually through analytical study of two or three versions selected from the possible set by predicted decisions [2]. The errors of these decisions are inverse proportional to the designer experience.

The modern mathematical disciplines as operational research give to the designer a vast apparatus of scientific analysis in optimal decisions establishing [3], [4], [5], [6], [7], [8], [9].

The mathematical theory and planning of multistage decision processes, the term was introduced by Richard Bellman in 1957 [10, 11, 12]. It may be regarded as a branch of mathematical programming or optimization problems formulated as a sequence of decision.

Traditional optimization algorithms have been applied to the minimum cost optimal design problem, such as linear programming [13], first introduced by Labye [14] for open networks.

Dynamic programming [15] is useful however for optimizing temporal processes such as those typical in system operation problems. Sterling and Coulbeck [16], Coulbeck [17], Sabel and Helwig [18], and Lansey and Awumah [19] applied dynamic programming to determine optimal pumping operation for minimization of costs in a water system. Dynamic programming is also used primarily to solve tree - shaped networks [20, 21] and could be extended to solve to looped systems [22]. Also, Sarbu [23] has been applied graph theory to establish optimal path for branched water supply networks.

In this context, this paper develops two deterministic mathematical models for optimization of water adduction main path, based on techniques of sequential operational calculus, implemented in a computer program. Using these optimization models could be obtained an optimal solution for selection of source location and of water adduction main path based on graph theory and dynamic programming. The results of few numerical applications show the effectiveness and efficiency of the proposed optimization models.

2. OPTIMIZATION MODELS

2.1. Optimization model based on dynamic programming

Sequential decision problem stated above is typical of dynamic programming, which is a procedure for optimizing a multistage decision at each stage. The technique is based on the simple Bellman's principle of optimality [10, 11], which states that "an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

To solve a dynamic programming problem, it is necessary to evaluate both immediate and long-term consequence costs for each possible state at each stage. This evaluation is done through the development of the following recursive equation (for minimization):

$$\min Z = \sum_{i=1}^{N} V_i(X_{i-1}, X_i) \quad (1)$$

considering that each size X_i can vary in a field that depends on X_0 and X_{i+1}, $\forall\, i = 1, 2,..., N$.

Optimization formula (1) is generalized for case if the terms X_i are vectors with n components:

$$X_i = \{x_{1i}\ x_{2i} \ldots x_{ni}\} \quad (2)$$

The form (1) imposed for function Z and the nature of variable variation domains allow use of a system of N phases for which V_i (X_{i-1}, X_i); (i = 1, 2,..., N) is the value function attached to each phase, and Z- value function attached for phase crowd.

Finding the minimum of the function Z requires solving at each step the functional equation:

$$f_{0,i}(X_0, X_i) = \min_{\{X_{i-1}\}}[V_i(X_{i-1}, X_i) + f_{0,i-1}(X_0, X_{i-1})] \quad (3)$$

where the notation $\{X_{i-1}\}$ means that X_{i-1} belongs to a values set which depend only on X_0 and X_i.

Calculation procedure consists in successively solving of minimization problems (3) for i = 1, 2,..., N, with discrete variation of X_0 and storing intermediate results. Thus, it are calculated successive optimal sub-policies for phases 1 and 2 together, then phase 1, 2 and 3 together, ..., for phases $\overline{1, N}$ together, i.e., optimal policies:

$$\min Z = f_{0,N}(X_0, X_N) = \min_{\{X_{n-1}\}}[V_N(X_{N-1}, X_N) + f_{0,N-1}(X_0, X_{N-1}) \quad (4)$$

in which $f_{0,N}(X_0,X_N)$ is optimal policy value from X_0 to X_N; $V_N(X_{N-1},X_N)$ - sub-policy value from X_{N-1} to X_N; $f_{0,N-1}(X_0, X_{N-1})$ - optimal policy value from X_0 to X_{N-1}.

A numerical example which illustrates the application of this optimization model shall be presented below.

2.2. Optimization model based on graph theory

Mathematical model of dynamic processes of discrete and, determinist type can be simplified using graph theory.

The modeling of stated problem is realized by plotting oriented connected graph G = (X, U) which consists of source as origin, paths as edges and critical points as vertices. Each edge $u_j^i \in U$ is assigned with a number $\lambda(u_j^i) \geq 0$, in conventional units, depending on the adopted optimization criterion. Optimal path is given by minimum value path in the graph, which is determined by applying the Bellman-Kalaba algorithm [23], [24].

The graph G = (X, U) has attached a matrix M whose elements m_{ij} are:

$$m_{ij} = \begin{cases} \lambda(u_j^i) - \text{ edge value from } x_i \text{ to } x_j; \\ \infty - \text{if vertices } x_i \text{ and } x_j \text{ are not adjacent;} \\ 0 - \text{for } i = j. \end{cases} \quad (5)$$

Optimal path is given by path μ of graph, which has the total value:

$$\lambda(\mu) = \sum_{u_j^i \in \mu} \lambda(u_j^i) \rightarrow \min \quad (6)$$

If V_i is the minimum value of existing path $\mu_n^i, (i = \overline{0, n})$ from at vertex x_i to vertex x_n:

$$V_i = \lambda(\mu_n^i), \quad (i = \overline{0, n}) \quad (7)$$

so

$$V_n = 0 \quad (8)$$

then in accordance with the principle of optimality:

$$V_i = \min_{j \neq i}(V_j + m_{ij}), \ (i = \overline{0, n-1}; j = \overline{0, n}) \text{ and } \quad V_n = 0 \quad (9)$$

The system (9) is solved iteratively, noting with V_i^k the value of V_i obtained from iteration k, namely:

$$V_i^{\text{o}} = m_{in} \quad (i = \overline{0, n-1}); \quad V_n^{\text{o}} = 0 \quad (10)$$

It is calculated:

$$V_i^1 = \min_{j \neq i}(V_j^{\text{o}} + m_{ij}), \ (i = \overline{0, n-1}; \quad j = \overline{0, n}); \quad V_n^1 = 0 \quad (11)$$

and then:

$$V_i^k = \min_{j \neq i}(V_j^{k-1} + m_{ij}), \ (i = \overline{0, n-1}; \ j = \overline{0, n}); \ V_n^k = 0 \quad (12)$$

Order k of iteration expressed by relation (12) gives finite values only for paths with length at most k-1, arriving at x_n, choosing between them minimal ones.
From ones iteration to the next:

$$V_i^k \leq V_i^{k-1}, \quad \forall j. \quad (13)$$

Numbers $V_i^k \ (i \neq n; \ k = 0, 1, ...)$ form monotone decreasing strings that necessarily reach a minimum after a finite number of iterations that not exceed n-1.

So, the algorithm stops when it comes to an iteration k such that $V_i^k = V_i^{k+1}, \ (i = \overline{0, n})$ and the value of the shorted path between vertex. x_0 and x_n is $V_0^k = V_0^{k+1}$.

In order to identify the paths that have found minimum values, shall be deducted from (12) that along them, at the last iteration:

$$V_i^k = m_{ij} + V_j^{k-1} = m_{ij} + V_j^k \quad (14)$$

Based on the described optimization model a computer program OPTRAD was performed in FORTRAN programming language for PC compatible microsystems, with the flow chart shown in Figure-1, where: N is the graph order; *M (I, J)* - matrix associated to the graph; *V (I, J)* - column vector built for each iteration k; *X (I)* - vertices succession of path with minimum value; *VAL* - value of the shorted path in graph.

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Bellman-Kalaba algorithm is not but the expression in another language of the optimality principle and analytical procedure for applying functional equations (3) and (4) is only a special case, more elementary of this algorithm.

2.3. Sensitivity of optimal solution

At a complex analysis that involve several optimization criteria in a sequentially program, is necessary to determine the optimal solution sensitivity. Exploration of optimal solution neighborhoods leads to the problem of k-optimal solutions, i.e., at the determination of the solutions very closed of optimal one (quasi optimally).

It is sometimes necessary to optimize a single criterion when multiple optimal solutions are obtained. The choice of optimal solution can be done but calling another criteria or even several optimization criteria.

3. NUMERICAL APPLICATIONS

3.1. Application of optimizing model based on dynamic programming

For example, a water adduction main for a village L starting from two source locations S_1 and S_2 is considered (Figure-2). Possible paths pass through critical points A, B, C, D, E, forming three sectors. Could be applied dynamic programming model to solve the selection problem of source location and of the path for this water adduction main.

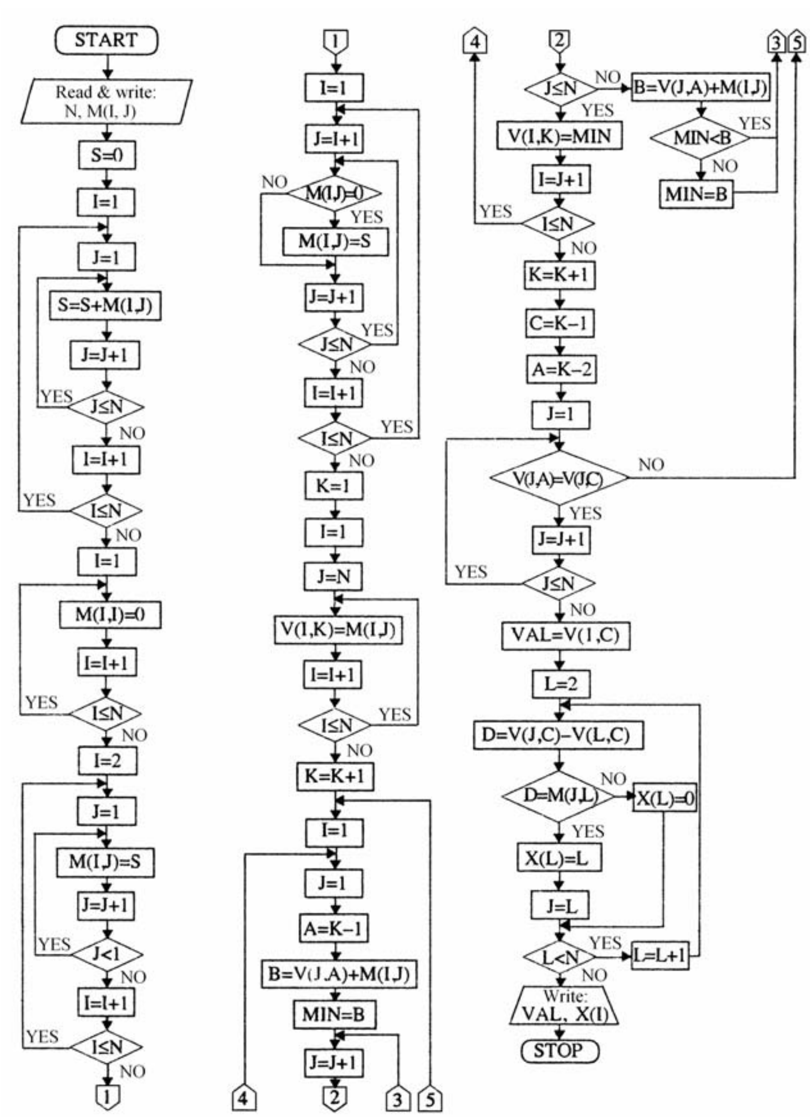

Figure-1. Flow chart of computer program OPTRAD.

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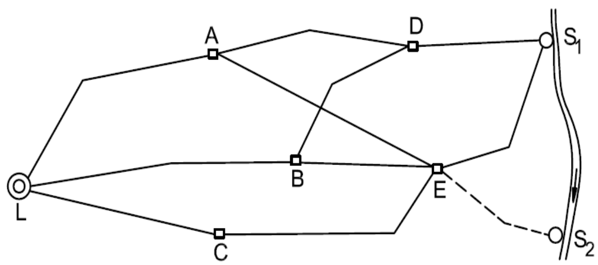

Figure-2. Variants of adduction path.

Adopting as optimization criterion the minimum total investment cost, partial investment is determined for each path and sequentially graph is plotted in Figure-3, in which each edge have associated a cost in conventional units. They noted with X_0, X_1, X_2 and X_3 decision variables related to the each sector. These variables will not take numerical values, but will be vertices in that graph which are on the same alignment.

The cost of sector 1 is noted V_I (X_0, X_1). This depends on values of X_0 and X_1 (in this case, X_0 could be only L). Identically are noted V_{II} (X_1, X_2) and V_{III} (X_2, X_3). The total value of adduction system is expressed by:

$$Z = V_{\mathrm{I}}\left(X_0, X_1\right) + V_{\mathrm{II}}\left(X_1, X_2\right) + V_{\mathrm{III}}\left(X_2 + X_3\right) \quad (15)$$

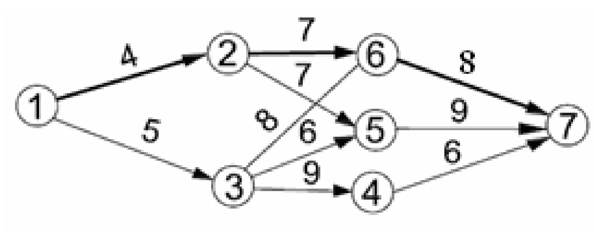

Figure-3. Sequential graph of possible adduction paths.

Initially, the adduction main is considered developed on a single sector (I). Noting with f_I (X_1) the minimum cost of sector I, for each of critical points A, B, C, equation (3) becomes:

$$f_{\mathrm{I}}(A) = V_{\mathrm{I}}(L, A) = 8; \quad f_{\mathrm{I}}(B) = V_{\mathrm{I}}(L, B) = 9; \quad f_{\mathrm{I}}(C) = V_{\mathrm{I}}(L, C) = 6 \quad (16)$$

The operator"min" is missing because step zero does not exist.

It is considered that the adduction main is formed of two sections (I, II), is denoted by $f_{I,\,II}$ (X_2) the minimum cost for sectors I and II together, for different values of X_2 equation (3) becomes:

$$\begin{aligned} f_{\mathrm{I,\,II}}(D) &= \min_{X_1 = A,B,C}\left[V_{\mathrm{II}}\left(X_1, D\right) + f_{\mathrm{I}}\left(X_1\right)\right] \\ f_{\mathrm{I,\,II}}(E) &= \min_{X_1 = A,B,C}\left[V_{\mathrm{II}}\left(X_1, E\right) + f_{\mathrm{I}}\left(X_1\right)\right] \end{aligned} \quad (17)$$

Giving successively to X_1 the values A, B, C and taking into account that for inexistent links are considered ∞ value, from previous relationships results:

$$\begin{aligned} f_{\mathrm{I,\,II}}(D) &= \min[\underset{X_1=A}{7+8}, \ \underset{X_1=B}{7+9}, \ \underset{X_1=C}{\infty + 6}] = 15, \ \text{for } X_1 = A; \\ f_{\mathrm{I,\,II}}(E) &= \min[\underset{X_1=A}{8+8}, \ \underset{X_1=B}{6+9}, \ \underset{X_1=C}{9+6}] = 15, \ \text{for } X_1 = C \end{aligned} \quad (18)$$

At the last step is considered that the adduction main is developed on sectors I, II and III. According with optimality principle the minimum cost for sectors I, II and III together, corresponding to values S_1 and S_2 for X_3, is written as:

$$\begin{aligned} f_{\mathrm{I,\,II,\,III}}\left(S_1\right) &= \min_{X_2 = D,E}[V_{\mathrm{III}}\left(X_2, S_1\right) + f_{\mathrm{I,II}}\left(X_2\right)]; \\ f_{\mathrm{I,\,II,\,III}}\left(S_2\right) &= \min_{X_2 = D,E}[V_{\mathrm{III}}\left(X_2, S_2\right) + f_{\mathrm{I,II}}\left(X_2\right) \end{aligned} \quad (19)$$

obtaining for the given example:

$$\begin{aligned} f_{\mathrm{I,\,II,\,III}}\left(S_1\right) &= \min[\underset{X_2=D}{4+15}, \quad \underset{X_2=E}{5+15}] = 19, \ \text{for } X_2 = D; \\ f_{\mathrm{I,\,II,\,III}}\left(S_2\right) &= \min[\underset{X_2=D}{\infty + 15}, \quad \underset{X_2=E}{6+15}] = 21, \ \text{for } X_2 = E \end{aligned} \quad (20)$$

It results as optimal solution the source location in S_1 and the water adduction main path: S_1, D, A, L.

Because of the analytical computation procedure is less intuitive a table calculation procedure is presented. So, in Table-1 are shown in column 0 the sector, in column 1 the vertex which marks the start of sector, in column 2 the vertex which marks the end of sector, in column 3 the value of corresponding edge, in column 4 the value of previous minimum sub-policy, and in column 5 the sum of columns 3 and 4.

Table-1. Calculation procedure for optimal path.

Sector	X_n^i	X_{n+1}^i	$\lambda\left(X_n^i, X_{n+1}^i\right)$	Previous min. sub-policy value	(3+4)
0	1	2	3	4	5
I	*A*	L	8	0	8
	B	L	9	0	9
	C	L	6	0	6
II	*D*	A	7	8	15
	D	B	7	9	16
	E	A	8	8	16
	E	B	6	9	15
	E	C	9	6	15
III	*S_1*	D	4	15	**19**
	S_1	E	5	15	20
	S_2	E	6	15	21

After fulfilling the entire table, is read the minimum value from column 5 of the sector III, which is 19, the bolded number. From this, it starts with a horizontal arrow to column 4 and is obtained 15; from here it starts again back in column 5 and up, to the position where he was transferred this number, and so on. The arrows show the optimal solution. The path is read from

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the column 1 with the first vertex S_1, continue in column 2 on the position of horizontal arrow: S_1, D, A, L (vertices that were underlined in table).

3.2. Application of optimizing model based on graph theory

It is presented a numerical example of application of optimizing model based on the graph theory to determine the optimal path of water adduction main for village L, starting from source S_1.

It is plotted in Figure-4, the oriented connected graph of order n = 7, formed of source as origin, paths as edges and critical points as vertices.

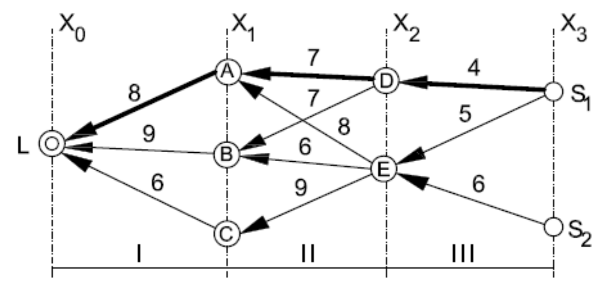

Figure-4. Graph of adduction paths.

The matrix M assigned to graph has the elements m_{ij} defined with the relations (5):

$$M = \begin{bmatrix} 0 & 4 & 5 & \infty & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty & 7 & 7 & \infty \\ \infty & \infty & 0 & 9 & 6 & 8 & \infty \\ \infty & \infty & \infty & 0 & \infty & \infty & 6 \\ \infty & \infty & \infty & \infty & 0 & \infty & 9 \\ \infty & \infty & \infty & \infty & \infty & 0 & 8 \\ \infty & \infty & \infty & \infty & \infty & \infty & 0 \end{bmatrix} \quad (21)$$

Using computer program OPTRAD it was determined minimum path in graph: (1, 2, 6, 7), having the value 19. It results that the optimal path of water adduction main is S_1, D, A, L.

4. CONCLUSIONS

In the presented optimization models were described two techniques of operational research for solving sequential programs. Applying the dynamic programming in determinist case at built of an adduction main could be established optimal path for more possible variants. The elaborated computer program based on the graph theory, for compatible microsystems, allows performing a quick and efficient computation.

REFERENCES

[1] T. M. Walski, D. V. Chase, D. A. Savic, W. Grayman, S. Beckwith and E. Koelle. 2003. Advanced water distribution modeling and management. Haestad Press, Waterbury, USA.

[2] Sarbu. 2010. Numerical modeling and optimization in building services (in Romanian). Politehnica Publishing House, Timisoara.

[3] A. Kaufmann. 1963. Methods and models of operations research. Prentice Hall Inc., Englewood Cliffs, N. J.

[4] R. Diestel. 2005. Graph theory. Springer-Verlag, New York, USA.

[5] L. R. Foulds. 1992. Graph theory applications. Springer-Verlag, New York, USA.

[6] J. L. Gross and T. W. Tucker. 2001. Topological graph theory. John Wiley and Sons, Portland.

[7] S. Pemmaraju and S. Skiena. 2003. Computational discrete mathematics: Combinatory and graph theory with mathematics. Cambridge University Press, Cambridge, U.K.

[8] E. Polak. 1971. Computational methods in optimization. Academic Press, New York, USA.

[9] A. Stefanescu and C. Zidaroiu. 1981. Operational Researches (in Romanian). Teaching and Pedagogical Publishing House, Bucharest, Romania.

[10] R. Bellman. 1951. Dynamic programming. Princeton Univ. Press, Princeton, N.J.

[11] R. E. Bellman. 2003. Dynamic Programming. Dover Publications, New York, USA.

[12] S. Dreyfus. 2002. Richard Bellman on the birth of dynamic programming. Operations Research. 50(1): 48-51.

[13] M. S. Bazaraa., J. J. Jarvis and H. D. Sherali. 1990. Linear programming and network flows. Wiley, New York, USA.

[14] Y. Labye. 1966. Étude des procédés de calcul ayant pour but le rendre minimal la coût d'un réseaux de distribution d'eau sous pression, La Houille Blanche. 5: 577-583.

[15] D. P. Bertsekas. 2005. Dynamic programming and optimal control. Athena Scientific, Belmont, Massachusetts, USA.

[16] M. J. Sterling and B. Coulbeck. 1973. A dynamic programming solution to optimization of pumping costs. Proceedings of Institute of Civil Engineers. 59(2): 813.

[17]B. Coulbeck. 1984. Optimization of water networks, Transactions of Institute of Measurements and Control. 6(5): 271.

[18]M. H. Sabel and O.J. Helwing. 1985. Cost effective operation of urban water supply system using dynamic programming. Water Resources Bulletin. 21(1): 75.

[19]K. E. Lansey and K. Awumah. 1994. Optimal pump operation considering pump switches. Journal of Water Resources Planning and Management, ASCE. 120(1): 17.

[20]T. Liang. 1971. Design conduit system by dynamic programming. Journal of Hydraulics Division, ASCE. 97(HY3): 383-393.

[21]P. Yang, T. Liang and I.P. Wu. 1975. Design of conduit system with diverging branches. Journal of Hydraulics Division, ASCE. 101(HY1): 167-188.

[22]Q. W. Martin. 1980. Optimal design of water conveyance systems. Journal of Hydraulics Division, ASCE. 106(HY9).

[23]Sarbu. 1997. Path optimization of a branched water supply network (in French). Scientific Bulletin of P.U. Timisoara, Romania. 42(1): 120-127.

[24]C. Berge. 1962. The theory of graphs and its applications. Wiley, New York, USA.