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PRESENTING A NEW LAMINATION METHOD FOR THE ELASTIC ANALYSIS OF MULTILAYER CYLINDRICAL SHELLS

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ABSTRACT

In this article, by using the lamination theory, three hypothetical equations with unknown coefficients have been considered for the modeling of the displacement behavior of each layer of a shell in the cylindrical coordinate system, which completely satisfy the natural and geometrical boundary conditions. In this article, complete continuity has been assumed between the layers of a multilayer shell at their common boundaries, and this continuity is modeled by considering the same behavioral equations at the interface of two adjacent layers. By using the first law of thermodynamics to obtain the total potential energy of the layers, and by taking the derivative of this total energy equation with respect to the unknown coefficients of displacement equations and solving the resulting system, these coefficients are determined. It is obvious that, by knowing a shell's displacement equations, the other quantities needed for the analysis of the shell can be calculated as well.

Keywords: shell analysis, lamination theory, potential energy, first law of thermodynamics.

INTRODUCTION

Today, there is a high demand for effective and efficient materials in various engineering practices and industries, and these advanced materials are greatly needed in production processes. The utilization of materials with high efficiency and low relative weight and price may be the most vital necessity. Included in these materials are shell types of composites, which are used extensively in aircraft structures. Of different theories presented for the analysis of multilayer shells, the lamellar theory can appropriately model the behavior of these types of structures. The lamellar theory has been divided into two parts: The partial lamellar theory, which uses the expansion of intra-plate components and disregards the displacement outside the plate, and the complete lamellar theory, which employs three displacement components [1]. The partial and complete lamellar theories can be used for the analysis of thick sheets. The lamellar theory can properly model the intra-plate zigzag displacements along the thickness. Whitney used the second-degree lamellar changes in order to improve the behavior modeling of multilayer sheets [2]. Swift and Heller employed the lamellar method to solve a laminated beam by assuming constant shear strains and continuous shear displacement between layers along the thickness [3].

Considering the same theory, Solecki solved the problems of 2-ply and 3-ply isotropic sheets. Reddy and Robbins applied the lamellar-displacement theory to model the behavior of a laminated piezoelectric beam [5]. Reddy's lamellar theory [6] may be one of the most common theories used by the researchers for the analysis of sheets and laminated sheets. Other lamellar theories have also been presented by Mau [7], Chou *et al.*, [8] and Ren [9]. Using the lamellar sheet theory, Ferreira *et al.*, [10] analyzed the deformation and vibration of sandwiched sheets.

Consider an M-layer shell where each layer has been made of a different material. This shell can deform under loading along the three directions of the cylindrical coordinate system (r, θ, z). An example of the coordinate system considered in this article for a single-layer cylinder has been depicted in Figure-1.

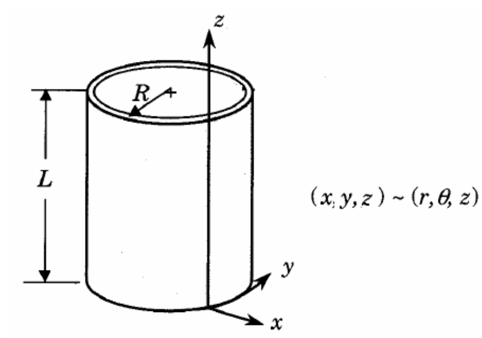

Figure-1. Coordinate system used for the shell.

Three displacement equations are considered for each shell layer in equation set (1).

$$u^{rk} = \sum_{i=1}^{P_{u^r}} \phi_i^{u^{rk}}(\theta, z)\Psi_i^{u^r}(r)$$

$$u^{\theta k} = \sum_{i=1}^{P_{u^\theta}} \phi_i^{u^{\theta k}}(\theta, z)\Psi_i^{u^\theta}(r)$$

$$u^{zk} = \sum_{i=1}^{P_{u^z}} \phi_i^{u^{zk}}(\theta, z)\Psi_i^{u^z}(r) \quad (1)$$

In the above equations, P_{u^r}, P_{u^θ} and P_{u^z} are the number of nodes chosen in the direction of each shell

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layer. Operators $\Psi_i^{u^r}(r)$, $\Psi_i^{u^\theta}(r)$ and $\Psi_i^{u^z}(r)$ indicate the behavior of the sheet along the radius which, considering the number of nodes along the thickness of each shell layer, could be linear, of second degree, or of higher degrees. $\varnothing_i^{u^{rk}}(x,y)$, $\varnothing_i^{u^{\theta k}}(x,y)$ and $\varnothing_i^{u^{zk}}(x,y)$ are the hypothetical equations for the modeling of the abovementioned displacements, which are calculated as follows:

$$\phi_i^{u^{r^k}}(\theta,z)=\sum_{n=1}^{P_{u^r}}\sum_{l=1}^{P_{pu_r^\theta}}\sum_{m=1}^{P_{pu_r^z}}\alpha_{lm}^{P^k_{u^r}}\,E_{lm}^{u^r}(\theta,z)\varphi^{u^r}(x,y)$$

$$\phi_i^{u^{\theta^k}}(\theta,z)=\sum_{n=1}^{P_{u^\theta}}\sum_{l=1}^{P_{pu_\theta^\theta}}\sum_{m=1}^{P_{pu_\theta^z}}\alpha_{lm}^{P^k_{u^\theta}}\,E_{lm}^{u^\theta}(\theta,z)\varphi^{u^\theta}(x,y)$$

$$\phi_i^{u^{z^k}}(\theta,z)=\sum_{n=1}^{P_{u^z}}\sum_{l=1}^{P_{pu_z^\theta}}\sum_{m=1}^{P_{pu_z^z}}\alpha_{lm}^{P^k_{u^z}}\,E_{lm}^{u^z}(\theta,z)\varphi^{u^z}(x,y) \tag{2}$$

In the above equations, $\varphi^{u^r}(x,y)$, $\varphi^{u^\theta}(x,y)$ and $\varphi^{u^z}(x,y)$ are the functions that satisfy the boundary conditions (natural and geometrical) in directions r, θ and z, and expressions $\alpha_{lm}^{p^k_{u^r}}$, $\alpha_{lm}^{p^k_{u^\theta}}$ and $\alpha_{lm}^{p^k_{u^z}}$ are the unknown coefficients of the hypothetical equations considered for the displacement equations. $E_{lm}^{u^r}((\theta,z)$ and $E_{lm}^{u^\theta}\theta,z)E_{lm}^{u^z}(\theta,z)$ are the terms that model the behavior of the sheet on the θ-z plane, which in this article, for all the displacement functions of the plate, have been considered uniformly as the following polynomial:

$$E(\theta,z)=\theta^{l-1}z^{m-1} \tag{3}$$

$P_{pu_r^\theta}$ and $P_{pu_r^z}$ are whole numbers that respectively denote the exponents assigned to terms θ and z in function $E(\theta,z)$. By knowing the three displacement equations of the layer, the strain equation for each sheet is obtained. It should be mentioned that a uniform displacement equation is considered in the common boundaries of layers. The potential energy of each layer is determined from the following formula:

$$\Pi^k=\amalg^k-W^k \tag{4}$$

where $\amalg^k$ and W^k are the strain energy and work done in each layer, respectively, which are obtained by [11-13]:

$$\amalg^k=\int_v \sigma_{ij}\varepsilon_{ij}dv \tag{5}$$

$$W^k=\int_S u^{rk}q^k(\theta,z)ds \tag{6}$$

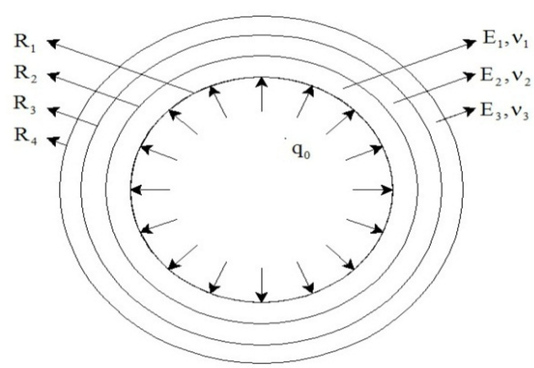
In the above relations, $q^k(\theta,z)$ is the loading applied on the investigated cylinder. The total energy of layers is obtained from the following formula:

$$\Pi_{tot}=\sum_{k=1}^{M}\Pi^k \tag{7}$$

By taking a derivative from the total potential energy with respect to the unknown coefficients of displacement equations and solving the resulting system, the coefficients of equations are determined. By obtaining each layer's displacement coefficients, the other quantities needed for the analysis of sheet in each layer can be calculated.

RESULTS

Consider a 3-layer cylindrical shell, like the one shown in Figure-2. The layers of this shell have been made of isotropic material, and their coefficients of elasticity and Poisson's ratios have been listed next to the figure.



$$R_1=12\text{in},\ E_1=1\text{psi},\ \nu_1=0.3$$
$$R_2=15\text{in},\ E_2=5\text{psi},\ \nu_2=0.3$$
$$R_3=16\text{in},\ E_3=10\text{psi},\ \nu_3=0.3$$
$$R_4=16.3\text{in}$$

Figure-2. Three-layer cylindrical shell with internal loading.

The above structure is subjected to a loading of $q_0=1N$ in its most interior layer. The analytical results of this cylinder and the behavior of its layers along the radius have been presented below by using the lamellar-displacement method.

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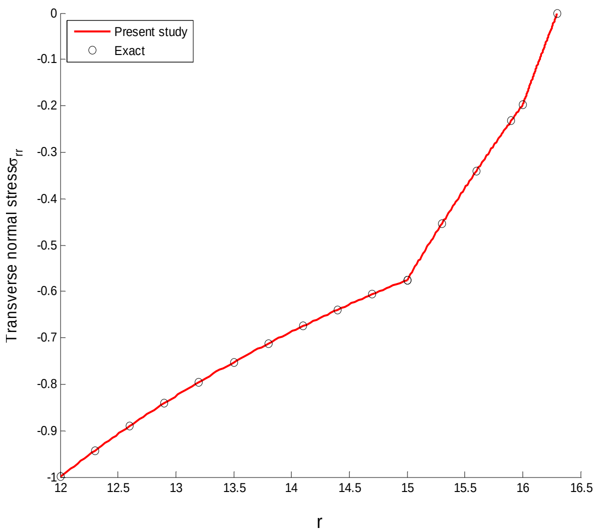

Figure-3. Normalized displacement of the multilayer cylinder along the radius.

Based on the assumption in the lamellar theory, displacement at the common boundaries of layers should be uniform and continuous, which is affirmed by the obtained result.

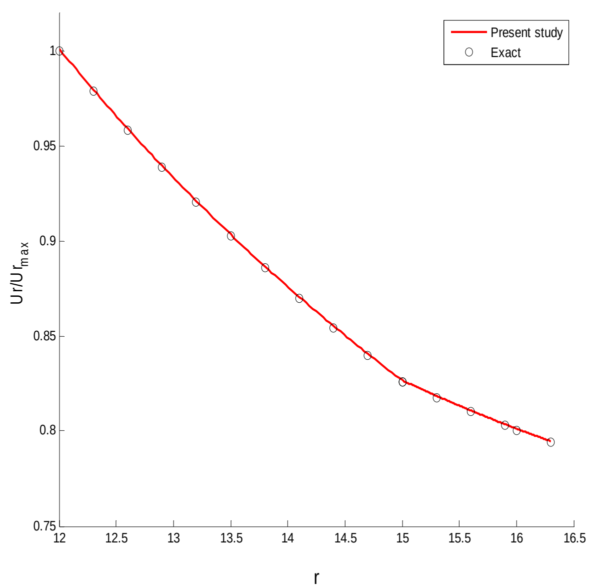

Figure-4. Normalized radial stress distribution along the radius of the multilayer cylinder.

Due to the action and reaction behavior by the adjacent layers, the amount of stress along the radius is the same in both layers. The diagram of this stress should indicate continuity along the radius, which is demonstrated and confirmed by the obtained diagram.

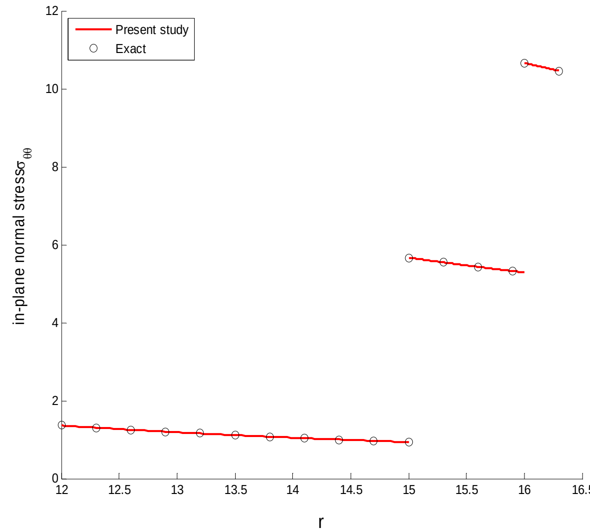

Figure-5. Normalized circumferential stress distribution along the radius of the multilayer cylinder.

Due to the different materials used in different layers, it is expected for the circumferential stress to display a discrete behavior; and this fact is demonstrated by the resulting diagram.

Solving the covered inhomogeneity problem in the Cartesian coordinates by using the lamellar-displacement method

Figure-6 shows a rectangular plate with covered inhomogeneity, being loaded by q_0 at infinity. The analysis results of this problem by means of the lamellar-displacement method have been given below.

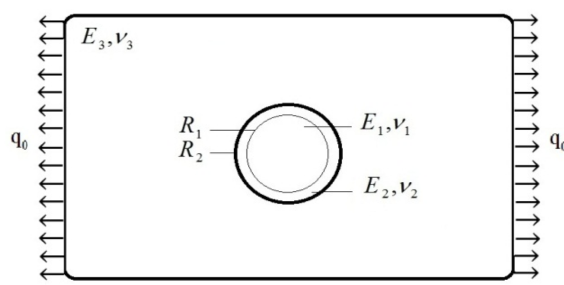

Figure-6. Plate with covered inhomogeneity and uniform loading of q_0 at infinity.

The boundary conditions for the presented problem exist at $\sigma_{xx} = q_0$, which by transforming this problem to cylindrical coordinate system, the new boundary conditions are defined as $\sigma_{rr} = q_0 \times \cos(2\times\theta)^2$ and $\sigma_{r\theta} = \dfrac{-q_0}{2} \times \sin(2\times\theta)$. The figure corresponding to this problem has been

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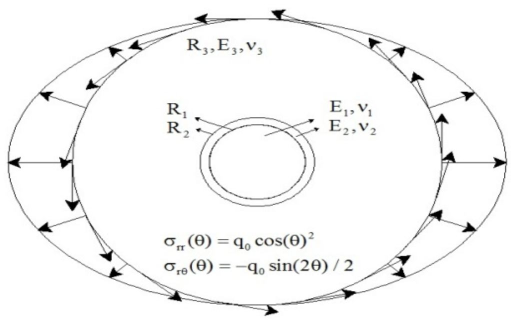
illustrated below, and the material properties have been listed next to the figure.



$$R_1 = 2\text{in},\ E_1 = 9.4736\text{psi},\ \nu_1 = 0.1842$$
$$R_2 = 4\text{in},\ E_2 = 22.5\,\text{psi},\ \nu_2 = 0.125$$
$$E_3 = 2.25\,\text{psi},\ \nu_3 = 0.125$$

Figure-7. Covered inhomogeneity transformed to the cylindrical coordinate system.

The analysis results of this sheet and the behavior of studied parameters along the radius have been provided in the following section.

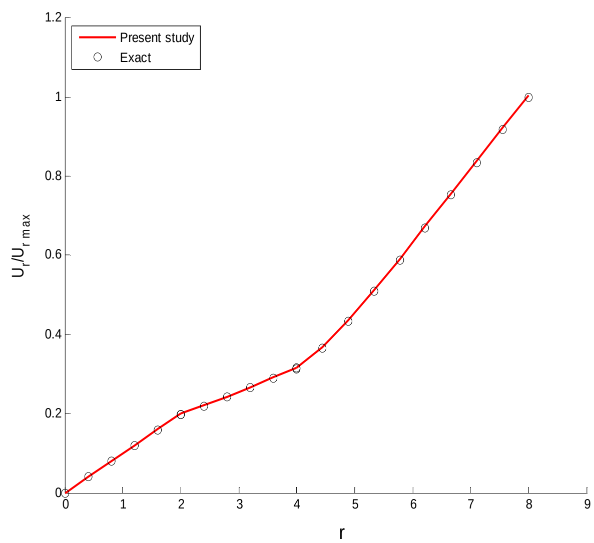


Figure-8. Normalized displacement of the rectangular plate with covered in homogeneity in cylindrical coordinates obtained by using the lamellar-displacement method along the radius.

In the analysis of this problem, inhomogeneity has been modeled similar to a shell with different material layers and therefore, considering the assumption of the lamellar method, displacement continuity is expected along the radius. The resulting diagram indicates the correctness of the solution and the conformity of the response to the basic theoretical hypothesis.

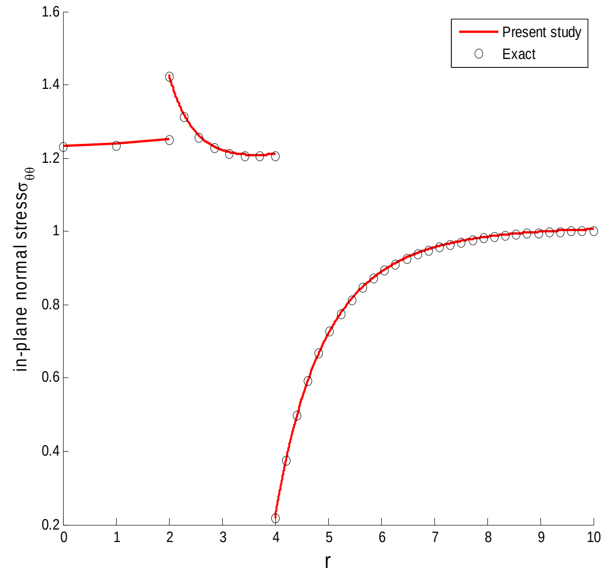


Figure-9. Normalized circumferential stress distribution of the rectangular plate with covered inhomogeneity in cylindrical coordinates obtained by using the lamellar-displacement method along the radius.

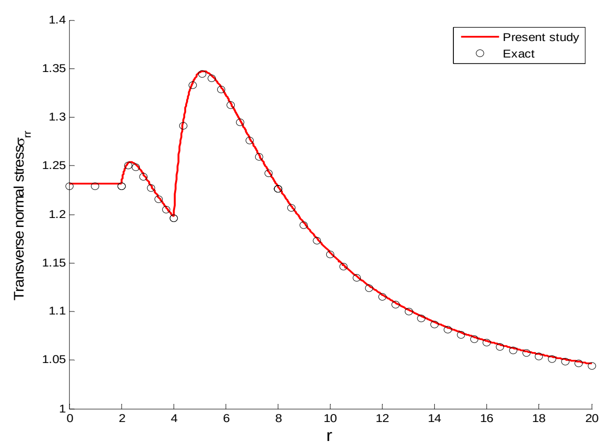


Figure-10. Normalized radial stress distribution of the rectangular plate with covered in homogeneity in cylindrical coordinates obtained by using the lamellar-displacement method along the radius.

CONCLUSIONS

The lamellar method, by assuming continuity between layers and similarity of intra-plate strains for adjacent layers, is highly capable of analyzing multilayer shells. And the hypothetical functions of θ^{l-1} and z^{m-1} model the behavior of the function on the θ-z plane very well. Also in this theory and method, the boundary conditions are appropriately satisfied by the relevant functions and the results of this method have a good agreement with the exact solution results.

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