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OPTIMIZATION OF WATER DISTRIBUTION NETWORKS PATH

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ABSTRACT

The determination of pipe network optimal path is important for an effective modeling and optimization of water-distribution systems. A principal application of the branched network optimal path is to evaluate the hydraulic performance of the distribution system based on selected schemes for many types of network analysis (e.g., design, operation, calibration). Already known algorithms for solving this problem usually determine a sole solution which in some cases proves to be suboptimal. This paper is a mathematical approach of the branched pipe network path optimization for water distribution. It is developed an algorithm based on graph-theory which generates all minimal trees of the graph comprising nodes where consumers are placed and links (pipes) between them and is implemented in a computer program for PC microsystems. Thus, can be determined all optimal solutions, for a given criterion. The optimization model is applicable to design of the distribution networks for hydro-urban and hydro-amelioration systems. Numerical example will be presented to demonstrate the accuracy and efficiency of the proposed optimization model. These show a good performance of the new model.

Keywords: water supply, branched networks, optimal path, graph-theory, computational algorithm, computer program.

1. INTRODUCTION

Distribution network is an essential part of all water supply systems and its cost may be equal to or greater than 60% of the entire cost of the project [1, 2].

Attempts should be made to reduce the cost and energy consumption of the distribution system through optimization in analysis and design. In some cases a distribution system is initially built to supply with low water flow rates a little territory and is extended over time as water consumption increases. In this case is realized originally a branched network which is then transformed into a looped network with increased capacity and security. As a consequence, the efficient design of branched network involves several optimization processes among which an important place is held by their path optimization. That problem is approached also on the looped network design for to determine an independent loop system (the virtual branched network) [1].

Traditionally, the choice of the optimal solution is made through analytical study of two or three versions selected from the possible set by predicted decisions [3]. The errors of these decisions are inverse proportional to the designer experience.

The modern mathematical disciplines as operational research give to the designer a vast apparatus of scientific analysis in optimal decisions establishing [4-9].

Traditional optimization algorithms have been applied to the minimum cost optimal design problem, such as linear programming [10], first introduced by Labye [11] for open networks. Dynamic programming is useful however for optimizing temporal processes [12] such as those typical in system operation problems. Sterling and Coulbeck [13], Coulbeck [14], Sabel and Helwig [15], and Lansey and Awumah [16] applied dynamic programming to determine optimal pumping operation for minimization of costs in a water system. Dynamic programming is also used primarily to solve tree-shaped networks [17, 18] and could be extended to solve to looped systems [19]. Also, Sarbu and Valea [20] has been applied graph theory to establish optimal path for water adduction mains.

In pressurized pipe networks the nodes with water consumers are first selected and pressure device (pumping station, reservoir), then the network path that connects these nodes is optimized in several steps. This optimization problem can have one or more solutions. Already knows algorithms (Sollin, Kruskal, etc.) [3, 6, 21] to solve the problem determine usually a single solution that in certain situations [22] proves to be cvasioptimal.

This paper is a mathematical approach of the branched pipe network path optimization for water distribution. It is developed an algorithm based on graph-theory which generates all minimal trees of the graph comprising nodes where consumers are placed and links (pipes) between them and is implemented in a computer program for PC microsystems. Thus, all optimal solutions can be determined, for a given criterion. Numerical example will be presented to demonstrate the accuracy and efficiency of the proposed optimization model.

2. FORMULATION OF OPTIMIZATION MODEL

2.1. Computational algorithm based on graph-theory

A distribution network may be represented by a directed connected graph *G* comprising a finite number of edges (pipes) connected to one another by vertices. At the end of each edge are vertices with known energy grade (fixed-grade nodes) or external water consumption (junction nodes). Water flow through the edges and can enter or exit the graph at any vertex.

The main optimization criteria that can be used are:

- Minimum capital cost ($\Sigma c_{ij}L_{ij} \rightarrow \min$);
- Minimum length of the path ($\Sigma L_{ij} \rightarrow \min$);

- Minimum transport work ($\Sigma L_{ij}Q_{ij} \rightarrow$ min);
- Minimum head loss ($\Sigma R_{ij}Q_{ij}^2 \rightarrow$ min),

in which L_{ij}, Q_{ij} are the length and the flow rate of the pipe *ij* (between nodes *i* and *j*); $c_{ij}$is the specific capital cost; R_{ij} is the hydraulic resistance of pipe *ij*.

- It is considered undirected connected graph $G = (X, U)$, where $X = \{1, 2, ..., n\}$ represent the set of vertices indexes and U – the set of edges. Each edge $u_j^i \in U$ has an associated value $\lambda(u_j^i) > 0$, in conventional units, according to the adopted optimization criterion. This graph has attached a matrix C of order *n* whose elements are:

$$c_{ij} = \begin{cases} \lambda(u_j^i), & \text{if } u_j^i \in U \\ \infty, & \text{if } u_j^i \notin U, \text{ or } i = j \end{cases} \tag{1}$$

where u_j^i is the edge *u* with one end in vertex *i* and the other one in vertex *j*.

If all $\lambda(u_j^i)$ values for $u_j^i \in U$ are distinct, the problem has a single solution. If there are values $\lambda(u_j^i)$ equal to more edges, the problem may have several solutions.

2.1.1. Partial graph determination of the minimal trees

The indexes of edge vertices u_j^i, belonging to the partial graph of the minimal trees, and the corresponding values c_{ij} are retained in a matrix M with 3 columns and n–1 rows, because a minimal tree that has n–1 edges. This matrix is performed in the following steps:

S1) Minimal elements from each row of the matrix C are determined.

S2) It is chosen one of the rows for which minimal element is unique and is denoted by *r*, *s* its indexes.

S3) Values r, s, c_{rs} are stored on the first row of matrix M, and c_{rs}, c_{sr} elements from matrix C are marked.

S4) It is determined the minimum unmarked element of C matrix rows on which exist at least a marked element, and is denoted by *r*, *s* the indexes of the minimum element or of one of them if there are more.

S5) If on row *s* of the matrix C there are marked elements, are marked also elements c_{rs}, c_{sr} and proceed to step S6. If on row *s* of the matrix C there are not marked elements, for each c_{is} element located on a row marked and equal with c_{rs} is added to matrix M a new row which consists of the values i, s, c_{is}. Shall be marked elements c_{is}, c_{si} from matrix C, then proceed to step S6.

S6) Proceed to step S4 or stop calculations as still exist or not in the matrix C rows with no unmarked element.

2.1.2. Generation of minimal trees

If for the matrix M result a row number bigger than considered graph order, the minimal tree problem has several solutions.

In this case first the rows of the matrix M are permuted so that the second column elements are ordered in ascending order. In the second column will find only indexes of n–1 vertices of the graph. If are denoted absolute frequencies of these indexes on the second column of the matrix M with F_1, F_2, ..., F_n, and added frequencies with $F_1^*, F_2^*, ..., F_n^*$ (i=1, 2,..., n–1), the number n_a of minimal trees of the graph is given by:

$$n_a = \prod_{i=1}^{n-1} F_i \tag{2}$$

To identify the n_a minimal trees are performed a matrix A with n–1 rows and n_a columns in the following steps:

S7) It is associated to each absolute frequency F_i a variable V_i whose values are $N_i = \left(F_{i-1}^*, F_i^*\right] \cap \{N\}$, ($i$ = 1, 2, ..., n–1) set elements, where $\{N\}$ is the natural number system, and $F_0^* = 0$.

S8) On each row *i* of the matrix A are inscribed n_a/F_i times the elements V_i of the set N_i, such as matrix A finally obtained have the columns lexicographically ordered.

On each column of the matrix A are the indexes of M matrix rows containing edge characteristics of one optimal tree.

If obtained solution is multiple, the choosing of optimal solution is performed taking into account other criteria.

2.2. Computer program OTREDIRA

Based on above developed algorithm OTRE-DIRA computer program in FORTRAN programming language for PC Microsystems was elaborated. This has flowchart in Figure-1, where: *N* is the graph order attached to the network; **C***(I,J)* is the criterion matrix associated to the topological graph of the network; *MIN(I)* is the minimal element from row *I*; *NEM(I)* is the number of minimal elements from the row *I*; *MAR(I)* is the number of marked elements from the row *I*; *MINLM* is the minimum of the marked elements from the rows containing marked elements A*(I,J)* is the helpful matrix in identifying minimal trees; M*(K,L)* is the matrix of optimal edge vertices and of corresponding values.

3. NUMERICAL APPLICATION

It is exemplified application of proposed optimization model to determine the optimal path of a pipe network with possible path graph of order n=9 (Figure-2). This graph has attached C capital cost matrix with the elements expressed in conventional units:

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$$\mathbf{C} = \begin{bmatrix} \infty & 98 & 83 & 88 & \infty & \infty & \infty & \infty & \infty \\ 98 & \infty & 85 & \infty & \infty & 73 & 68 & \infty & \infty \\ 83 & 85 & \infty & 33 & 33 & 48 & 90 & \infty & \infty \\ 88 & \infty & 33 & \infty & 33 & \infty & \infty & 93 & \infty \\ \infty & \infty & 33 & 33 & \infty & 33 & \infty & 43 & \infty \\ \infty & 73 & 48 & \infty & 33 & \infty & 38 & 43 & \infty \\ \infty & 68 & 90 & \infty & \infty & 38 & \infty & 98 & 40 \\ \infty & \infty & \infty & 93 & 43 & 43 & 98 & \infty & 95 \\ \infty & \infty & \infty & \infty & \infty & \infty & 40 & 95 & \infty \end{bmatrix} \quad (3)$$

Using OTREDIRA computer program, the following results were obtained:

- The number of minimal trees $n_a = 2$.
- The matrix M of optimal edge vertices and the corresponding values:

$$\mathbf{M} = \begin{bmatrix} 7 & 2 & 68 \\ 1 & 3 & 83 \\ 3 & 4 & 33 \\ 5 & 4 & 33 \\ 3 & 5 & 33 \\ 5 & 6 & 33 \\ 6 & 7 & 38 \\ 5 & 8 & 33 \\ 7 & 9 & 40 \end{bmatrix} \quad (4)$$

- The matrix A on whose columns are found the indexes for the rows of the ordered matrix M containing the edges of one of the minimal trees:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \\ 5 & 5 \\ 6 & 6 \\ 7 & 7 \\ 8 & 8 \\ 9 & 9 \end{bmatrix} \quad (5)$$

Optimal path solutions, applying the minimum capital cost criterion, are illustrated in Figure-3.

Since obtained optimal solution is multiple, decision to adopt one of these alternatives is made considering other criteria too.

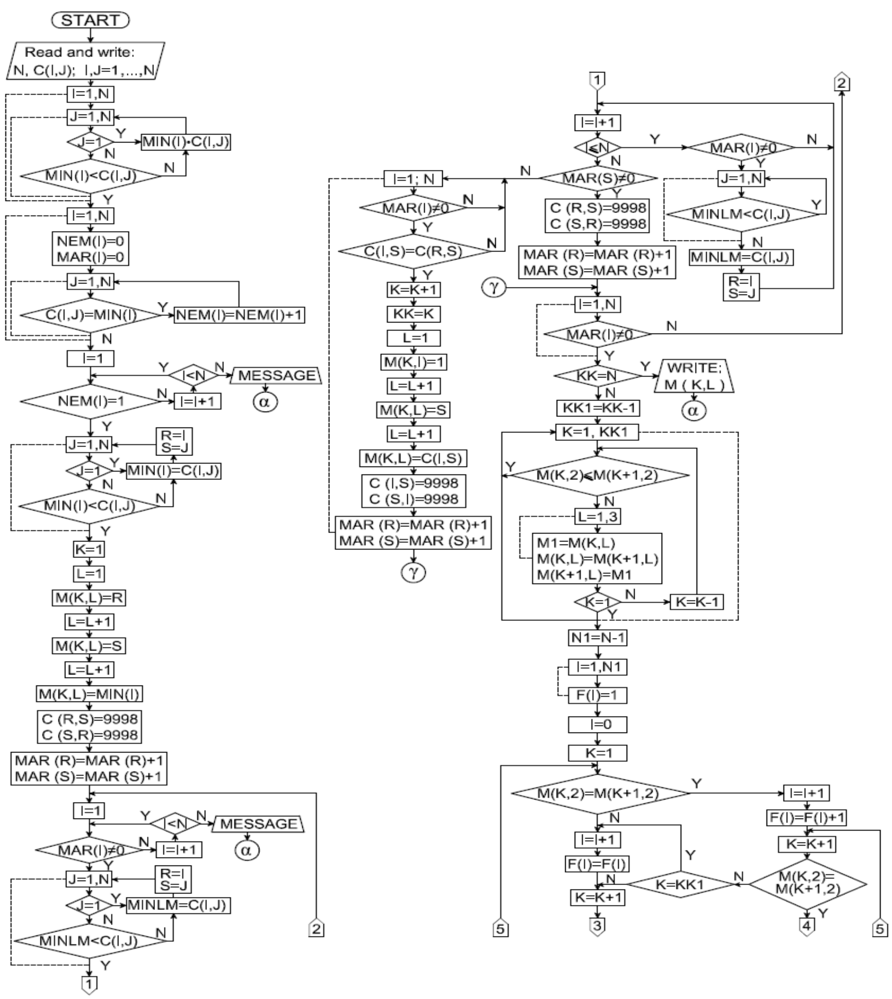

Figure-1. Flow chart of computer program OTREDIRA.

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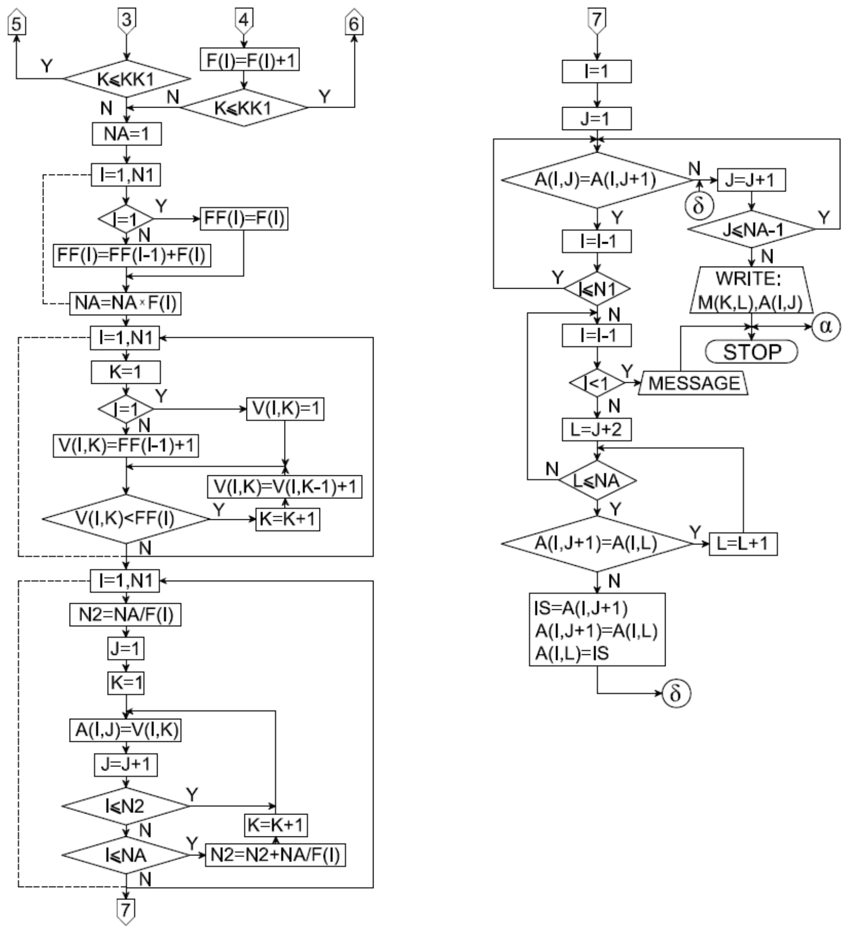

Figure-1. Flow chart of computer program OTREDIRA (continuation).

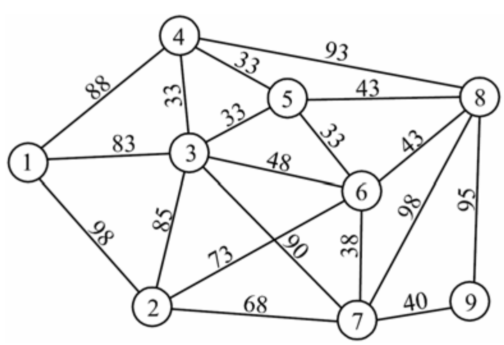

Figure-2. Graph of network paths.

4. CONCLUSIONS

The determination of optimal path for branched pipe networks is important for an effective modelling and optimization of water-distribution systems. The choice of the path optimal solution must be performs taking into account several criteria.

The optimization model is applicable to design of the distribution networks for hydro-urban and hydro-amelioration systems.

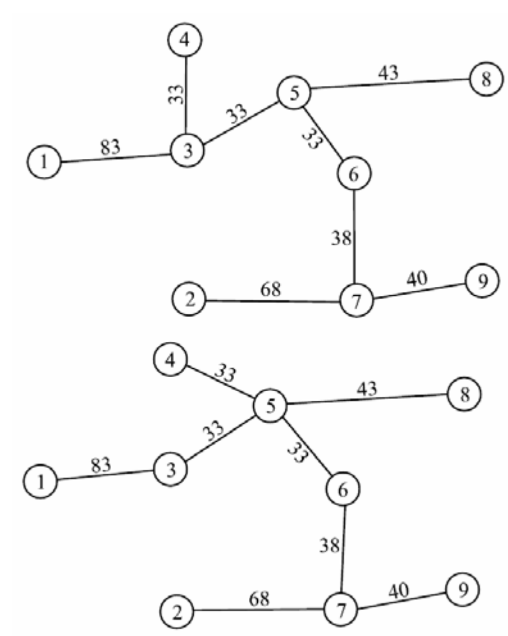

Figure-3. Optimal paths of distribution network.

Using the proposed optimization model leads to savings of the pipelines, earthworks, and electricity due to shortening the path and a more even distribution of the pressures in the system.

The elaborated computer program based on the optimization model allows performing a quick and efficient computation.

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