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EFFECTS OF SLIP CONDITION AND MULTIPLE CONSTRICTIONS ON COUPLE STRESS FLUID FLOW THROUGH A CHANNEL OF NON UNIFORM CROSS SECTION

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ABSTRACT

Steady incompressible couple stress fluid flow through a non-uniform channel with two stenoses is investigated. Assuming the stenoses to be mild and using the slip boundary condition, the equations governing the flow of the proposed model are solved and closed form expressions for the flow characteristics (resistance to flow and wall shear stress) are derived. Both the resistance to flow and the wall shear stress increase with the heights of the stenoses and slip parameter but decrease with Darcy number. The effects of wall exponent parameter on the flow characteristics also have been studied.

Keywords: couple stress fluid, stenoses, slip condition, non-uniform channel, resistance to flow.

INTRODUCTION

Stenosis is the term used for the abnormal and unnatural growth in the lumen of the arteries carrying blood from heart to various parts of the body. It is caused mainly due to intravascular atherosclerotic plaques, which develop at arterial wall, thus occluding the lumen of the artery partly or fully. The formation of stenosis disturbs the normal blood flow and there is considerable evidence to show that the hydro dynamical factors such as wall shear stress, resistance to flow etc. can play a significant role in the development of this pathological condition. Hence, a detailed knowledge of the flow in the stenosed tube may lead to a better understanding of flow situation in physiological system.

In view of this, several authors have considered theoretical and experimental studies of blood flow in stenotic region (Young [1], Lee and Fung [2], Radhakrishnamacharya and Srinivasa Rao [3]). In these studies, blood is assumed to behave like a Newtonian fluid. However, it is known that blood behaves like a non-Newtonian fluid under certain conditions, particularly, at low shear rates and in small diameter vessels (Majhi and Nair [4], Blair and Spanner [5], Shukla *et al*., [6]).

In most of the studies in the literature, the artery is assumed to be of uniform cross-section in which only a single stenosis has developed. However, in general multiple stenoses may develop in series along the length of the artery whose cross-section varies slowly i.e., the tube converges or diverges slowly (Schneck and Ostrach [7]). Maruthi Prasad and Radhakrishnamacharya [8] discussed blood flow through an artery having multiple stenoses with non-uniform cross-section, considering blood as a Herschel-Bulkley fluid.

Couple stress fluids consist of rigid, randomly oriented particles suspended in a viscous medium. Couple stress fluid is known to be a better model for bio-fluids, such as blood, lubricants containing small amount of high polymer additive, electro-rheological fluids and synthetic fluids (Valanis and Sun [9]). The main feature of couple stress fluids is that the stress tensor is anti-symmetric and their accurate flow behavior cannot be predicted by the classical Newtonian theory. Stokes [10] generalized the classical model to include the effects of the presence of the couple stresses. Sankad and Radhakrishnamacharya [11], Srinavasacharya and Srikanth [12] and Mekheimer and Shehawey [13] studied the flow of couple stress fluid under different conditions.

The existence of slip phenomenon at the boundaries and interfaces has been observed in the flows of rarefied gasses, hypersonic flows of chemically reacting binary mixtures and flows of polymeric liquids (Bhatt and Sacheti [14]). Several investigators considered the effect of slip (Mehta and Tiwari [15], Kwang *et al*., [16], Srinivas *et al*., [17], Hakeem *et al*., [18]).

The effect of slip condition on a fluid flow in a channel with stenoses has not received any attention. Hence, in this paper, a mathematical model for couple stress fluid flow through a channel with non-uniform cross section and two stenoses is considered. Using slip boundary condition and assuming that the stenoses are mild, closed form solutions have been obtained. Expressions for the resistance to flow and the wall shear stress have been derived and the effects of various parameters on these flow variables have been studied and shown graphically.

MATHEMATICAL FORMULATION

We consider steady and incompressible couple stress fluid flow in a non-uniform channel with two stenoses. The non-stenosed parts consist of regions with uniform cross section of half width d_0 and slowly varying cross section characterized by $y^*(x)$. Cartesian coordinate system is chosen so that the x-axis coincides with the center line of the channel and the y-axis normal to it. The stenoses are supposed to be mild and develop in a symmetric manner. The geometry of the wall is taken as (Maruthi Prasad and Radhakrishnamacharya [8]).

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$$\eta(x)=\begin{cases} d_0 & :0\le x\le d_1 \\ d_0-\frac{\delta_1}{2}\left(1+\cos\frac{2\pi}{L_1}(x-d_1-\frac{L_1}{2})\right) & :d_1\le x\le d_1+L_1 \\ d_0 & :d_1+L_1\le x\le B_1-\frac{L_2}{2} \\ d_0-\frac{\delta_2}{2}\left(1+\cos\frac{2\pi}{L_2}(x-B_1)\right) & :B_1-\frac{L_2}{2}\le x\le B_1 \\ y^*(x)-\frac{\delta_2}{2}\left(1+\cos\frac{2\pi}{L_2}(x-B_1)\right) & :B_1\le x\le B_1+\frac{L_2}{2} \\ y^*(x) & :B_1+\frac{L_2}{2}\le x\le B \end{cases} \quad (1)$$

where B is the length of the channel, L_i and δ_i $(i=1,2)$ are the lengths and maximum heights of two mild stenoses. (Figure-1).

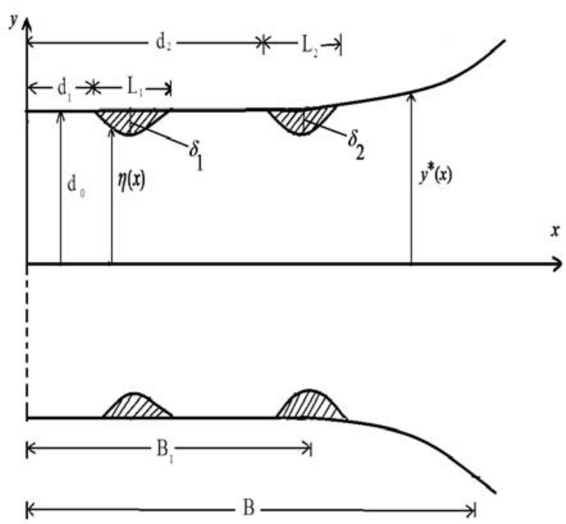

Figure-1. Flow geometry of the channel with multiple stenoses.

The equations governing the flow of steady and an incompressible couple stress fluid for the present problem (neglecting the body forces and body couples) are: (Alemayehu and Radhakrishnamacharya [19]).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (2)$$

$$-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}-\eta'\frac{\partial^4 u}{\partial y^4}=0 \quad (3)$$

$$-\frac{\partial p}{\partial y}=0 \quad (4)$$

where u and v are the velocity components along the x and y directions respectively, p is the pressure, μ is the viscosity coefficient of the classical fluid dynamics and η' is the couple stress fluid viscosity.
The boundary conditions are given by

$$u=\frac{-d_0\sqrt{D_a}}{\alpha_1}\frac{\partial u}{\partial y} \qquad at \quad y=\pm\eta \quad (5)$$

$$\frac{\partial^2 u}{\partial y^2}=0 \qquad at \qquad y=\pm\eta \quad (6)$$

Here equation (5) is the Saffman's slip boundary condition (Bahtt and Sacheti [14]) and equation (6) indicates the vanishing of couple stress. Further, Da is the permeability parameter (or Darcy number) and α_1 is the slip parameter.

Taking the restriction for mild stenosis (Young [1]) and solving equations (2)-(4) under the boundary conditions (5) and (6), the velocity is given as:

$$u(y)=-\frac{1}{\mu}\frac{\partial p}{\partial x}[\frac{\eta^2}{2}-\frac{y^2}{2}+\frac{\cosh(m^* y)}{(m^*)^2\cosh(m^*\eta)} -\frac{1}{(m^*)^2}+\frac{d_0\sqrt{D_a}}{\alpha_1}\left(\eta-\frac{\tanh(m^*\eta)}{m^*}\right)] \quad (7)$$

where $m^*=\left(\frac{\mu}{\eta'}\right)^{\frac{1}{2}}$

ANALYSIS

The flux Q of the fluid is given by

$$Q=2\int_0^\eta u\ dy=\frac{-2}{\mu}\frac{\partial p}{\partial x}[\frac{1}{3}\eta^3-\left(\frac{1}{(m^*)^2}-\frac{\eta\, d_0\sqrt{Da}}{\alpha_1}\right)\times \left(\eta-\frac{\tanh(m^*\eta)}{m^*}\right)] \quad (8)$$

Introducing the following dimensionless quantities

$$\delta_1'=\frac{\delta_1}{d_0},\ (x',d_1',L_1',L_2',B_1')=\frac{(x,d,L_1,L_2,B_1)}{B}, \quad \delta_2'=\frac{\delta_2}{d_0},\ (y^*(x))'=\frac{y^*(x)}{d_0},\ H=\frac{\eta}{d_0} \quad (9)$$

In equation (1) and equation (8), we get (after dropping primes)

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$$Q = -\frac{2}{\mu}\frac{\partial p}{\partial x}d_0^{\,3}\left[\frac{1}{3}H^3 - \left(\frac{1}{m^2} - \frac{H\sqrt{Da}}{\alpha_1}\right)\times \left(H - \frac{\tanh(mH)}{m}\right)\right] \tag{10}$$

where $m(= d_0 m^* = d_0(\mu/\eta')^{\frac{1}{2}})$ is a couple stress parameter.
From equation (10)

$$\frac{\partial p}{\partial x} = -\frac{Q\mu}{2d_0^{\,3}}\left(\frac{1}{\frac{1}{3}H^3 - \left(\frac{1}{m^2} - \frac{H\sqrt{Da}}{\alpha_1}\right)\left(H - \frac{\tanh(mH)}{m}\right)}\right) \tag{11}$$

Integrating equation (11) with respect to x, we get pressure difference Δp along the total length of the channel as:

$$\Delta p = \frac{Q\mu}{2d_0^{\,3}}\int_0^1\left(\frac{1}{\frac{1}{3}H^3 - \left(\frac{1}{m^2} - \frac{H\sqrt{Da}}{\alpha_1}\right)\left(H - \frac{\tanh(mH)}{m}\right)}\right)dx \tag{12}$$

The resistance to flow, denoted by λ, is defined by

$$\lambda = \frac{\Delta p}{Q} \tag{13}$$

Using equation (12) in equation (13), we get

$$\lambda = \frac{\mu}{2d_0^{\,3}}\int_0^1\left(\frac{1}{\frac{1}{3}H^3 - \left(\frac{1}{m^2} - \frac{H\sqrt{Da}}{\alpha_1}\right)\left(H - \frac{\tanh(mH)}{m}\right)}\right)dx \tag{14}$$

The pressure drop in the case of no stenosis (H=1), denoted by Δp_n, is obtained from equation (12) as:

$$\Delta p_n = \frac{Q\mu}{2d_0^{\,3}}\int_0^1\left(\frac{1}{\frac{1}{3} - \left(\frac{1}{m^2} - \frac{\sqrt{Da}}{\alpha_1}\right)\left(1 - \frac{\tanh(m)}{m}\right)}\right)dx \tag{15}$$

The resistance to flow in the absence of stenosis, λ_n, is defined by

$$\lambda_n = \frac{\Delta p_n}{Q} \tag{16}$$

Using equation (15) in equation (16), we obtain

$$\lambda_n = \frac{\mu}{2d_0^{\,3}}\int_0^1\left(\frac{1}{\frac{1}{3} - \left(\frac{1}{m^2} - \frac{\sqrt{Da}}{\alpha_1}\right)\left(1 - \frac{\tanh(m)}{m}\right)}\right)dx \tag{17}$$

The normalized resistance to the flow, $\bar{\lambda}$, is given by

$$\bar{\lambda} = \frac{\lambda}{\lambda_n} \tag{18}$$

The shear stress acting on the wall of the channel is given by

$$\tau_w = -\mu\left.\frac{\partial u}{\partial y}\right|_{y=\eta} \tag{19}$$

Using equation (7), we get

$$\tau_w = \frac{Q\mu}{2d_0^{\,2}}\left(\frac{1}{\frac{1}{3}H^3 - \left(\frac{1}{m^2} - \frac{H\sqrt{Da}}{\alpha_1}\right)\left(H - \frac{\tanh(mH)}{m}\right)}\right)\times\left(H - \frac{\tanh(mH)}{m}\right) \tag{20}$$

The shear stress at the wall in the absence of stenosis (H=1), denoted by $(\tau_w)_n$, can be obtained from equation (20) as

$$(\tau_w)_n = \frac{Q\mu}{2d_0^{\,2}}\left(\frac{1}{\frac{1}{3} - \left(\frac{1}{m^2} - \frac{\sqrt{Da}}{\alpha_1}\right)\left(1 - \frac{\tanh(m)}{m}\right)}\right)\times\left(1 - \frac{\tanh(m)}{m}\right) \tag{21}$$

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The normalized shear stress at the wall, $\overline{\tau_w}$, is given by

$$\overline{\tau_w} = \frac{\tau_w}{(\tau_w)_n} \tag{22}$$

RESULTS AND DISCUSSIONS

The resistance to the flow and the wall shear stress are the two important characteristics in the study of fluid flow through a stenosed artery. The expressions for resistance to the flow and wall shear stress, given by equations (18) and (22) respectively have been numerically evaluated using MATHEMATICA software for different values of relevant parameters and presented graphically.

Further, it is assumed (Maruthi Prasad and Radhakrishnamacharya [8]).

$$\frac{y^*(x)}{d_0} = \exp[k'(x - B_1)^2]$$

Where $d_1 = 0.2, L_1 = 0.2, L_2 = 0.2, B_1 = 0.8$ and $k' = \alpha B^2$.

The effects of heights of stenoses δ_1 and δ_2, the wall exponent parameter k', Darcy number Da, the slip parameter α_1 and couple stress parameter m on the resistance to flow $\overline{\lambda}$ are shown in Figures (2)-(11).

It is observed that the resistance to the flow $\overline{\lambda}$ increases with the slip parameter α_1 (Figures 2-7) but decreases with the couple stress parameter m (Figures 8-9) and Darcy number Da (Figures 10-11). Also the resistance to the flow increases with the heights of the stenoses (δ_1 and δ_2) and the number of stenoses (Figures 2-11). The effect of α_1 is not very significant in the case of single stenosis and when the height of the stenosis (δ_1 or δ_2) is less than 0.1 (Figures 2, 4 and 6). Further, for a given value of α_1, the resistance to the flow is more for the convergent channel ($k' < 0$) compared to its values for a divergent channel ($k' > 0$) (Figures 4-7). This observation agrees with the experimental findings of Talukder *et al*., [20].

The wall shear stress acting over the length of the second stenosis is shown graphically in Figures (12-19) for different values of Da, α_1, δ_2 and k'. It can be seen that the wall shear stress increases with the height of the second stenosis δ_2 (Figures 12-13) and the slip parameter α_1 (Figures 14-15) but decreases with Darcy number Da (Figures 16-17). Further, it is observed that for fixed values of Da and α_1, the wall shear stress is more for convergent channel ($k' < 0$) compared to its corresponding value for divergent channel ($k' > 0$) but the difference is not very significant(Figures 14-17)). Taking δ_2 =0, it is interesting to note that the wall shear stress increases with the slip parameter for a convergent channel ($k' < 0$) (Figure-18). However, the shear stress decreases with the slip parameter for a divergent channel ($k' > 0$) (Figure-19).

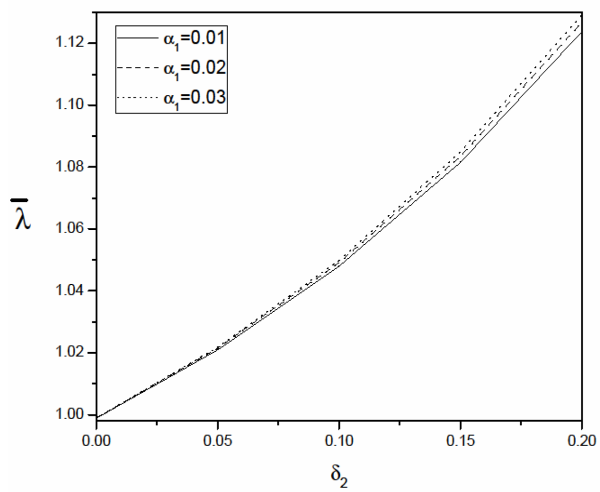

Figure-2. Effect of α_1 on $\overline{\lambda}$ $(m=0.3, Da=0.002, \delta_1=0.0, k'=0.1)$.

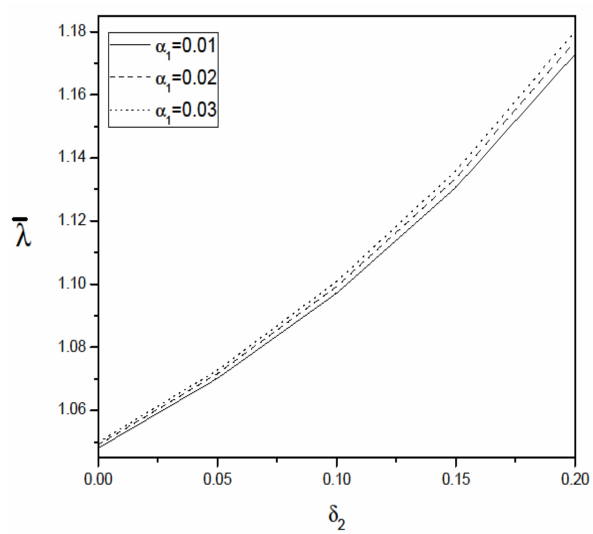

Figure-3. Effect of α_1 on $\overline{\lambda}$ $(m=0.3, Da=0.002, \delta_1=0.1, k'=0.1)$.

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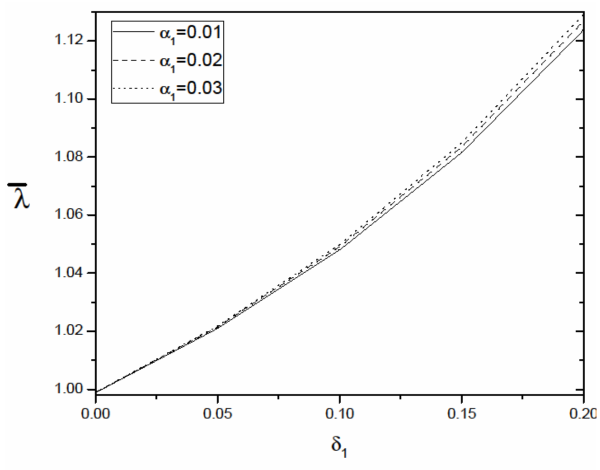

Figure-4. Effect of α_1 on $\bar{\lambda}$ $(m=0.3, Da=0.002, \delta_2=0.0, k'=0.1)$.

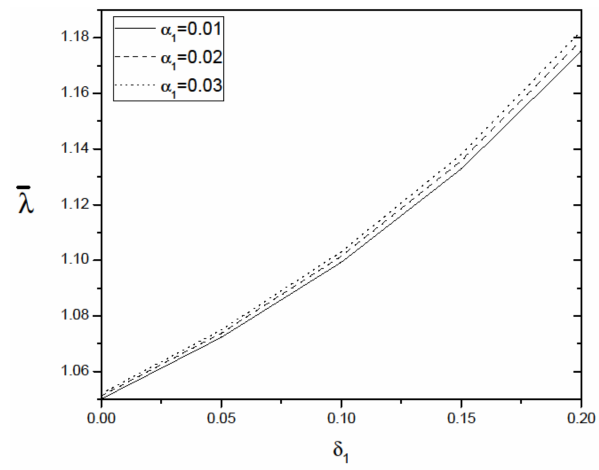

Figure-7. Effect of α_1 on $\bar{\lambda}$ $(m=0.3, Da=0.002, \delta_2=0.1, k'=-0.1)$.

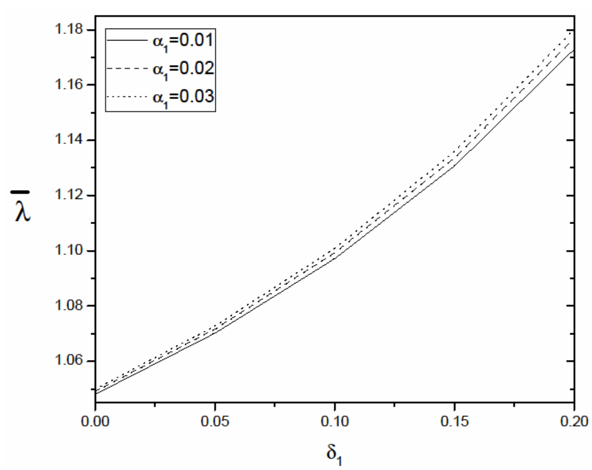

Figure-5. Effect of α_1 on $\bar{\lambda}$ $(m=0.3, Da=0.002, \delta_2=0.1, k'=0.1)$.

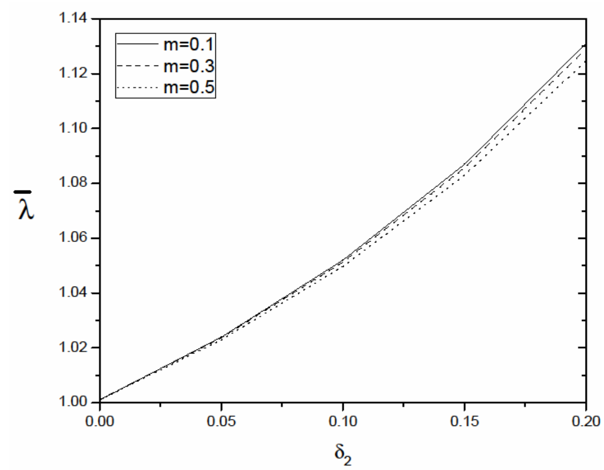

Figure-8. Effect of m on $\bar{\lambda}$ $(\alpha_1=0.02, Da=0.002, \delta_1=0.0, k'=-0.1)$.

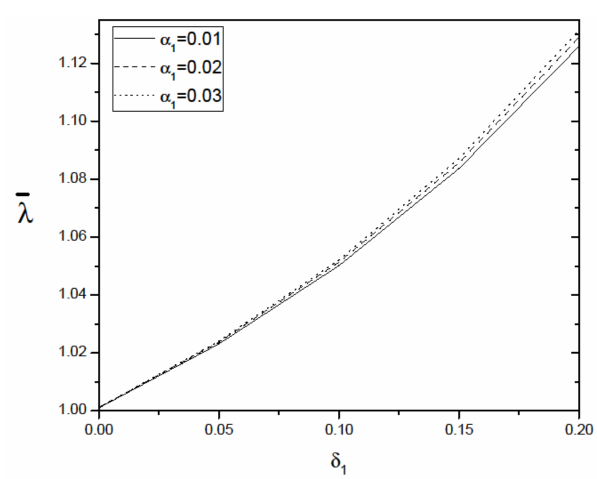

Figure-6. Effect of α_1 on $\bar{\lambda}$ $(m=0.3, Da=0.002, \delta_2=0.0, k'=-0.1)$.

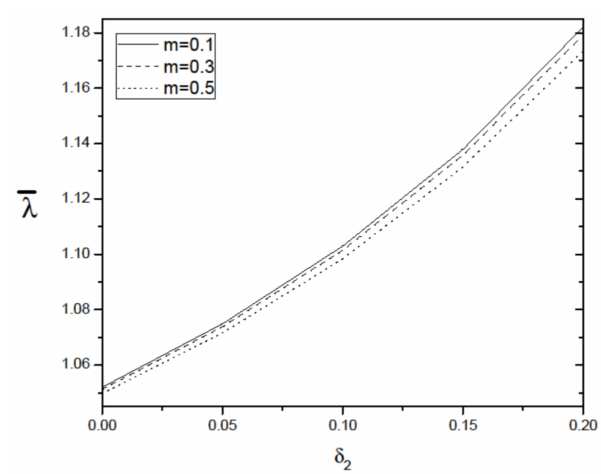

Figure-9. Effect of m on $\bar{\lambda}$ $(\alpha_1=0.02, Da=0.002, \delta_1=0.1, k'=-0.1)$.

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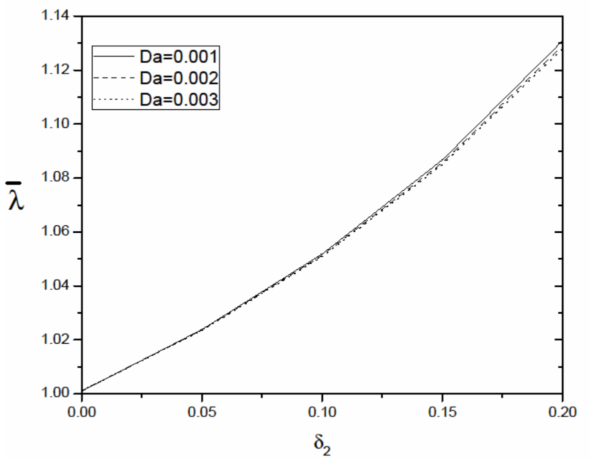

Figure-10. Effect of Da on $\bar{\lambda}$ ($\alpha_1=0.02, m=0.3, \delta_1=0.0, k'=-0.1$).

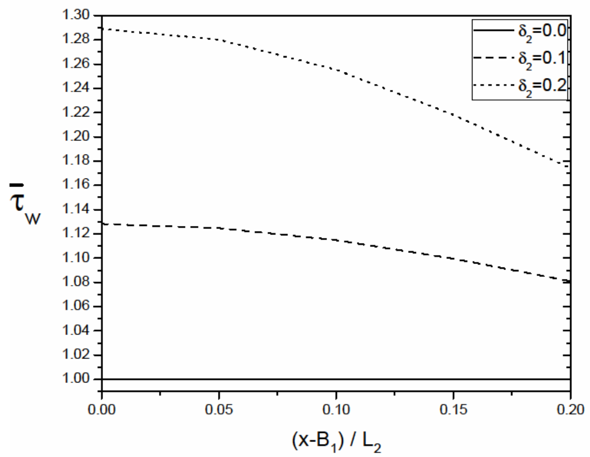

Figure-13. Effect of δ_2 on $\bar{\tau}_w$ ($\alpha_1=0.02, m=0.3, Da=0.002, k'=-0.1$).

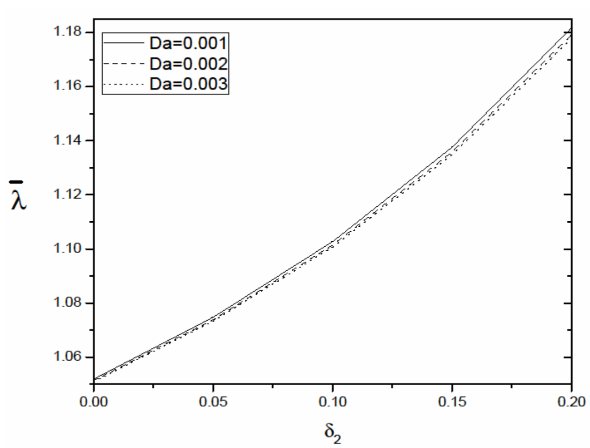

Figure-11. Effect of Da on $\bar{\lambda}$ ($\alpha_1=0.02, m=0.3, \delta_1=0.1, k'=-0.1$).

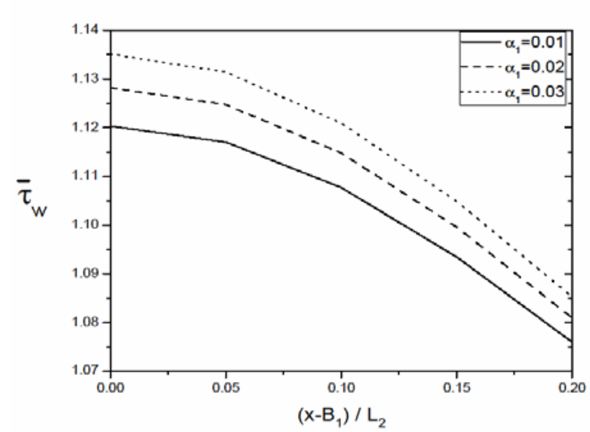

Figure-14. Effect of α_1 on $\bar{\tau}_w$ ($\delta_2=0.1, m=0.3, Da=0.002, k'=-0.1$).

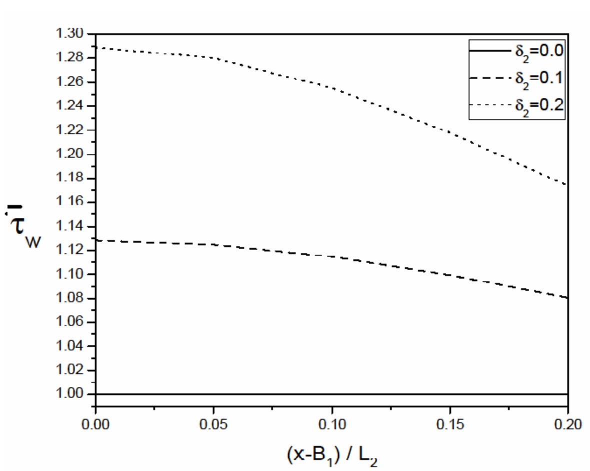

Figure-12. Effect of δ_2 on $\bar{\tau}_w$ ($\alpha_1=0.02, m=0.3, Da=0.002, k'=0.1$).

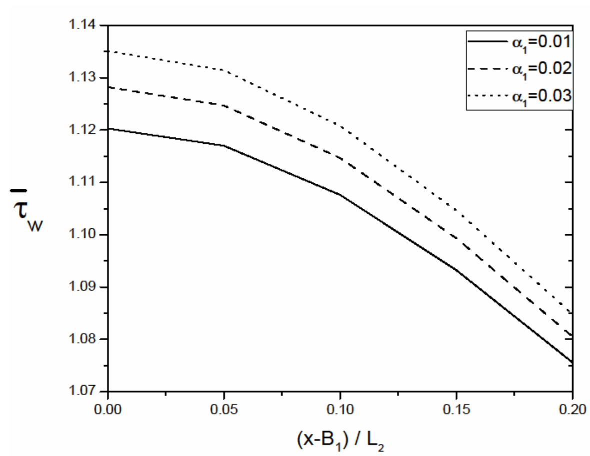

Figure-15. Effect of α_1 on $\bar{\tau}_w$ ($\delta_2=0.1, m=0.3, Da=0.002, k'=0.1$).

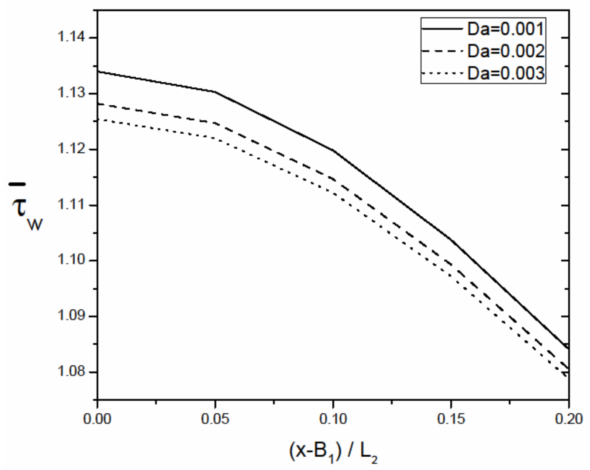

Figure-16. Effect of Da on $\bar{\tau}_w$ $(\delta_2=0.1, m=0.3, \alpha_1=0.02, k'=0.1)$.

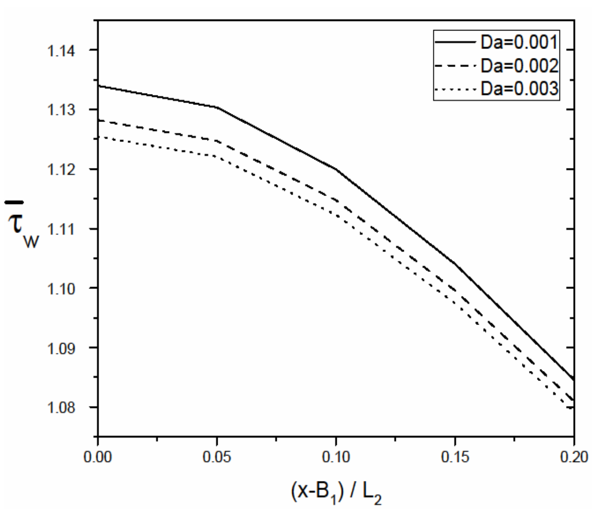

Figure-17. Effect of Da on $\bar{\tau}_w$ $(\delta_2=0.1, m=0.3, \alpha_1=0.02, k'=-0.1)$.

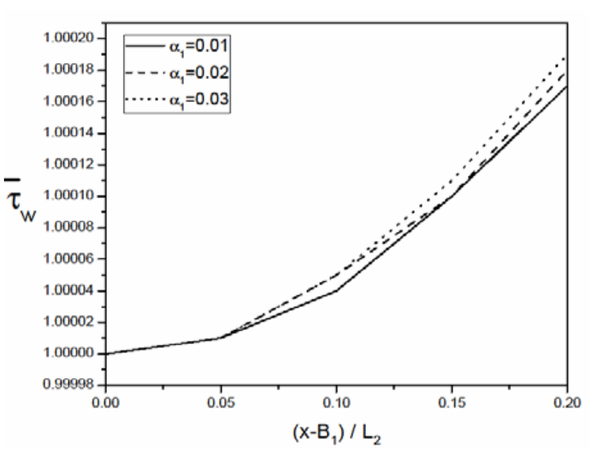

Figure-18. Effect of α_1 on $\bar{\tau}_w$ $(\delta_2=0.0, m=0.3, Da=0.002, k'=-0.1)$.

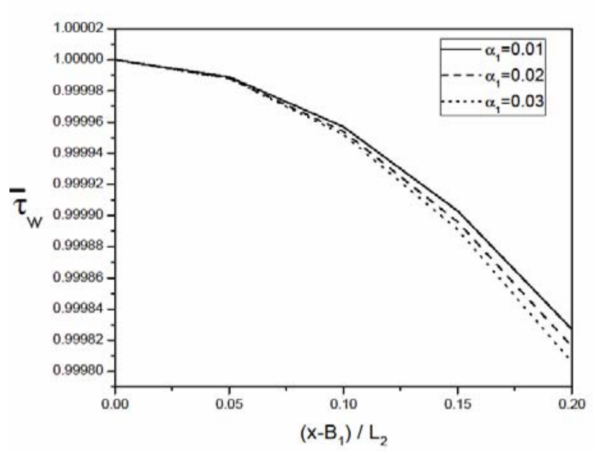

Figure-19. Effect of α_1 on $\bar{\tau}_w$ $(\delta_2=0.0, m=0.3, Da=0.002, k'=0.1)$.

CONCLUSIONS

The flow of steady and an incompressible couple stress fluid through a non-uniform channel and having two stenoses has been presented. Solutions have been obtained for mild stenoses and using slip boundary condition. It has been observed that the resistance to the flow increases with the heights of stenoses and slip parameter but it decreases with couple stress parameter and Darcy number.

It is also observed that the shear stress acting on the second stenosis increases with the height of stenosis and slip parameter but decreases with Darcy number. Further, the resistance to flow is more for convergent channel ($k' < 0$) compared to its value for divergent channel ($k' > 0$).

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