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PRODUCTIVITY EQUATION FOR A HORIZONTAL WELL INSIDE A CLOSED-ANISOTROPIC BOX-SHAPED RESERVOIR UNDER PSEUDOSTEADY-STATE CONDITIONS

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ABSTRACT

Beyond doubt, the productivity index is one of the most important key parameters in reservoir evaluation and characterization. It is a screening guide to establish the reservoir size, the presence on an aquifer and the connectivity degree. Thereby, its accurate determination is a challenge for engineers. This paper presents a productivity equation of a horizontal well in pseudo-steady state in a closed anisotropic box-shaped reservoir, using a uniform line sink model. A new equation for calculating pseudo skin factor due to partial penetration is also proposed. Compared with the equations for horizontal wells in pseudo-steady state in the literature, the new equations are more practical and easy to use in the field practice.

Keywords: horizontal well, productivity index, anisotropy, finite reservoir, flow rate, drainage area.

1. INTRODUCTION

Due to their great efficiency in producing higher flow rates per unit pressure drawdown, horizontal wells have currently become the most popular alternative for the development of hydrocarbon fields around the world. So far, most of the introduced correlations for the estimation of their productivity index have shown certain differences among their results. This does not allow us to properly establish which one of them provides the closest value to the actual one, since there is no evidence of a trustable enough reference point.

Throughout the years, several investigations for the determination of horizontal-well productivity index have been carried out. These researches have been focused on the determination of steady-state solutions for the above-mentioned parameter, therefore, a diverse number of correlations have been introduced. These correlations were introduced by such well-known researchers as, Giger (1983), Merkulov (Borisov, 1954), Renard and Dupuy (1990), and Joshi (Borisov, 1954; Joshi, 1988 and 1991; and Penmatcha, Arbab, and Azz., 1997). However, there are remarkable differences among their results which do not allow us to clearly establish which one matches closely the actual value since not accurate comparison point has been given. Much later, Escobar and Montealegre-M (2008) presented a more accurate correlation which behaves steadier for a wider range of application and presents the lowest deviation error among the others above mentioned. According to the proximity to the simulated results, they also found that Joshi's correlation is the second option to estimate the productivity index in horizontal wells.

To determine the economic feasibility of drilling a horizontal well, the engineers need reliable methods to estimate its expected productivity. There have been several attempts to describe and estimate horizontal well productivity indexes. Lu (2001), Lu (2003) and Lu *et al*. (2003) presented steady-state productivity equations for a horizontal well in an infinite extension, finite thickness reservoir and in a circular cylinder drainage volume reservoir.

Babu and Odeh (1989) introduced a complex equation to calculate productivity of horizontal wells which required that the drainage volume be approximately box-shaped, and all the boundaries of the drainage volume be sealed. They reduced their original infinite series solution into equations for shape factor and partial penetration skin.

2. MATHEMATICAL DEVELOPMENT

2.1. Horizontal well model

Figure-1 is a schematic of a horizontal well.

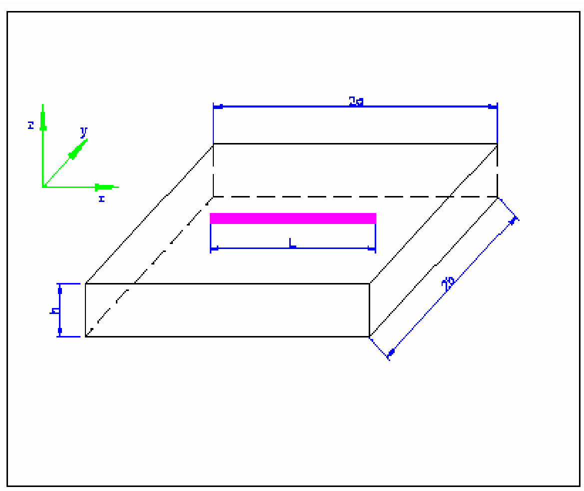

Figure-1. Horizontal well model.

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A horizontal well of length, *L*, drains a box-shaped reservoir with height, *H*, length (*x* direction), 2*a*, and width (*y* direction), 2*b*. The well is parallel to the *x* direction with a length $L \leq 2a$.

The following assumptions are made:

a) The reservoir is a horizontal, homogeneous, anisotropic, and has constant k_x, k_y, k_z permeabilities, thickness *H*, porosity ϕ. All the boundaries of the box-shaped drainage volume are sealed.
b) The reservoir pressure is initially constant. At time t = 0 pressure is uniformly distributed in the reservoir, equal to the initial pressure P_i.
c) The production occurs through a horizontal well of radius, r_w, represented in the model by a uniform line sink located at a distance z_w from the lower boundary, the length of the well is *L*.

As Figure-1 shows, the horizontal well is a uniform line sink in three dimensional space, the coordinates of the two ends are ($-L/2$, 0, z_w) and ($L/2$, 0, z_w).

Let,

$$\Omega = (-a,a)\times(-b,b)\times(0,H) \tag{1}$$

Ω is the box-shaped drainage volume.

All the boundaries of the box-shaped drainage volume are sealed; i.e., the reservoir has the following outer boundary condition:

$$\frac{\partial P}{\partial N}\Big|_{\Gamma} = 0 \tag{2}$$

where $\partial P/\partial N|_{\Gamma}$ is the external normal derivative of pressure on the surface of box-shaped drainage volume $\Gamma = \partial\Omega$.

The reservoir pressure is at initial condition,

$$P|_{t=0} = P_i \tag{3}$$

The geometric average permeability, k_a, is:

$$k_a = \sqrt[3]{k_x k_y k_z} \tag{4}$$

The vertical and horizontal permeabilities are respectively

$$k_v = k_z$$
$$k_h = (k_x k_y)^{1/2} \tag{5}$$

Let β be the permeability anisotropic factor

$$\beta = \sqrt{k_h / k_v} \tag{6}$$

And η be the partially penetration factor

$$\eta = L/(2a) \tag{7}$$

2.2. Horizontal well productivity

A horizontal well flow rate in a sealed box-shaped reservoir in pseudo-steady state can be calculated from:

$$q_w = \frac{0.001127(k_y k_z)^{1/2} L(P_a - P_w)/(\mu B)}{\Lambda + s_p} \tag{8}$$

where P_a is average reservoir pressure throughout the box-shaped drainage volume, and,

$$\Lambda = \left(\frac{b_D}{6H_D}\right) - \left(\frac{1}{2\pi}\right)\ln\{4\sin(\pi z_{wD}/H_D)\sin[\pi r_{wD}/(2H_D)]\} \tag{9}$$

The pseudo skin factor due to partial penetration s_p is:

$$\begin{aligned} s_p &= \left(\frac{a_D L_D}{12 b_D H_D}\right)\left(1 - \frac{L_D}{a_D} + \frac{L_D^2}{4a_D^2}\right) + \left(\frac{b_D^2}{H_D L_D \pi^3}\right) \\ &\times \left\{\sum_{m=1}^{M}\left(\frac{1}{m^3}\right)\left\{\frac{\cosh[(a_D m\pi/b_D)(1-L_D/a_D)]}{\sinh(a_D m\pi/b_D)} - \coth(a_D m\pi/b_D)\right\}\right\} \\ &+ \left\{\left(\frac{a_D^3}{b_D H_D L_D \pi^3}\right)\sum_{n=1}^{M}\cos^2(n\pi z_{wD}/H_D) \right. \\ &\times \left. \sum_{m=0}^{M}\left(\frac{1}{d_m \mu_{mn}^3}\right)\left\{\frac{\cosh[(\mu_{mn}\pi)(1-L_D/a_D)]}{\sinh(\mu_{mn}\pi)} - \coth(\mu_{mn}\pi)\right\}\right\} \end{aligned} \tag{10}$$

It has been shown that in Equation (10), the integer number $M = 100$ is sufficiently big enough to reach the engineering accuracy for all practical purpose, and,

$$m = 0, d_m = 1; m > 0, d_m = 1/2 \tag{11}$$

$$\mu_{mn} = [(m a_D / b_D)^2 + (n a_D / H_D)^2]^{1/2} \tag{12}$$

For a fully penetrating horizontal well, $L = 2a$, then

$$s_p = 0 \tag{13}$$

And Equation (8) reduces to,

$$q_w = \frac{0.001127(k_y k_z)^{1/2} L(P_a - P_w)/(\mu B)}{\Lambda} \tag{14}$$

where Λ has the same meaning as in Equation (9). For a given well in a closed box, the flow rate reaches the maximum value when it is located at the middle height of the payzone. In order to calculate s_p we may separate Equation (10) into three parts

$$s_p = \Psi_1 + \Psi_2 + \Psi_3 \tag{15}$$

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where

$$\Psi_1 = \left(\frac{a_D L_D}{12 b_D H_D}\right)\left(1 - \frac{L_D}{a_D} + \frac{L_D^2}{4a_D^2}\right) \quad (16)$$

$$\Psi_2 = \left(\frac{b_D^2}{H_D L_D \pi^3}\right)\left\{\sum_{m=1}^{M}\left(\frac{1}{m^3}\right)\left\{\frac{\cosh[(a_D m\pi / b_D)(1-L_D/a_D)]}{\sinh(a_D m\pi / b_D)} - \coth(a_D m\pi / b_D)\right\}\right\} \quad (17)$$

$$\Psi_3 = \left(\frac{a_D^3}{b_D H_D L_D \pi^3}\right)\left\{\sum_{n=1}^{M}\cos^2(n\pi z_{wD} / H_D) \times \sum_{m=0}^{M}\left(\frac{1}{d_m \mu_{mn}^3}\right)\left\{\frac{\cosh[(\mu_{mn}\pi)(1-L_D/a_D)]}{\sinh(\mu_{mn}\pi)} - \coth(\mu_{mn}\pi)\right\}\right\} \quad (18)$$

3. EXAMPLES AND EFFECTS OF SOME CRITICAL PARAMETERS

Examples are given below to compare the proposed equation with the equations of Babu and Odeh (1989) and Helmy and Wattenberger (1998). These equations are also used to investigate the effects of some critical parameters on productivity index (PI).

3.1. Example-1

Equation (8) is used to calculate productivity index of a horizontal well in pseudo-steady state in a sealed box-shaped drainage volume. Using the reservoir and fluid properties data reported in Table-1, we have

a = 1000 ft, b = 2000 ft

Values of k_a = 125.99 md, β = 2 and η = 0.5 were obtained using Equations (4), (6) and (7), respectively.

Table-1. Reservoir and fluid properties for example-1.

Parameter	Value
Pay zone thickness, H, ft	100
Reservoir length, *RESL*, ft	4000
Reservoir width, *RESW*, ft	2000
Well vertical location, z_w, ft	50
Wellbore radius, r_w, ft	0.25
Permeability in X direction, k_x, md	200
Permeability in Y direction, k_y, md	200
Permeability in Z direction, k_z, md	50
Oil viscosity, μ, cp	1.0
Formation volume factor, B, rb/STB	1.0

With the definition of the dimensionless parameters given in the appendix, the following information is obtained:

$L_D = a_D = 0.7937$
$b_D = 1.5874$
$z_{wD} = 0.07937$
$h_D = 0.15874$
$a_D = L_D$, $1 - L_D / a_D = 0$
a_D / L_D,=0.5, $z_{wD} / h_D = 0.5$

Application of Equation (12) yields:

$$\mu_{mn} = \sqrt{m^2 / 4 + 25n^2}$$

Equation (9) allows to estimate a value of Λ = 2.392. Also, Equations (16) through (18) are used to respectively calculate Ψ_1=0.00521, Ψ_2=-0.54606 and Ψ_3=0.00495. The partial penetration skin was calculated with Equation (10) to be -0.5458. Finally, the productivity index of this well, i.e., the flow rate per unit pressure drop, is calculated as 59.54 (STB/D/psi).

It is interesting to find that s_p is negative in the above calculations. The denominator of Equation (8) for a partially penetrating well is smaller than the denominator for a fully penetrating well, (s_p=0)). But the flow rate of the partially penetrating well still decreases due to smaller well length L in the numerator of Equation (8).

3.2. Example-2

In this example, only fluid properties, reservoir width and length are constants, other reservoir parameters are variables. The effects of some critical parameters on productivity index are investigated here.

Table-2. Reservoir and fluid properties for example-2.

Parameter	Value
Reservoir length, *RESL*, ft	4000
Reservoir width, *RESW*, ft	2000
Wellbore radius, r_w, ft	0.25
Oil viscosity, μ, cp	1.0
Formation volume factor, B, rb/STB	1.0

a) Effect of pay zone thickness on *PI*

Table-3 and Figure-2 presents the effect of payzone thickness on *PI* calculated by the proposed Equation (8) and Babu and Odeh (1989) and Helmy and the equations proposed by Babu and Odeh (1989) and Helmy and Wattenberger (1998), with the following constant parameters:
L = 1000 ft, $k_x = k_y = k_z$ = 200 md, $k_z = k_v$ = 50 md and the horizontal well is at mid-height of the payzone, $z_w = h/2$.

Table-3 and Figure-2 show the existence of no significant differences among the *PIs* calculated by the three equations. If the payzone thickness h is not very big (h<50 ft), *PI* increases rapidly with the increasing of h, but when h>50 ft, *PI* increases slowly with the increasing of h. From h = 10 to 50 ft, *PI* increases about 4 times; from h= 50 to 100 ft, *PI* increases about 1.5 times. For this reason, horizontal wells are believed to perform better than their vertical counterparts in thin reservoirs.

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Table-3. Effect of payzone thickness on *PI.*

	Method		
***PI*, STB/psi**	**(1)**	**This work**	**(2)**
H=10 ft	8.425	9.325	9.609
H=20 ft	16.161	17.799	17.358
H=30 ft	23.156	25.376	23.886
H=40 ft	29.448	32.115	29.527
H=50 ft	35.096	38.101	34.490
H=60 ft	40.165	43.421	38.971
H=70 ft	44.717	48.156	42.911
H=80 ft	48.811	52.379	46.546
H=90 ft	52.499	56.155	49.880
H=100 ft	55.828	59.538	52.958

(1) Babu and Odeh
(2) Helmy and Wattenberger

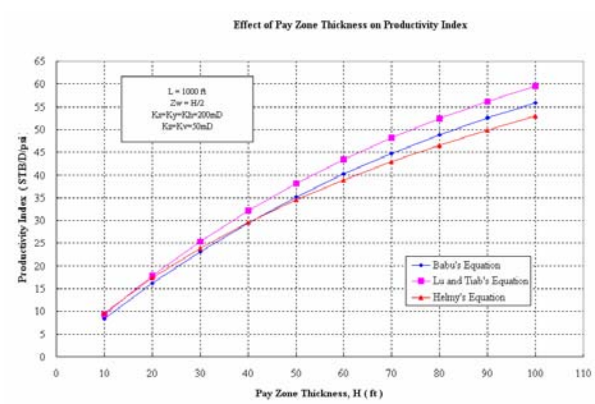

Figure-2. Effect of payzone thickness on *PI.*

b) Effect of well location in vertical direction on *PI*

Table-4 and Figure-3 present the effect of well location in vertical direction on *PI*, with the following constant parameters:

L = 1000 ft, k_x= k_y= k_z = 200 md, k_z = k_v = 50 md, h = 100 ft.

Table-4. Effect of well location in vertical direction on *PI.*

	Method		
***PI*, STB/psi**	**(1)**	**This work**	**(2)**
H=10 ft	51.097	54.453	50.807
H=20 ft	53.583	57.156	51.962
H=30 ft	54.910	58.573	52.556
H=40 ft	55.608	59.308	52.862
H=50 ft	55.828	59.538	52.958
H=60 ft	55.608	59.308	52.862
H=70 ft	54.910	58.573	52.556
H=80 ft	53.583	57.156	51.962
H=90 ft	51.097	54.453	50.807

(1) Babu and Odeh
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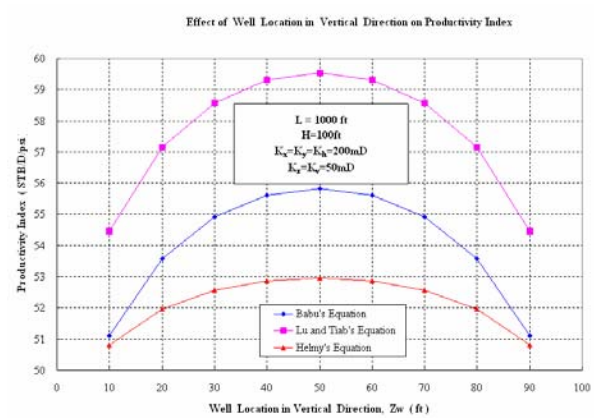

Figure-3. Effect of well location in vertical direction on *PI.*

Table-4 and Figure-3 show that the location of horizontal well in vertical direction does not have significant effect on *PI*. For maximum *PI*, the well should be located at the mid-height the pay zone.

c) Effect of Well Length on *PI*

Table-5 and Figure-4 present the effect of well length on *PI*, with the following constant parameters:

L = 1000 ft, k_x= k_y= k_z = 200 md, k_z = k_v = 50 md, h = 100 ft, z_w = 50 ft.

Table-5. Effect of well length on *PI.*

	Method		
***PI*, STB/psi**	**(1)**	***PI*, STB/psi**	**(2)**
L=250 ft, □=0.125	21.222	22.342	17.994
L=500 ft, □=0.250	34.306	36.214	30.472
L=750 ft, □=0.375	45.444	48.277	41.800
L=1000 ft, □=0.500	55.828	59.538	52.958
L=1250 ft, □=0.625	65.870	70.183	64.189
L=1500 ft, □=0.750	75.710	79.994	75.278
L=1750 ft, □=0.875	85.337	88.428	85.608
L=2000 ft, □=1.000	94.637	94.335	94.259

(1) Babu and Odeh
(2) Helmy and Wattenberger

Table-5 and Figure-4 show that no significant differences among the *PI* values calculated by the three equations. *PI* increases slowly with the increasing *L*. For every increase in *L* of 250 ft, *PI* only increases about 10 STB/D/psi. From 250 to 1000 ft, *PI* increases 4 times.

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From 1000 to 2000 ft, *PI* only increases about 1.6 times. So *PI* is not sensitive to *L* and η.

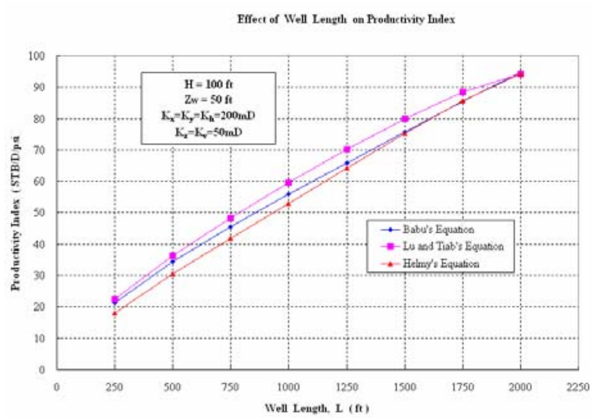

Figure-4. Effect of well length on *PI.*

Table-6 presents the pseudo skin factor s_p due to partial penetration calculated by the Equations (15) through (18). For a partially penetrating horizontal well, Ψ_1 is positive while Ψ_2 and Ψ_3 are negative.

From Table-6 we find that when η increases, Ψ_1 decrease, the absolute values of Ψ_2 and Ψ_3 also decreases. When $\eta \geq 0.25$, the absolute value of Ψ_3 is very small compared with Ψ_1 and the absolute value of Ψ_2. So, in many cases, Ψ_3 are negligible when s_p is calculated.

Table-6. Pseudo skin factor calculations.

	Ψ_1	Ψ_2	Ψ_3	Ψ_4
L=250 ft, □=0.125	0.03988	- 1.15038	- 0.02031	- 1.13080
L=500 ft, □=0.250	0.05859	- 0.88431	- 0.01015	- 0.83587
L=750 ft, □=0.375	0.06104	- 0.69534	- 0.00677	- 0.64107
L=1000 ft, □=0.500	0.05208	- 0.54607	- 0.00500	- 0.49898
L=1250 ft, □=0.625	0.03662	- 0.41720	- 0.00406	- 0.38464
L=1500 ft, □=0.750	0.01953	- 0.29477	- 0.00338	- 0.27862
L=1750 ft, □=0.875	0.00570	- 0.16434	- 0.00291	- 0.16155
L=2000 ft, □=1.000	0.00000	0.00000	0.00000	0.00000

d) Effect of horizontal permeability on *PI*

Table-7 and Figure-5 present the effect of horizontal permeability (k_x= k_y= k_z) on *PI*, with the following constant parameters:

L = 1000 ft, k_x= k_y= k_h, k_z = k_v = 25 md, h = 100 ft, z_w = 50 ft.

Table-7 and Figure-5 show that *PI* is a strong function of horizontal permeability (assuming k_x= k_y= k_h). Horizontal permeability increases 8 times, while β only increases 2.8 times and PI calculated by the three equations increase about 5.5 times.

Table-7. Effect of horizontal permeability on *PI.*

	Method		
***PI*, STB/psi**	**(1)**	***PI*, STB/psi**	**(1)**
k_h=25 md, β=1.000	8.626	9.379	8.562
k_h=50 md, β=1.414	15.744	16.952	15.193
k_h=75 md, β=1.732	22.094	23.653	21.071
k_h=100 md, β=2.000	27.914	29.769	26.479
k_h=125 md, β=2.236	33.333	35.451	31.550
k_h=150 md, β=2.449	38.431	40.790	36.361
k_h=175 md, β=2.646	43.264	45.848	40.963
k_h=200 md, β=2.828	47.870	50.669	45.390

(1) Babu and Odeh
(2) Helmy and Wattenberger

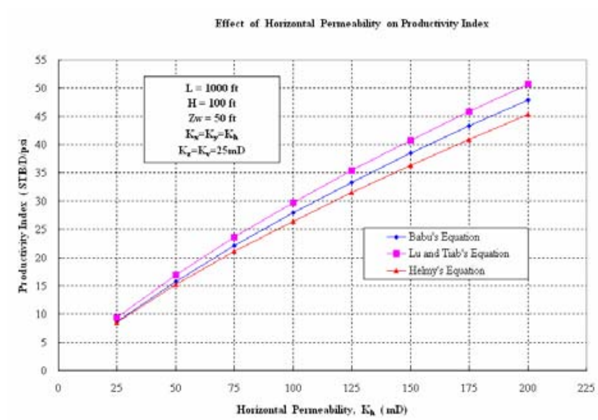

Figure-5. Effect of horizontal permeability on *PI*, (k_x= k_y= k_h).

e) Effect of vertical permeability on *PI*

Table-8 and Figure-6 present the effect of vertical permeability on PI, with the following constant parameters:

L = 1000 ft, k_x = k_y = k_z = 100 md, k_z = k_v = 50 md, h = 100 ft, z_w = 50 ft.

Table-8 and Figure-6 show that PI is a weak function of vertical permeability, vertical permeability increases 10 times, (β decreases about 3 times) *PIs* calculated by the three equations increase about 1.6 times.

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Table-8. Effect of vertical permeability on *PI.*

	Method		
***PI*, STB/psi**	**(1)**	***PI*, STB/psi**	**(2)**
k_v=10 md, β=3.162	22.609	23.890	21.528
k_v=20 md, β=2.236	26.666	28.361	25.240
k_v=30 md, β=1.826	28.902	30.897	27.501
k_v=40 md, β=1.581	30.394	32.622	29.124
k_v=50 md, β=1.414	31.489	33.905	30.385
k_v=60 md, β=1.291	32.340	34.913	31.412
k_v=70 md, β=1.195	33.028	35.733	32.276
k_v=80 md, β=1.118	33.599	36.420	33.020
k_v=90 md, β=3.162	34.085	37.007	33.671
k_v=100 md, β=1.000	34.504	37.516	34.248

(1) Babu and Odeh
(2) Helmy and Wattenberger

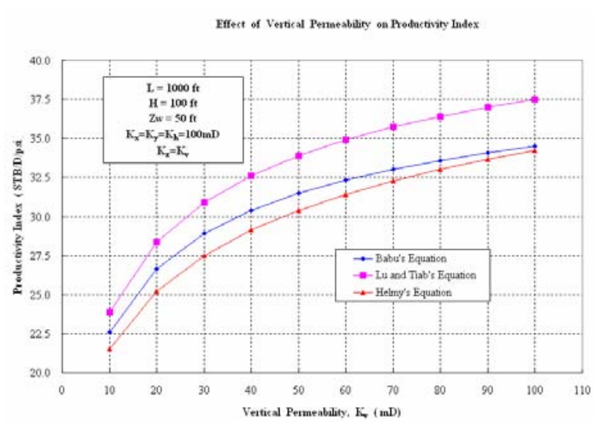

Figure-6. Effect of vertical permeability on *PI.*

f) Effect of permeability perpendicular to horizontal well on *PI*

Table-9 and Figure-7 present the effect of permeability perpendicular to the well in horizontal plane (k_y) on *PI*, with the following constant parameters:

L = 1000 ft, k_x = 50 md, $k_z = k_v$ = 25 md, h = 100 ft, z_w = 50 ft.

Table-9 and Figure-7 show that *PI* is a strong function of k_y when $k_y \leq 25$ md. From 5 to 25 md, k_y increases 5 times, and *PI* calculated by the three equations also increase about 5 times; but *PI* is a weak function of k_y when $k_y > 25$ md. From 25 to 250 md, k_y increases 10 times, but *PI* values calculated by the three equations only increase about 5 times.

Table-9. Effect of permeability perpendicular to horizontal well on *PI.*

	Method		
***PI*, STB/psi**	**(1)**	***PI*, STB/psi**	**(2)**
k_y=5 md, β=0.795	2.527	2.609	2.425
k_y =15 md, β=1.047	6.316	6.626	6.035
k_y =25 md, β=1.189	9.420	9.979	9.021
k_y =50 md, β=1.414	15.744	16.950	15.193
k_y =75 md, β=1.565	20.938	22.787	20.339
k_y =100 md, β=1.682	25.459	27.940	24.868
k_y =125 md, β=1.778	29.518	32.614	28.966
k_y =150 md, β=1.861	33.233	36.926	32.739
k_y =175 md, β=1.934	36.678	40.950	36.255
k_y =200 md, β=2.000	39.905	44.736	39.560
k_y =225 md, β=2.060	42.949	48.322	42.687
k_y =250 md, β=2.115	45.838	51.734	45.662

(1) Babu and Odeh
(2) Helmy and Wattenberger

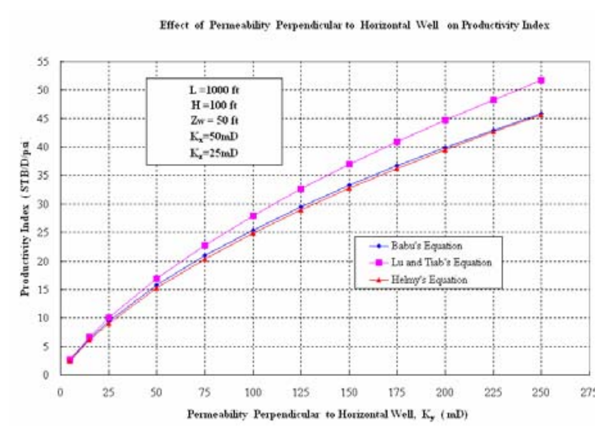

Figure-7. Effect of k_y on *PI.*

g) Effect of permeability parallel to horizontal well on *PI*

Table-10 and Figure-8 present the effect of permeability parallel to horizontal well (k_x) on *PI*, with the following constant parameters:

L = 1000 ft, k_y = 50 md, k_z = 25 md, h = 100 ft, z_w = 50 ft.
Table-10 and Figure-8 show that *PI* is a weak function of k_x. When k_x increases 50 times, (β increases about 3 times) *PI* values calculated by the three equations increases about 1.6 times.

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Table-10. Effect of permeability parallel to horizontal well on *PI*.

	Method		
***PI*, STB/psi**	**(1)**	***PI*, STB/psi**	**(2)**
k_y=5 md, β=0.795	9.606	11.773	10.009
k_y =15 md, β=1.047	12.939	14.660	13.055
k_y =25 md, β=1.189	14.200	15.736	14.122
k_y =50 md, β=1.414	15.744	16.950	15.193
k_y =75 md, β=1.565	16.542	17.531	15.641
k_y =100 md, β=1.682	17.056	17.889	15.894
k_y =125 md, β=1.778	17.426	18.136	16.057
k_y =150 md, β=1.861	17.710	18.320	16.171
k_y =175 md, β=1.934	17.937	18.463	16.256
k_y =200 md, β=2.000	18.124	18.579	16.322
k_y =225 md, β=2.060	18.282	18.674	16.374
k_y =250 md, β=2.115	18.418	18.755	16.417

(1) Babu and Odeh
(2) Helmy and Wattenberger

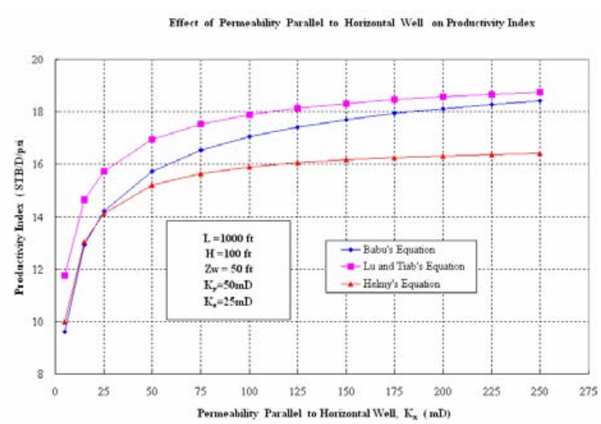

Figure-8. Effect of k_x on *PI*.

3.3. Example-3

In this example, the effect of reservoir size on productivity index (*PI*) is investigated. Fluid properties and other reservoir parameters are the same as in Table-1.

h) Effect of reservoir width on PI

Table-11 and Figure-9 present the effect of reservoir width (*RESW*, the size of reservoir parallel to horizontal well) on *PI*, with the following constant parameters:

L = 1000 ft, $k_x = k_z = k_h$ = 200 md, $k_z = k_v$ = 50 md h = 100 ft, z_w = 50 ft.

And reservoir length (*RESL*) is a constant, and it is equal t0 4000 ft.

Table-11 and Figure-9 show that *PI* is a weak function of reservoir width (*RESW*), from *RESW*=1000 to 2200 ft, (the partially penetrating factor η drops from 1.0 to 0.455), *PIs* calculated by the three equations increase slowly; from *RESW* = 2200 to 4000 ft, (the partially penetrating factor η drops from 0.455 to 0.250) *PIs* calculated by the three equations decrease slowly.

Table-11. Effect of reservoir width on *PI*.

	Method		
***PI*, STB/psi**	**(1)**	***PI*, STB/psi**	**(2)**
RESW=1000 ft, □=1.00	47.318	47.168	47.129
RESW=1300 ft, □=0.769	52.205	54.073	50.792
RESW=1600 ft, □=0.625	54.646	57.576	52.397
RESW=1900 ft, □=0.526	55.682	59.246	52.926
RESW=2200 ft, □=0.455	55.913	59.857	52.876
RESW=2500 ft, □=0.400	55.682	59.853	52.504
RESW=2800 ft, □=0.357	55.190	59.490	51.953
RESW=3100 ft, □=0.323	54.553	58.921	51.301
RESW=3400 ft, □=0.294	53.842	58.234	50.596
RESW=3700 ft, □=0.270	53.099	57.486	49.865
RESW=4000 ft, □=0.250	52.349	56.707	49.125

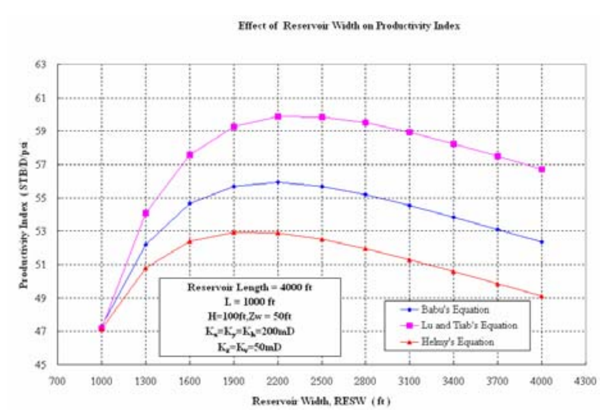

Figure-9. Effect of reservoir width on *PI*.

Table-12 presents the pseudo skin factor s_p due to partial penetration caused by the different reservoir widths. We obtain the same conclusion as that in Table-6 in Example 2. For a partially penetrating horizontal well, Ψ_1 is positive while Ψ_2 and Ψ_3 are negative. When $\eta \geq$

0.25, the absolute value of Ψ_3 is very small compared with Ψ_1 and the absolute value of Ψ_2.

Table-12. Pseudo skin factor calculations.

***PI*, STB/psi**	Ψ_1	Ψ_2	Ψ_3	s_p
RESW=1000 ft, □=1.000	0.00000	0.00000	0.00000	0.00000
RESW=1300 ft, □=0.769	0.00721	- 0.30983	- 0.00508	- 0.30769
RESW=1600 ft, □=0.625	0.02344	- 0.45286	- 0.00508	- 0.43450
RESW=1900 ft, □=0.526	0.04441	- 0.52898	- 0.00508	- 0.48966
RESW=2200 ft, □=0.455	0.06818	- 0.57218	- 0.00508	- 0.50908
RESW=2500 ft, □=0.400	0.09375	- 0.59761	- 0.00508	- 0.50894
RESW=2800 ft, □=0.357	0.12054	- 0.61292	- 0.00508	- 0.49746
RESW=3100 ft, □=0.323	0.14819	- 0.62226	- 0.00508	- 0.47916
RESW=3400 ft, □=0.294	0.17647	- 0.62801	- 0.00498	- 0.45653
RESW=3700 ft, □=0.270	0.20524	- 0.63157	- 0.00498	- 0.43132
RESW=4000 ft, □=0.250	0.23438	- 0.63378	- 0.00498	- 0.40439

i) Effect of reservoir length on *PI*

Table-13 and Figure-10 present the effect of reservoir length (*RESL*, the size of reservoir perpendicular to horizontal well) on *PI*, with the following constant parameters:

L = 1000 ft, k_x= k_z= k_h = 200 md, k_z = k_v = 50 md H = 100 ft, z_w = 50 ft, *RESW* = 2000 ft.

Table-13. Effect of reservoir length on *PI.*

	Method		
***PI*, STB/psi**	**(1)**	***PI*, STB/psi**	**(1)**
RESL=1000 ft,	78.011	86.885	81.143
RESL=1500 ft,	75.223	81.777	74.390
RESL=2000 ft,	70.336	76.269	68.654
RESL=2500 ft,	66.045	71.288	63.810
RESL=3000 ft,	62.248	66.889	59.671
RESL=3500 ft,	58.863	62.994	56.091
RESL=4000 ft,	55.828	59.528	52.958
RESL=4500 ft,	53.090	56.423	50.188
RESL=5000 ft,	50.608	53.625	47.720

(1) Babu and Odeh
(2) Helmy and Wattenberger

Table-13 and Figure-10 show that *PI* is a weak function of reservoir length (*RESL*).

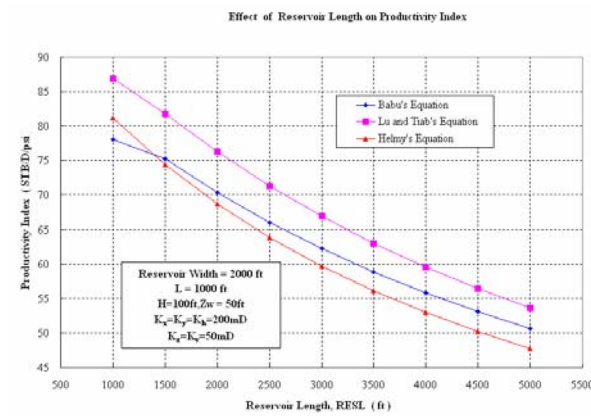

Figure-10. Effect of reservoir length on *PI.*

CONCLUSIONS

a) The proposed productivity equation for a horizontal well in a sealed box-shaped drainage system in pseudo-steady state is based on solving three dimensional partial differential equations, and has a solid mathematical base in the theory of flow in porous media; it is reliable and easy to use in oil field.
b) The horizontal well productivity is a weak function of well length, well location, vertical permeability, the permeability parallel to the well in the horizontal plane, and reservoir size. But it is a strong function of pay zone thickness, the permeability perpendicular to the well in the horizontal plane. For maximum productivity, the horizontal well should be located at the mid-height of the pay zone.

Nomenclature

B	Volumetric factor, RB/STB
c_f	Fluid compressibility, 1/psi
h	Formation thickness, ft
k	Permeability, md
L	Horizontal wellbore length, ft
P	Pressure, psi
PI	Well productivity index, STB/D/psi
q_w	Well flow rate, STB/D
r_w	Wellbore radius, hr
RESL	Reservoir length, ft
RESW	Reservoir width, ft
s_p	Pseudoskin facfor due to partial penetration, dimensionless

Greeks

β	Permeability anisotropic factor, fraction
η	Partially penetrating factor, fraction
μ	Fluid viscosity, cp
ϕ	Porosity, fraction
Λ	A function defined by Equation (8)
Ψ_1	A function defined by Equation (16)
Ψ_2	A function defined by Equation (17)
Ψ_3	A function defined by Equation (18)

Suffices

a	Average
D	Dimensionless
e	External
h	Horizontal
i	Initial
v	Vertical
w	Wellbore
x,y,z	Coordinate indicators

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APPENDIX

Definition of dimensionless variables

$$x_D = \frac{x}{L}\sqrt{\frac{k_a}{k_x}}$$
$$y_D = \frac{y}{L}\sqrt{\frac{k_a}{k_y}} \qquad \text{(A.1)}$$
$$z_D = \frac{z}{L}\sqrt{\frac{k_a}{k_z}}$$

$$a_D = \frac{a}{L}\sqrt{\frac{k_a}{k_x}}$$
$$b_D = \frac{b}{L}\sqrt{\frac{k_a}{k_y}} \qquad \text{(A.2)}$$
$$z_{wD} = \frac{z_w}{L}\sqrt{\frac{k_a}{k_z}}$$

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$$L_D = \sqrt{\frac{k_a}{k_x}} \qquad H_D = \frac{H}{L}\sqrt{\frac{k_a}{k_z}} \qquad \text{(A.3)}$$

$$t_D = \frac{k_a t}{\phi \mu c_t L^2} \qquad \text{(A.4)}$$

$$P_D = \frac{k_a L(P_i - P)}{q \mu B} \qquad \text{(A.5)}$$

$$r_{wD} = [\sqrt[4]{k_x / k_z} + \sqrt[4]{k_z / k_x})] r_w / (2L) \qquad \text{(A.6)}$$