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PRESSURE DRAWDOWN EQUATIONS FOR MULTIPLE-WELL SYSTEMS IN CIRCULAR-CYLINDRICAL RESERVOIRS

Jing Lu[1], Tao Zhu[1], Anh Dinh[2], Djebbar Tiab[3] and Freddy H. Escobar[4]
[1]Department of Petroleum Engineering, the Petroleum Institute, Abu Dhabi, United Arab Emirates
[2]Schlumbeger, Midland, TX
[3]The University of Oklahoma, Norman, OK, USA
[4]Universidad Surcolombiana/CENIGAA, Av. Pastrana-Cra 1, Neiva, Huila, Colombia
E-Mail: fescobar@usco.edu.co

ABSTRACT

Currently, in the oil industry, a field usually contains several wells producing from the same drainage domain, and each well will have an effect on the pressure at other wells. For an infinite-acting multiple-wells system, pressure drawdown equation is already obtained by using superposition principle. This paper presents pressure drawdown equations of a multiple-wells system in a circular cylindrical reservoir with constant pressure outer boundary. The proposed equations provide fast analytical tools to evaluate the performance of multiple wells, which are located arbitrarily in a circular cylinder reservoir, and are producing at different rates. Here, it is also examined the pressure drawdown response of a specific well located in a system of producing wells. The interference effects of nearby producing wells on the pressure drawdown response are investigated. The proposed equations are illustrated by numerical examples. It is concluded that, for a given multiple-wells system in a circular cylindrical reservoir, well pattern, well spacing, skin factor, flow rates and well off-center distances have significant effects on single well pressure drawdown behavior. Because the reservoir is under edge water drive, the outer boundary is at constant pressure, when producing time is sufficiently long; steady-state is definitely reached.

Keywords: multiple-wells, superposition principle, exponential function, reservoir shape, pressure drop.

1. INTRODUCTION

It is rare to find a reservoir being produced from only a single well. A field usually contains several wells producing from the same drainage domain, and each well will have an effect on the pressure at other wells. If we have one well producing at a constant rate, the flowing bottomhole pressure in that well is a function of its own production as well as the production from surrounding wells. For an infinite-acting multiple-wells system, pressure drawdown equation is already obtained by using superposition principle (Lee, *et al*., 2003).

It is not unusual during drilling, completion, or workover operations for materials such as mud filtrate, cement slurry, or clay particles to enter the formation and reduce the permeability around the wellbore. This effect is commonly referred to as "wellbore damage" and the region of altered permeability is called the "skin zone." This zone can extend from a few inches to several feet from the wellbore and causes an additional pressure loss in the formation. Because the radius of skin zone is small, we can always assume steady-state radial flow in this altered zone and the steady-state radial flow equations for the pressure drops are applicable in this region.

As given by Lee, Rollins, and Spivey, (2003), if we refer to the pressure drop in the skin zone as ΔP_S, it yields,

$$\Delta P_s = \frac{q\mu B}{2\pi k_r h} s \tag{1}$$

If a fully penetrating off-center vertical well is located in an anisotropic circular cylinder reservoir with constant pressure outer boundary, and the off-center distance is r_o, the drainage radius is r_e, in the constant pressure outer boundary dominated flow regime, the dimensionless flowing bottom hole pressure is given by Owayed and Lu (2009):

$$P_{wD} = 2\left\{ \begin{array}{l} Ei-\left(\dfrac{r_{eD}^2 - r_{oD}^2}{4t_D}\right) - Ei\left(\dfrac{-r_{wD}^2}{4t_D}\right) \\ \quad -\left(\dfrac{2r_{oD}^2 - r_{eD}^2}{r_{oD}^2}\right)[\exp\left(\dfrac{r_{oD}^2 - r_{eD}^2}{4t_D}\right) \\ \quad -\exp\left(\dfrac{-r_{eD}^2}{4t_D}\right) \end{array} \right\} + 4s \tag{2}$$

Where Ei is exponential integral function, S is mechanical skin factor. The definitions of dimensionless variables are given in Appendix A.

Rearrange Equation (2), the flowing bottomhole pressure drawdown equation is obtained,

$$P_{ini} - P_w = \frac{q\mu B}{4\pi k_r h}\left\{ \begin{array}{l} Ei\left(-\dfrac{r_{eD}^2 - r_{oD}^2}{4t_D}\right) - Ei\left(\dfrac{-r_{wD}^2}{4t_D}\right) - \left(\dfrac{2r_{oD}^2 - r_{eD}^2}{r_{oD}^2}\right) \\ \left[\exp\left(\dfrac{r_{oD}^2 - r_{eD}^2}{4t_D}\right) - \exp\left(\dfrac{-r_{eD}^2}{4t_D}\right)\right] + 2s \end{array} \right\} \tag{3}$$

where Equation (1) is used, and P_{ini} is initial reservoir pressure.

The goal of this study is to present pressure drawdown equations of a multiple-wells system in a circular cylinder reservoir with constant pressure outer boundary. Taking fully penetrating vertical wells as uniform line sinks, reasonably accurate pressure drawdown equations are obtained for a multiple-wells system in the boundary dominated flowing period. The interference effects of nearby producing wells on a specific well response are investigated.

2. WELL AND RESERVOIR MODELS

Figure-1 is a schematic of a multiple-wells system, a number of producing wells drain an anisotropic circular cylinder reservoir with height h and radius r_e, and the j-th well is located r_j away from the center of the drainage circle, in polar coordinate system, the j-th well is located (r_j, θ_j).

The following assumptions are made:

a) At time t = 0, pressure is uniformly distributed in the reservoir, equal to the initial pressure P_{ini}, all wells begin to produce at time t = 0, and the boundary dominated flow is reached at the same time for each well.
b) The wells are parallel to the z direction with a producing length equal to thickness h, the top and bottom reservoir boundaries are impermeable. The outer boundary pressure P_e is always equal to initial pressure P_{ini}.
c) A single phase fluid, of small and constant compressibility c_f, constant viscosity μ, and formation volume factor B, flows from the reservoir to the wells. Fluid properties are independent of pressure. Gravity forces are neglected.
d) Only the mechanical skin factor due to formation damage or stimulation is considered, other additional pressure drops are ignored.
e) The reservoir has constant radial permeability k_r, vertical permeability k_v, and thickness h. In any given time interval, the number of wells, n, their locations (r_j, θ_j), and the skin factors, s_j, are considered constant, and the wellbore radio, R_{wj} are identical for each well.

2.1. Equations for initial condition and boundary conditions

The initial reservoir pressure is constant,

$$P|_{t=0} = P_{ini} \tag{4}$$

And the top and bottom reservoir boundaries are impermeable.

$$\frac{\partial P}{\partial z}\Big|_{z=0} = 0, \frac{\partial P}{\partial z}\Big|_{z=h} = 0 \tag{5}$$

The outer boundary pressure is always equal to initial reservoir pressure P_{ini}.

$$P(r = r_e) = P_{ini} \tag{6}$$

2.2. Dimensionless equations for multiple-wells system

In a multiple-wells system as shown in Figure-1, the number of wells is n, assume the wellbore radii are identical, then the dimensionless flowing bottomhole pressure of the observation well (assume it is k-th well) is

$$P_{wD,k} = \sum_{j=1}^{n} W_{kj}, \qquad (k = 1,2,3,...,n) \tag{7}$$

where

$$\tau_j = \frac{q_j}{q_{ref}}, \qquad (j = 1,2,3,...,n) \tag{8}$$

When $j = k$, then

$$W_{kk} = 2\tau_k \left\{ \frac{Ei\left(-\frac{r_{eD}^2 - r_{kD}^2}{4t_D}\right) - Ei\left(\frac{-r_{wD}^2}{4t_D}\right) - \left(\frac{2r_{kD}^2 - r_{eD}^2}{r_{kD}^2}\right)}{\left[\exp\left(\frac{r_{kD}^2 - r_{eD}^2}{4t_D}\right) - \exp\left(\frac{-r_{eD}^2}{4t_D}\right)\right]} \right\} + 4s_k \tag{9}$$

where s_k is mechanical skin factor of k-th well (the observation well), and

When $j \neq k$, then

$$W_{kj} = \tau_j \left\{ \frac{2 \times RE\left\{Ei\left(\frac{r_{jD}r_{kD}\exp[i(\theta_j - \theta_k)] - r_{eD}^2}{4t_D}\right)\right\} - \left(\exp\left(-\frac{r_{eD}^2}{4t_D}\right)\Big/ 2t_D h_D \chi\right)(r_{jD}^2 + r_{kD}^2 - r_{eD}^2) \times RE\left\{\frac{\exp[\chi \times \exp[i(\theta_j - \theta_k)]] - 1}{\exp[i(\theta_j - \theta_k)]}\right\} - }{2Ei\left\{-\frac{[r_{jD}^2 + r_{kD}^2 - 2r_{jD}r_{kD}\cos(\theta_j - \theta_k)]}{4t_D}\right\}} \right\} \tag{10}$$

Where $i = \sqrt{-1}$, θ_j is the wellbore location angle of j-th well, r_j is off - center distance of j-th well, RE is real part operator, (if a_1, a_2, b_1, b_2 are real numbers, then $z = (a_1 + b_1 i)(a_2 + b_2 i)$ is a complex number, $RE(z) = a_1 a_2 - b_1 b_2$) and

$$\chi = \frac{R_{jD}R_{kD}}{4t_D} \tag{11}$$

The definitions of dimensionless variables in the above and following equations are the same as given in Appendix-A.

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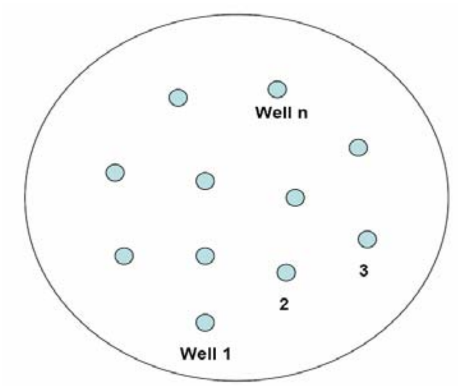

Figure-1. Multiple-wells system.

At early time, (t_D □ 5) the flowing bottomhole pressure of the observation well is only a function of its own production as if surrounding wells do not exist; and because

$$Ei\left(-\frac{r_{eD}^2 - r_{kD}^2}{4t_D}\right) \approx 0, \qquad (12)$$
$$\left(\frac{2r_{kD}^2 - r_{eD}^2}{r_{kD}^2}\right)\left[\exp\left(\frac{r_{kD}^2 - r_{eD}^2}{4t_D}\right) - \exp\left(\frac{-r_{eD}^2}{4t_D}\right)\right] \approx 0$$

Thus, Equation (7) can be simplified below:

$$P_{wD,k} = -2\tau_k Ei\left(-\frac{r_{wD}^2}{4t_D}\right) + 4s_k \approx 2\tau_k\left[\ln(t_D) + \ln\left(\frac{4}{1.781R_{wD}^2}\right)\right] + \qquad (13)$$
$$4s_k = 2\tau_k\left[\ln(t_D) - 2\ln(r_{wD}) + 0.8091\right] + 4s_k$$

If $\tau_k = 1$, the dimensionless pressure derivative at early time is,

$$\frac{dP_{wD,k}}{d(\ln t_D)} = t_D\frac{dP_{wD,k}}{dt_D} = 2 \qquad (14)$$

If all wells have the identical off-center distance, and if $n \geq 3$, the wells are located at vertexes of a regular polygon inside a circular reservoir, as shown in Figure-2, we then assume:

$$r_j = r_o, \qquad (j = 1,2,3,...,n) \qquad (15)$$

Thus when $j = k$, Equation (9) reduces to

$$W_{kk} = 2\tau_k\left\{\begin{matrix} Ei\left(-\frac{r_{eD}^2 - r_{oD}^2}{4t_D}\right) - Ei\left(\frac{-r_{wD}^2}{4t_D}\right) - \left(\frac{2r_{oD}^2 - r_{eD}^2}{r_{oD}^2}\right) \\ \left[\exp(\frac{r_{oD}^2 - r_{eD}^2}{4t_D}) - \exp\left(\frac{-r_{eD}^2}{4t_D}\right)\right]\end{matrix}\right\} + 4s_k \qquad (16)$$

And when $j \neq k$, Equation (10) reduces to

$$W_{kj} = 2\tau_j\left\{\begin{matrix} RE\left\{Ei\left(\frac{r_{oD}^2\exp[i(\theta_j - \theta_k)] - r_{eD}^2}{4t_D}\right)\right\} - \\ Ei\left(\frac{r_{oD}^2\cos(\theta_j - \theta_k) - r_{oD}^2}{2t_D}\right) \\ -\left(\frac{2r_{oD}^2 - r_{eD}^2}{r_{oD}^2}\right)\exp\left(-\frac{r_{eD}^2}{4t_D}\right)\left\{\begin{matrix}\exp\left[\left(\frac{r_{oD}^2}{4t_D}\right)\cos(\theta_j - \theta_k)\right] \\ \times\cos[(\theta_j - \theta_k) - \\ \frac{r_{oD}^2}{4t_D}\sin(\theta_j - \theta_k)] - \cos(\theta_j - \theta_k)\end{matrix}\right\}\end{matrix}\right\} \qquad (17)$$

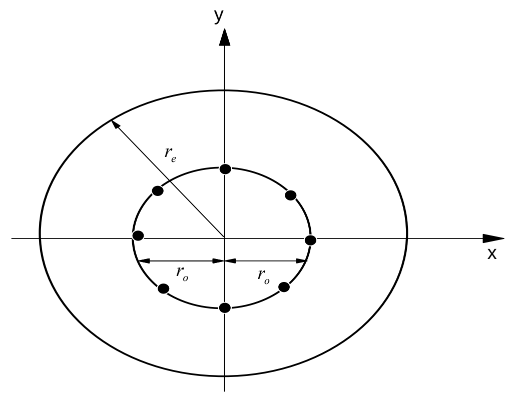

Figure-2. Multiple wells located at vertexes of a regular polygon.

2.3. Dimensionless equations for two-wells system

If only two wells are located in the reservoir with identical off-center distance r_o, without losing generality, we may always assume the first well is the observation well, which is located at (r_o, 0); the second well is located at (r_o, θ), then

$$P_{wD,1} = 2\tau_1\left\{\begin{matrix} Ei\left(\frac{r_{eD}^2 - r_{oD}^2}{4t_D}\right) - Ei\left(-\frac{r_{wD}^2}{4t_D}\right) - \\ \frac{2r_{oD}^2 - r_{eD}^2}{r_{oD}^2}\left[\begin{matrix}\exp\left(\frac{r_{oD}^2 - r_{eD}^2}{4t_D}\right) - \\ \exp\left(\frac{-r_{eD}^2}{4t_D}\right)\end{matrix}\right]\end{matrix}\right\} + 4s_1$$
$$+2\tau_2\left\{\begin{matrix} RE\left[Ei\left(\frac{r_{oD}^2\exp(i\theta) - r_{eD}^2}{4t_D}\right)\right] - Ei\left(\frac{r_{oD}^2\cos(\theta) - r_{oD}^2}{2t_D}\right) \\ -\frac{2r_{oD}^2 - r_{eD}^2}{r_{oD}^2} \\ \exp\left(-\frac{r_{eD}^2}{4t_D}\right)\left\{\begin{matrix}\exp\left[\left(\frac{r_{oD}^2}{4t_D}\right)\cos(\theta)\right] \\ \times\cos\left[\theta - \left(\frac{r_{oD}^2}{4t_D}\right)\sin(\theta)\right] - \cos(\theta)\end{matrix}\right\}\end{matrix}\right\} \qquad (18)$$

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Where s_1 is the mechanical skin factor of the first well (observation well).

If $\theta = \pi / 2$, then Equation (18) reduces to

$$P_{wD,1} = 2\tau_1 \left\{ \begin{array}{l} Ei\left(-\frac{r_{eD}^2 - r_{oD}^2}{4t_D}\right) - Ei\left(-\frac{r_{wD}^2}{4t_D}\right) \\ -\frac{2r_{oD}^2 - r_{eD}^2}{r_{oD}^2}\left[\exp\left(\frac{r_{oD}^2 - r_{eD}^2}{4t_D}\right) - \exp\left(\frac{-r_{eD}^2}{4t_D}\right)\right] \end{array} \right\} + 4s_1 + 2\tau_2 \left\{ \begin{array}{l} RE\left[Ei\left(\frac{ir_{oD}^2 - r_{eD}^2}{4t_D}\right)\right] - Ei\left(\frac{-r_{oD}^2}{2t_D}\right) \\ -\left(\frac{2r_{oD}^2 - r_{eD}^2}{r_{oD}^2}\right)\exp\left(-\frac{r_{eD}^2}{4t_D}\right)\sin\left(\frac{r_{oD}^2}{4t_D}\right) \end{array} \right\} \quad (19)$$

And if the two wells are located symmetric with respect to the circular center, i.e., the two wells are on a diameter with identical off-center distance r_o, ($\theta = \pi$) as shown in Figure-3, then Equation (18) reduces to

$$P_{wD,1} = 2\tau_1 \left\{ \begin{array}{l} Ei\left(-\frac{r_{eD}^2 - r_{oD}^2}{4t_D}\right) - Ei\left(-\frac{r_{wD}^2}{4t_D}\right) \\ -\frac{2r_{oD}^2 - r_{eD}^2}{r_{oD}^2}\left[\exp\left(\frac{r_{oD}^2 - r_{eD}^2}{4t_D}\right) - \exp\left(\frac{-r_{eD}^2}{4t_D}\right)\right]\} + 4s_1 \\ +2\tau_2\{Ei\left(-\frac{r_{oD}^2 + r_{eD}^2}{4t_D}\right) - Ei\left(\frac{-r_{oD}^2}{t_D}\right) \\ +\frac{2r_{oD}^2 - r_{eD}^2}{r_{oD}^2}\left[\exp\left(-\frac{r_{oD}^2 + r_{eD}^2}{4t_D}\right) - \exp\left(\frac{-r_{eD}^2}{4t_D}\right)\right] \end{array} \right\} \quad (20)$$

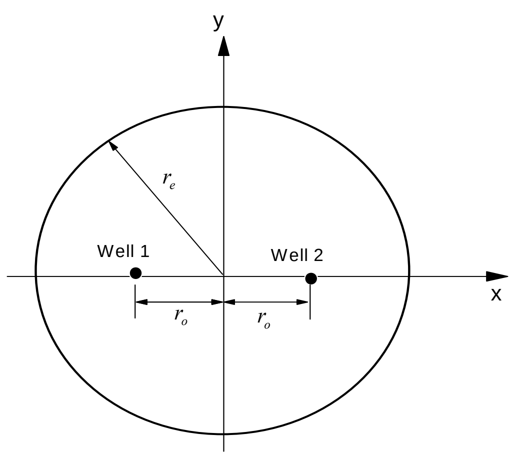

Figure-3. Symmetric two-wells system.

At late time, ($t_D \gg 100$), it is obtained,

$$\begin{array}{l} Ei\left(-\frac{r_{eD}^2 - r_{oD}^2}{4t_D}\right) - Ei\left(-\frac{r_{wD}^2}{4t_D}\right) \approx \ln\left(\frac{r_{eD}^2 - r_{oD}^2}{r_{wD}^2}\right), \\ \frac{2r_{oD}^2 - r_{eD}^2}{r_{oD}^2}\left[\exp\left(\frac{r_{oD}^2 - r_{eD}^2}{4t_D}\right) - \exp\left(\frac{-r_{eD}^2}{4t_D}\right)\right] \approx 0 \end{array} \quad (21)$$

$$\begin{array}{l} Ei\left(-\frac{r_{eD}^2 - r_{oD}^2}{4t_D}\right) - Ei\left(-\frac{r_{oD}^2}{4t_D}\right) \approx \ln\left(\frac{r_{eD}^2 - r_{oD}^2}{r_{oD}^2}\right), \\ \frac{2r_{oD}^2 - r_{eD}^2}{r_{oD}^2}\left[\exp\left(\frac{r_{oD}^2 - r_{eD}^2}{4t_D}\right) - \exp\left(\frac{-r_{eD}^2}{4t_D}\right)\right] \approx 0 \end{array} \quad (22)$$

Thus, if $\tau_1 = \tau_2$, when late is very long, Equation (20) reduces to,

$$P_{wD,1} = 2\ln\left(\frac{r_{eD}^4 - r_{oD}^4}{4r_{wD}^2 r_{oD}^2}\right) \quad (23)$$

3. PRESSURE DRAWDOWN ANALYSIS

In each case of the following examples, we always assume all the wells are fully penetrating with identical wellbore radius r_w, have identical off-center distance r_o, and identical mechanical skin factor s, and if $n \geq 3$, the wells are located at vertexes of a regular polygon as shown in Figure-2. In Examples 1, 2 and 4, the well flow rates are identical, thus every well has the same pressure behavior in each case, any well can be chosen as the observation well.

Example-1: $\boldsymbol{r_{wD}} = 0.01$, $r_{oD} = 10$, $r_{eD} = 20$, $s = 2$, $\tau = 1$

Figure-4 shows that the number of wells has significant effects on dimensionless flowing bottomhole pressure when all the wells are producing at the identical flowing rates, ($\tau = 1$). At early times, the flowing bottomhole pressure of the observation well is only a function of its own production as if surrounding wells do not exist; when producing time is long, the influence from surrounding wells appears. At a given time t, if the number of wells n increases, P_{wD} also increases, i.e., the wellbore pressure drop P_i - $P_w(t)$ at the observation well increases with n, which indicates the interference effect from surrounding wells is more pronounced. Because the outer boundary is at constant pressure, when the producing time is sufficiently long, steady state will be reached, P_{wD} becomes a constant, i.e., the flowing bottomhole pressure $P_w(t)$ reaches a constant value, does not change with time.

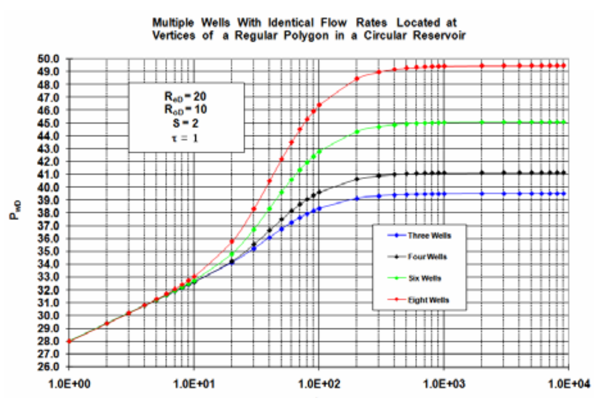

Figure-4. The effects of number of wells on P_{wD}.

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Example-2: $r_{wD} = 0.01$, $r_{eD} = 20$, $s = 2$, $\tau = 1$

Figure-5 shows that for a given number of wells and at a given time t, all the wells are producing at the identical flowing rates, ($\tau = 1$), if off-center distance r_o increases, P_{wD} decreases, i.e., the wellbore pressure drop P_i - P_w (t) at the observation well decreases when r_o increases, which indicates the effect from constant pressure outer boundary is more pronounced. At a given off-center distance r_o and at a given time t, if the number of wells n increases, P_{wD} also increases.

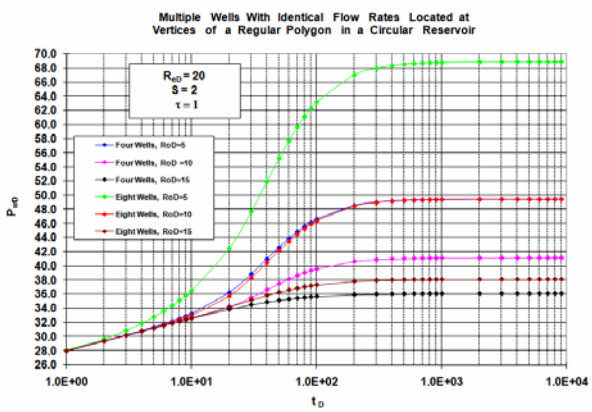

Figure-5. The Effects of off-center distance on $P_{wD.}$

Example-3: $r_{wD} = 0.01$, $r_{eD} = 20$, $s = 2$, $\theta = \pi / 2$

If two wells with identical off-center distance r_o, the first well is at (r_o, 0), the second well is at (r_o, π / 2), and define flow rate ratio $\tau = q_2 / q_1$. Figure-6 shows that at a given time t and at given off-center distance r_o, if τ increases, P_{wD} also increases, i.e., the wellbore pressure drop P_i - $P_w(t)$ at the first well increases with τ, which indicates the interference effect from the second well is more pronounced when its flow rate increases. At a given flow rate ratio τ and at a given time t, if the off-center distance r_o increases, P_{wD} decreases.

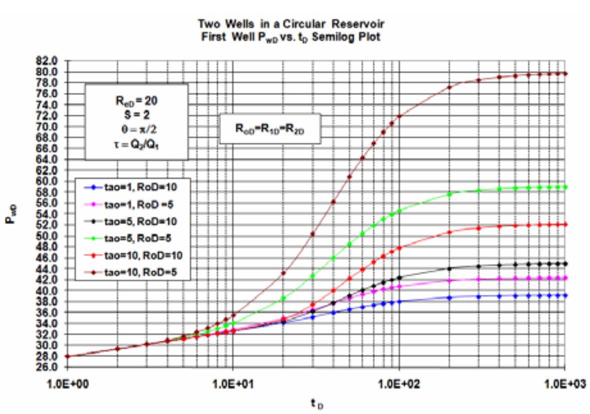

Figure-6. The effects of flow rate ratio on $P_{Wd.}$

Example-4: $r_{wD} = 0.01$, $r_{eD} = 20$, $r_{oD} = 10$, $S = 2$

If two wells with identical off-center distance R_o are producing at the identical flowing rates, (τ = 1), the first well is at (r_o, 0), the second well is at (R_o, θ). Figure-7 shows that at a given time t and at given off-center distance r_o, if θ decreases, P_{wD} increases, i.e., the wellbore pressure drop P_i - $P_w(t)$ at the first well increases when θ decreases, which indicates the interference effects from the second well is more pronounced.

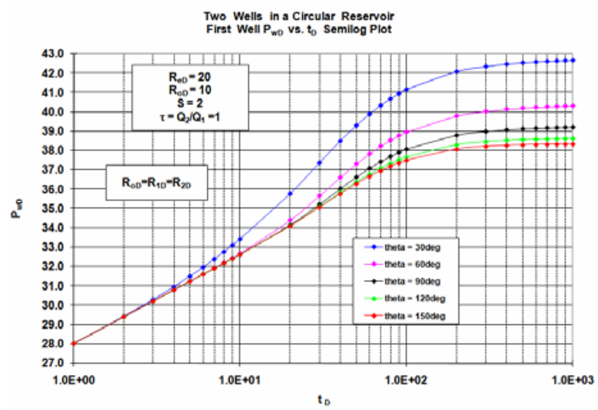

Figure-7. The effects of well location angle on $P_{wD.}$

CONCLUSIONS

Assuming no water encroachment, and no multiphase flow effects, and taking edge water as constant pressure boundary, and assuming top and bottom reservoir boundaries are impermeable, the following conclusions are reached:

a) The proposed pressure drawdown equations provide a reasonably accurate and fast analytical tool to evaluate the performance of multiple-wells system;
b) For a given number of wells, permeability, well pattern, well spacing, skin factor, flow rates and well off-center distances have significant effects on single well pressure transient behaviour;
c) With constant pressure outer boundary, when producing time is sufficiently long, steady-state is definitely reached;
d) If all the wells are fully penetrating with identical mechanical skin factor S, and are located at vertexes of a regular polygon, at a given time t, if the number of wells n increases, P_{wD} also increases; if off-center distance r_o increases, P_{wD} decreases; for the two-wells system, if flow rate ratio τ increases, P_{wD} increases, and if wellbore location angle θ decreases, P_{wD} increases.

Nomenclature

B	= formation volume factor, L^3 / L^3
h	= payzone thickness, L
i	= $\sqrt{-1}$
k	= effective permeability, L^2
n	= number of wells.
P_e	= reservoir outer boundary pressure, $m/ (Lt^2)$

P_{ini} = initial reservoir pressure, $m/(Lt^2)$
$P_{w,j}$ = flowing bottomhole pressure of j-th well in the multiple wells system, $m/(Lt^2)$
q_j = flow rate of j-th well in the multiple wells system, L^3/t
q_{ref} = reference flow rate, L^3/t
r_w = wellbore radius, L
r_e = radius of the circular cylinder reservoir, L
r_j = off - center distance of j-th well in the multiple wells system, L
RE = real part operator.
S = mechanical skin factor, dimensionless.
t = time, t

Greeks

θ_j = wellbore location angle of j-th well in the multiple wells system, radians.
μ = fluid viscosity, $m/(Lt)$
x = a function defined by Equation (11).
τ = well flow rate ratio defined by Equation (8)

Suffices

D = dimensionless
ini = initial
j,k = well index
r = radial
t = total
v = vertical
w = well
x, y, z = coordinate indicators

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APPENDIX-1. DEFINITION OF DIMENSION-LESS VARIABLES

$$x_D = \frac{2x}{h}, \quad y_D = \frac{2y}{h}, \quad z_D = \frac{2z}{h}\sqrt{\frac{k_r}{k_v}}, \quad h_D = 2\sqrt{\frac{k_r}{k_v}} \tag{A-1}$$

$$r_{oD} = \frac{2r_o}{h}, \quad r_{jD} = \frac{2r_j}{h}, \quad r_{eD} = \frac{2r_e}{H}, \quad r_{wD} = \frac{2r_w}{h} \tag{A-2}$$

$$t_D = \frac{4k_r t}{\phi \mu c_t h^2} \tag{A-3}$$

$$P_{wD} = \frac{8\pi k_r h(P_{ini} - P_w)}{q_{ref}\mu B} \tag{A-4}$$