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REVIEW ON VARIOUS STUDIES OF COMPOSITE LAMINATES WITH PLY DROP-OFF

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ABSTRACT

A review on various studies of composite laminates with ply drop-off is presented in this study. Tapered laminated structures, which are formed by dropping of some of the plies at discrete positions over the laminate, have received much attention from researchers because of their structural tailoring capabilities, damage tolerance, and their potential for creating significant weight savings in engineering applications. This study is a comprehensive review, devoted to static and dynamics analysis, buckling analysis, vibration analysis, delamination and interlaminar stress analysis of laminated composite plates and beams, tapered laminated structure with ply drop-off. Overall remarks drawn from the reviewed works are given in the final section of the paper.

Keywords: ply drop-off, buckling, vibration, interlaminar stress.

INTRODUCTION

The use of advanced composite laminates in the aerospace industry has led to structures with superior strength-to weight and strength-to-stiffness ratios. To further optimize the structure, tapered thicknesses have commonly been manufactured. Components made of composite laminates often vary in thickness from one part of the component to another because composite laminates are frequently made from prepregs of discrete thickness; it is not possible to taper a laminate continuously. Instead the usual way that tapering is achieved by terminating one or more of the internal plies during prepeg layup. This is known as ply drop-off. Figure-1 presents the geometry of a ply drop-off.

The laminate tapers from a thick section to a thin section as a result of the ply drop-off. Plies may be inserted, at access holes, lightening holes and at joints or connections to strengthen them. In all these applications use of ply drop-off results in significant saving in material and therefore, it is cost effective. However, ply drop-off causes a discontinuity within the laminate and therefore, it introduces structural difficulties like stress concentration at the drop station. This leads to failure of the components through delamination and/or failure of resin. The formation of interlaminar stresses at the drop-off may initiate failure long before the ultimate load carrying capacity of the laminate is reached. Hence the potential benefits in dropping piles may be compromised through a reduction of the strength of the laminate. Hence, the dropping-off of piles has to be done in a manner that does not affect the strength of the laminates to a great extent. Ply drop-off in laminated has to be identified as a stress rises from the very beginning. Over past decade several experimental and analytical studies has been reported regarding various aspects of this problem. A review of different analysis of laminated composite with ply drop-off is given in this paper.

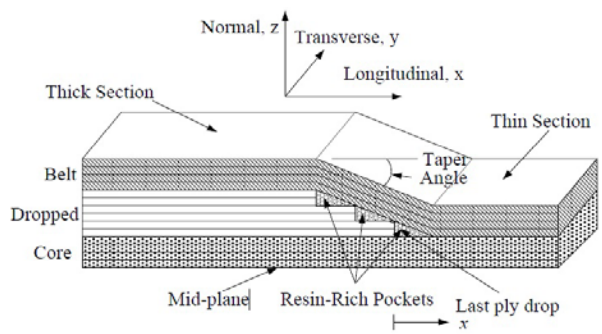

Figure-1. Schematic diagram of tapered specimen.

OVERVIEW

Tapered composite structures, formed by terminating some of plies, create geometry and material discontinuities that act as a source of delamination initiation and propagation. From earlier research work concerning composite laminate with ply drop-off, two major categories of work can be identified. The first is to understand failure mechanics induced by drop-off piles which include the determination of the state of interlaminar stresses in the vicinity of ply drop-offs and secondary the initiation and propagation of delamination. Daoust and Hoa [1] developed an extensive finite-element program using displacement formulation to study the effect of some parameters on the strength of the laminate. They concluded from their study that under tension, bending and torsion, internal ply drop-off is roughly twice as strong as external drop-off. Wu and Webber [2] applied a quasi- three dimensional isoparametric finite-element model for the linear elastic static analysis of tapered laminated plate of infinite width subjected to a uniform in-plane load. Numerical results were given for a single-step plate with various arrangements for the ply fibre directions. Very high peak stresses were predicted in the corner region of the step, but these were reduced when a resin fillet was included in the theoretical model for the step region. Following this analysis was a continuation

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work by Wu [3], in which non-linear material behavior was considered to account for the redistribution of stresses in the resin that would occur in the presence of the peak stresses. Compared with the linear results, the nonlinear ones show that the peak stresses are reduced by about half as a result of non-linear deformation of the resin and the non-linear model gave more realistic pre-diction of interlaminar stress distribution and failure mode at the ply drop-offs. Thomsen *et al*. [4] used a simple mechanical model to investigate the local bending effects of ply drop-offs in CFRP/honeycomb sandwich panels. The interaction between the core material and the face laminates was modeled using a two-parameter elastic foundation model. It was concluded from the examples given that the elastic response is strongly influenced by the presence of a supporting core material and that out-of-plane stiffness of the honeycomb core, the bending stiffness of the base-line face laminate and the bending stiffness of the dropped sub-laminates provide significant bending effects induced by ply drop-offs. Experimental investigation based on the use of electronic speckle-pattern interferometry (ESPI) was con- ducted to validate the simple model and it was shown that the theoretically predicted and measured out-of- plane detection profiles correlated well with respect to the local bending response induced by the ply drop-offs. The model was extended to apply to delamination failure analysis [5] and stress analysis for internal drop-off tapered laminates [6]. Miravete [7] presented a study of mechanical behavior of variable-thickness composite beams subjected to transverse load. A theoretical model based on a plane-strain finite-element theory was carried out to analyze the stress distribution near the areas of change of thickness, which is strongly dependent on thickness ratio. For low values of the angle of variation of thickness, the strength is outstanding and the variable-thickness effect does not alter the mechanical behavior of the plate. For high angle of thickness variation, the strength is lower because of the variable- thickness effect and failure occurs at the location where thickness varies. The delamination mechanism is a result of high interlaminar shear stress generated by the variable-thickness effect. The shear deformation element was modified to accommodate variable-element thickness with mid-plane layer drop-off. With the example problem, it is shown that the tapered element formulation in the QHD40 element, which was developed by Ochoa and Chan [8] for analysis and design of complex shape composite components, adequately models tapered and layered plates. The finite-element modeling, however, was simplified by negating the presence of small resin pockets caused by forming the terminated layers.

Most of the previous works with regard to delamination analysis of transverse-ply drop-off tapers were towards ideally two-, quasi-three- or full three-dimensional representation of the geometry of the taper. In general, they may consist of either 0, 90, ±45, ±150 plies or a certain combination of them with a tapering angle of less than 150, usually with 5.710 for a 10-to-1 taper ratio. Most of them are symmetric, while a few papers investigated asymmetric laminates that are typically used in applications where a flat surface is important, such as wing skins. Both numerically intensive finite-element models and simplified analyses have been attempted. Experimentally, tapered specimens have been manufactured from graphite/epoxy and glass/epoxy composites and tested under quasi-static loading conditions and fatigue.

Vizzini [9] used a quasi-three-dimensional finite-element approach for strength prediction for longitudinal ply drop-off tapers. In his analytical work, which correlated well with experimental evidence, Vizzini found that modelling a discontinuity with an associated resin pocket provides direct evaluation of the stresses in the region where failure occurs. Pogue and Vizzini [10] extended the structural tailoring techniques to the suppression of the delamination at the stress-free edge by dropping plies just around the edge of the laminates. This is one of four edge-alteration techniques applied to prevent the delamination induced by the stress-free edge of composite laminates. The introduction of the taper around the edge can change the state of stress at the free edge while introducing an internal discontinuity. It may help to decrease the interlaminar stresses which arise as a result of the stress-free edge; however, the newly introduced internal edge may be prone to internal damage that is difficult to detect non- destructively. The fact that benefit and detriment of tapering exist simultaneously shows that much care must be taken in choosing an appropriate tailoring technique. Ochoa and Chan [8] also examined the longitudinal taper under tensile, bending and torsion loads using finite elements in analyzing laminates with 900 plies dropped symmetrically just inside of the free edge. They found that under tensile loading, interlaminar stresses at the free edge were reduced significantly, while under bending and torsion loading only a small amount of increase of interlaminar stress values was found. Poon *et al*. [10, 11] presented a study of tapered laminated carbon-fibre/epoxy composite with a dropped ply subjected to in-plane bending is modeled in which the strain fields are predicted by a three-dimensional finite element method. The finite element results highlight the effect of strain concentrations at the ply drop-off and the free edge problems. The validation of the finite element analysis is performed by strain measurements on a tapered composite specimen using moiré interferometry. Tapered composite specimens were also tested in situ observation by SEM to establish the causes of failure. The implications of the finite element results together with the test results are discussed in relation to the failure mechanisms of tapered composites.

A detailed research survey on the study of tapered laminated composite structures has been provided by Hoe and Ganesan [12] emphasis on interlaminar stress analysis, delamination analysis and parametric study. Various approaches in modeling and design analysis are also reviewed. Varughese and Mukherjee [13] had done tremendous work composite structure with ply drop-off. In 1997 they develop a ply drop-off element (1D) for analysis of tapered laminated. They also provide the guidelines for ply drop-off in laminated composite structures to reduce stress concentration. Again in 1999 they developed 2D drop-off element. Vidyashanker and Murty [14] presented the complete 3D finite element analysis considering Tsai-Wu criterion. The study reveals the influence of various parameters (like effect of numbers of piles dropped, effect of taper angle, effect of fiber orientation and observed the following facts:

- The ply-drop zones contain high interlaminar stress gradients and hence potential sites for the onset and the subsequent growth of delamination.
- The size of resin pocket is shown to be crucial parameter affecting the nature of stress distribution.
- Dropping of 0ff-axis piles leads to higher failure factors due to the presence of significant in-plane shear stresses, where as the dropping of 900 piles has negligible effect on the stress distribution.

BUCKLING AND VIBRATION ANALYSIS

Relatively few researchers have investigated the vibration response tapered composite structures. Chopra and Durvasula [15] study the natural frequencies and modes of simply supported tapered skew plates having a linear variable thickness in one direction are considered. Kapania and Singhvi [16, 17] developed an efficient method for free vibration analysis of generally laminated composite skew plates having arbitrary edge conditions, where adjacent lamina will have longitudinal axes generally not parallel. A point to be noted is that a “tapered” implies that the root chord and the tip chord length are unequal and the plate is considered perfectly flat. Within the limitations of the classical lamination theory, the procedure consists of the Rayleigh-Ritz method utilizing the strain energy functional containing both bending and stretching effects and accommodating arbitrary ply stacking sequences. A set of Chebyshev polynomials is used as trial functions to represent the three components of the displacement at a given point. Rao and Ganesan [18] investigated the harmonic response of tapered composite beams using finite element based on a higher-order shear deformation theory. The taper model considered in this investigation consists of individual tapered plies that are laid up in the width axis. Free vibration of stepped Timoshenko composite beams was studied by Farghaly and Gadelrab [19]. They obtained the natural frequencies for a stepped composite beam based on the Timoshenko beam theory. Gupta and Rao [20] used finite element with two nodes at the ends and two degrees of freedom per node to obtain the stiffness and mass matrices for linearly-tapered and twisted beams. Cleghorn and Tabarrok [21] presented a finite element model for free vibration analysis of linear-tapered beams. Ganesan and Chen [22] study the effect of free vibration response of variable thickness composite beam.

An experimental and analytically investigation has been conducted on buckling and postbuckling behavior of laminated composite plates with ply drop-off under uniaxial compression is done by Dinardo [23]. The shell elements, solid elements and transition elements were developed by Davila and Johnson [24] to capture post- buckling response in the internally dropped laminates. The shell elements employed to model the majority of the laminate is a nine-node assumed natural strain (ANS) element with 5 degrees of freedom per node. The solid element used to model the ply drop-off region is a 20-node serendipity brick element with 3 displacement degrees of freedom per node. A transition element that has 15-node element with 51 degrees of freedom per node and permits the connection of shell and solid element was constructed by degenerating the 20-node solid element. The influences of the geometric non-linearity on the stress concentration and the delamination initiation were examined through the analysis by this advanced element. Rasul M. A. [25, 26, 27] developed an extensive finite element program for study the dynamic and buckling analysis of variable thickness laminated composite beams using conventional and advanced finite element formulation. The analysis results show that the mid-plane type of tapered gives low value of natural frequencies in dynamic analysis point of view.

Zabihollah [28] done the free vibration response and buckling of different types of tapered composite beams are analyzed using conventional and advanced finite element formulations. Conventional finite element formulation requires a large number of elements to obtain acceptable results. In addition, continuity of curvature at element interfaces cannot be guaranteed with the use of conventional formulation. As a result, stress distribution across the thickness is not continuous at element interfaces. In order to overcome these limitations, an advanced finite element formulation is developed based on classical laminate theory and first-order shear deformation theory. The developed formulation is applied to analysis of various types of tapered composite beams (NCT/301 graphite-epoxy). Shaikh M.S. [29] has done buckling analysis of different types of tapered composite plates. The analysis is based on classical laminate theory, first-order shear deformation theory and third-order shear deformation theory. The Ritz and finite element method is developed by Liu W. [30] for calculating dynamic instability regions. Solution is based on classical laminate theory have been developed first for vibration and buckling analysis of uniform and tapered composite plates with and without in-plane forces. The effect on the laminate stiffness of the composite plate caused by the taper angle has been considered.

A free vibration analysis of tapered composite beams using hierarchical finite element method is done by Chen Lin [31] with and without axial force is analyzed first using Ritz method, then the developed hierarchical finite element formulation. Both classical and first-order shear deformation theory are considered for the analysis of different types of tapered composite beams. A nonlinear finite element formulation is developed by liu D. [32] based on the first order shear deformation theory and geometric nonlinearity in the von Karman sense for the analysis of tapered composite laminated plates. The tensor polynomial form of failure criterion is used in conjunction with progressive failure simulation methodology to study the progressive failure. The first-ply failure load, the ultimate failure load, he buckling load, the associated maximum transverse displacements and the locations and mode failure of tapered laminated plates are determined for different types of tapered laminated plates. Vibration analysis of tapered composite beams is performed by Zabihollah [33]. A finite element formulation that includes the effect of different types of tapers based on a higher-order finite element. The most important conclusion is the Tapered beam (staircase-dispersed type) is the stiffest configuration and hence this model gives the highest natural frequencies for all types of boundary conditions, geometries. Ganesan and Zabihollah [34] presented free damped Vibration analysis of various tapered composite beams shown in Figure-2 and parametric study using a higher-order finite element.

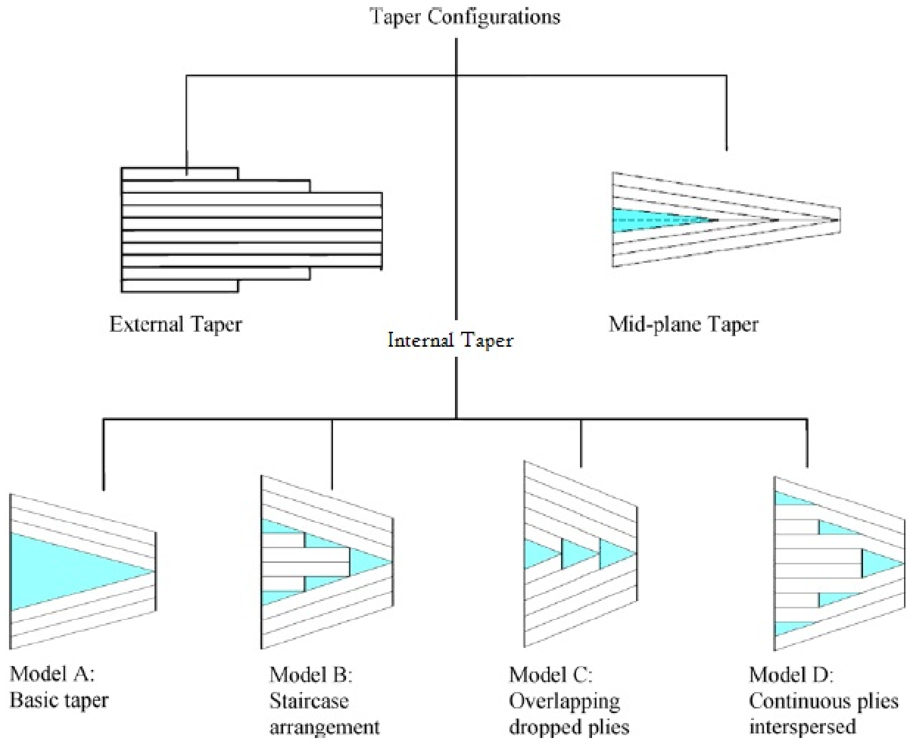

Figure-2. Schematic illustration of taper configurations.

The most important conclusions are given in the following: The tapered beam Model D (staircase-dispersed type), is the stiffest configuration, and hence this model gives the highest natural frequencies for all types of boundary conditions, geometries and laminate configurations. The tapered beams designed using Model A and Model C (overlapped-grouped type), shows that the resin pockets have the same effect on the stiffness in these two cases. The tapered beam Model A is the least-stiff (most flexible) model. Further, Models B and C take the second and third positions. Compressive response of Tapered curved composite plates is analyzed by Rasul S. M.A. [35]. For buckling analysis, four different analytical approaches are employed. Parametric studies are carried out in which the effects of boundary conditions, stacking sequence, tapered configurations, radius and geometric parameters of the plates are investigated. The important conclusion is that the tapered curved plates provide a better design option in terms of saving the material without any compromise of the strength. The cost of the engineering products can be minimized by reducing the weight of the products. Rasul S. A. [56] presented buckling analysis of tapered plates using ritz method based on first order shear deformation theory. A detailed parametric study has been conducted on various taper and layup configurations, all made of NCT/301 graphite epoxy, in order to investigate the effects of taper angle, length to height ratio, length to width ratio, boundary conditions and taper and layup configurations. Again in Rasul S. A. [57] presented a simplified methodology to predict the stability limit load that requires the consideration of only two load steps. The stability limit loads of the tapered curved plates are calculated using this methodology. Based on the first-ply failure analysis and the simplified non-linear buckling analysis, the critical sizes and parameters of the tapered curved plates that will not fail before global buckling are determined. The stability limit loads calculated using the present simplified methodology are shown to have good agreement with that calculated based on the non-linear load-deflection curve using the conventional non-linear buckling analysis methodology.

INTERLAMINAR STRESSES

The central part of an investigation of delamination analysis of tapered laminates lies in how to describe accurately the interlaminar stress state in the critical region of components. The finite-element method is the most prevalent and powerful tool in dealing with geometrically complex problems such as tapered composites, as applied by a large number of authors. However, some authors either aimed at developing a simple physical model to demonstrate stress-transfer mechanisms at the drop-off location or desired to develop a complicated model to find out the true stress distribution at the critical region.

A simple model for interlaminar stresses at the interface between the continuous plies and drop-off plies was developed by Armanios and Parnas [36] on the basis of equilibrium conditions on the continuous sublaminate (belt) and of local stiffness variations at the ply drop locations. In this model, the resin pockets were assumed as primarily shear-stress carriers and ply-drop locations as extensional and concentrated shear springs. The estimation of the interlaminar stresses was determined by application of a minimum complementary-energy principle. Although the interlaminar shear stress from their model was in qualitative agreement with a finite-element solution, it failed to capture the tensile nature of the interlaminar normal stress at the ply drops.

The work of kemp and Johnson [37] include quasi 3D approach, in their model the displacements normal to the plane of the model were still included and therefore these models have five or six non-zero components of strains. In refer to above four elements through the thickness in the plies in the vicinity of the drop-off were set up to perform the analysis as required to satisfy reasonably the continuity of intralaminar stresses and strains. The analyzed results show that the interlaminar stresses reach the maximum at the ply drop location. Fish and Lee [38] first introduced hybrid elements in the analysis of tapered composites. In their work, 3D assumed hybrid elements were used to develop a methodology for the prediction of delamination onset in tapered composite laminates containing multiple ply-drop steps. The model contained 433 eight-node brick elements and six-node pentahedral elements for a total of 2916 global degrees of freedom. The eight-node hexahedral elements were based on an assumed stress hybrid formulation and could provide more accurate stresses than the linear displacement element. The six-node pentahedral elements were based on the assumed displacement formulation. The influences of the sub-laminates above and below the ply-drop steps were investigated. Both experimental testing and finite-element modelling of the tapered region were conducted. The failure of the tapered laminates is a result of the interlaminar shear stress and occurs at the last ply-drop step. Curry *et al*. [39] conducted a global/ local approach to tapered composite analysis. The global analysis was performed using the general-purpose computer program STAGS, while the local analysis for determining the three-dimensional state of stress in the vicinity of the dropped plies was based on the generalized plane deformation. Their study shows that interlaminar normal stress in the interface or resin layer reaches a maximum at the end of the dropped plies, and at the same location where the interlaminar shear stress is close to its maximum value.

Based on the Hellinger and Reissner variational functional, Harrison and Johnson [40] developed a stress-based method of approximation for the prediction of interlaminar stresses in the vicinity around ply drops. The approach chosen was to follow Pagano's laminate structural theory which modeled the laminate by a series of layers with the stress field assumed within each layer. The stresses were assumed to be explicit functions of the thickness coordinate with stress variables as coefficients. These stress variables were functions of the longitudinal coordinate only. Substituting the assumed stress field into the Hellinger and Reissner variational principle and invoking the stationarity condition with respect to all admissible stresses and displacements led to a system of differential-algebraic equations (DAEs) that could be solved by the finite-difference method. The solution for interlaminar stresses in the modeled tapered laminates that were assumed to be under generalized plane deformation was found to be in good agreement with the finite-element solution. Mukherjee *et al*. [41] also made a global/local approach for the analysis of tapered composites. Considering that the drop-off need not pass through a nodal line in a global analysis, they developed drop-off elements that can be independent of the location of the drop-offs. The elements were used in a global analysis to reduce the size of the global structural matrix and showed more flexibility in the meshing division. An accurate stress distribution around the ply drop-offs was determined by local analysis with refined finite elements over the critical region and the input from the global analysis as the boundary conditions. Good correlation was found by comparing the results obtained using this approach with published results based on three-dimensional modelling.

Vizzini [42] employed the so-called shear-lag model to analyze interlaminar stresses in the region around drop-offs. In his model, resin layers were assumed to act as media carrying shear stresses only, and fiber layers as media carrying extensional stresses. The stress state at the ply drop region was modeled as a three-zone problem, in each of which force equilibrium was implemented. The resulting governing differential equations, subjected to satisfaction of constraints on displacement at the boundary by finite-element results from the global analysis, the inter-zone constraints on displacements and forces, and constraints resulting from degenerate cases involving zero thickness, were solved with assumed polynomials for the displacements in the fiber layers. In comparison with the finite-element results, he found that the shear-lag model can capture a majority of the load-transfer mechanisms about internally dropped plies under extensional stresses.

Mukherjee and Varughese [43] developed few guidelines for the design of tapered laminated composites by studying the effects of important parameters that

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determine strength of the laminate. He K [44] had done a numerical and experimental study on interlaminar stresses and delamination in tapered laminates. Numerical analyses performed involved development of partial hybrid stress finite elements needed to enhance computational efficiency, and development of physical concept-based modified shear-lag model that is based on the essential assumptions that both plies and resin layers are treated as carries of tensile stress and also to act as stress-transfer media. Experimental analysis was attempted to access the accuracy of numerical predictions. To perform strength and delamination analysis of tapered laminate, the laminate was modelled as a generalized plane deformation problem, where all the variables involved in the model are independent of the coordinate system. Stress based criteria that have proved to be effective in determination of critical location and load of delamination onset were utilized in his study to predict the delamination strength of the laminate. Evaluation of strain energy release rates of delamination occurring at the critical interfaces of the tapered laminate was carried out by using the J-Integral approach.

DELAMINATION

In order to predict delamination onset and growth and, hence the performance of the various laminates studied, some kind of failure analysis is applied. Two general approaches are strength-of- materials approach (stress-strength approach) and the strain-energy-release-rate approach (fracture-mechanics approach). In strength-of- materials approach, the local stress or strain state is compared to the material strength allowable. In application of this, usually more than one failure criterion was used to predict the weakest location over the whole structure. Frequently, different criteria were used for prediction of in-plane and out-of-plane failure of the plies as well as for out-of-plane failure between plies. In the strain-energy-release-rate approach, which is based on fracture mechanics, the laminate is assumed to fail when the available strain energy of a delamination crack in a ply interface exceeds the critical strain-energy-release rate for the material. This approach is a concept from fracture mechanics. It may be interpreted as the amount of work required to close a delamination by an incremental length. Much of the work has been done to calculate the modes of strain-energy-release rate using the finite-element method for delamination in tapered composites.

Kemp and Johnson [37] used the maximum stress criterion to predict the failure in the rich resin surrounding the dropped plies, while applying the Tsai-Wu criterion for intralaminar failure prediction. With these criteria under consideration, they found that majority of the first failure events in either tension or compression is a resin failure in a few cases for which failure occurred at the re-entrant corner. Fish and Lee [38] used a modified Tsai-Wu criterion to predict the out-of-plane failure of the composite laminates in their study. They introduced the average stress concept for the situations where the stress state is dominated by a single stress and applied it to the out-of-plane stress distributions obtained from the numerical analysis; thus, the maximum stress failure was considered. They found that the maximum stress criterion, using an interlaminar stress averaging distance of one ply thickness, provided consistent and accurate delamination onset predictions for the laminates investigated, which was also supported by the experimental observations.

Both interlaminar and intralaminar failure criteria were used by Curry *et al*. [39] for their analysis. The interlaminar criterion they used, which is based on a matrix failure mode developed by Hashin [45], was evaluated at all interfaces between plies with different fiber orientations in the local model, while the intralaminar criterion was a modification of Tsai-Wu, in which only the strength parameters that correspond to the failure mode were included. The failure analysis with the above criteria and finite analysis results indicated that the first major failure event for the laminate studied was a delamination at the interface between the dropped ply and continuous ply that appeared to initiate at the end of the dropped ply. These failure analyses, however, underestimated the experimental failure load by more than 30%.

On the basis of the assumption that the primary failure in the tapered composites is to be delamination and to occur in the interply resin layer, Vizzini [46] employed the von Mises stress criterion, an isotropic failure criterion, to be a measure of the overall stress state for a given configuration. The maximum von Mises stress in the realistic laminate with an ill-formed pocket modelled as to be four-sided rather than triangular as is usually assumed, or with unsymmetric ply drops, occurred around the last ply drop-offs. He found that the results agreed well with his finite-element analysis and, further, the von Mises stress for the laminate with a fracture void increased by more than 50%, which indicated that the presence of a void greatly affected the stress state around the ply drop and that the interlaminar stress criterion that excludes voids will over predict the onset of damage. The von Mises criterion was used to determine the strength of the resin pocket at the discontinuity. Falling within the scatter of the experimental data for delamination initiation, this resin pocket model predicted very well the initiation of damage for the laminates dominated by the internal edge failure.

Wisnom *et al*. [47] presented the results of the tests carried out on the rapidly tapered specimens with dropped ±450 and 00 plies in order to determine static and fatigue strengths. The failure modes for the three types of specimens studied are fiber breaks initiated near the first dropped ply or delamination occurring at the dropped 00 plies which are more susceptible to delamination than ±450 plies. Wisnom *et al*. [48] continued this analysis by comparison of tapered laminate delamination with delamination in internal cut plies under fatigue loading. They found that the delamination in the cut-ply specimens propagated in both directions from the cut, whereas for the dropped-ply specimens it propagated only to the thick end with a slower delamination rate than for the cut-ply specimens as a result of the effect of through-thickness compressive stresses in the region where delamination initiated. Normalization of strain-energy-release rates calculated from a simple equation was made by dividing the cyclic strain-energy-release-rate range ΔG by the fracture energy Gc deduced from a static tension delamination test, and obtained the similarity between the delaminations in the specimens under fatigue and static loadings.

Wisnom [49] also choose three asymmetrical composites with 0/±450 lay-ups loaded in tension to carry out an experimental investigation on the effects of the tapered geometry and the stiffness of the is continuous plies for asymmetrically tapered sections based on the conclusion that the strain-energy-release rate associated with the discontinuous plies was the critical factor controlling delamination into the thick section, with the effect of the tapered geometry being of secondary importance. In comparison with the previous results for the thick section delamination failure mechanisms in the symmetric tapers studied, they concluded that the asymmetry does not appear to have a significant effect on thick-section delamination. Existence of thin-section delamination induced by specimen's tapered geometry, which is less severe than thick-section delamination, however, showed a different delamination behavior from the previously tested symmetric specimens and, therefore, further investigation was encouraged to explain it. The taper angle and the degree of consolidation in the region around the dropped plies are likely the reasons suggested by the authors for this behavior. For analysis and testing were made from glass/epoxy or graphite/ epoxy, and were configured in multidirectional and symmetric form. The majority of the work was based on experimental programs wherein the test coupons were subjected to static and/or fatigue loading. Various finite- element modelling and non-finite-element approaches were used in the analysis of tapered laminates. The maximum interlaminar shear stress was found by most works to appear at the ply drop step, while the maximum interlaminar normal stress was found to appear at the ply drop step, and at the taper root. The final delamination will grow into the thick and thin sections simultaneously, but the location of delamination initiation was found to appear at the ply drop step in some works, and at the taper root in others.

Harrison and Johnson [40] used a delamination fraction concept, which was proposed by Brewer and Lagace [50], as a measure to investigate the effect of eccentricity and stiffness discontinuity on the tendency of laminates with dropped plies to delaminate. In combination with their mixed variational approach, they found that the highest delamination fraction value is contributed by both the interlaminar normal and shear stresses at the ply drop region and show that it is the stiffness discontinuity rather than eccentricity of the laminate that has a larger inuflence on the interlaminar stresses and eventual delamination. Murri *et al*. [51] examined the effect of combined tension/bending loading on glass/epoxy laminates with a non-linear taper and internal ply-drops. The delamination growth originating from the initial tension crack at the drop-off was simulated in the 2D finite- element model by releasing pairs ofmulti-point constraints at the critical interfaces, and the strain-energy-release rates were thus calculated using VCCT for a delamination of interlaminar stress distribution at ply drop-off positions, where the interlaminar stress peaks occur and by far exceed the interface material allowable at the load levels where delamination failure can be detected experimentally. Thus, it is impossible to provide physically meaningful prediction delamination initiation in laminated tapered composites by direct application of the calculated peak stresses together with some point- stress criteria. To overcome the difficulties induced by the inherent singularity, some other techniques like `stress averaging' or 'effective/characteristic length' were applied and proved to be effective in dealing with delamination initiation analysis of tapered laminates.

Murri *et al*. [52] The viability of a method for determining the fatigue life of composite rotor hub flexbeam laminates using delamination fatigue characterization data and a geometric non-linear finite element (FE) analysis was studied. Combined tension and bending loading was applied to non-linear tapered flexbeam laminates with internal ply drops. These laminates, consisting of coupon specimens cut from a full-size S2/E7T1 glass-epoxy flexbeam were tested in a hydraulic load frame under combined axial tension and transverse cyclic bending. The magnitude of the axial load remained constant and the direction of the load rotated with the specimen as the cyclic bending load was applied. The first delamination damage observed in the specimens occurred at the area around the tip of the outermost ply-drop group. Subsequently, unstable delamination occurred by complete delamination along the length of the specimen. Continued cycling resulted in multiple delaminations. A 2D finite element model of the flexbeam was developed and a geometrically non-linear analysis was performed. The global responses of the model and test specimens agreed very well in terms of the transverse displacement. The FE model was used to calculate strain energy release rates (G) for delaminations initiating at the tip of the outer ply-drop area and growing toward the thick or thin regions of the flexbeam, as was observed in the specimens. The delamination growth toward the thick region was primarily mode II, whereas delamination growth toward the thin region was almost completely mode I. Material characterization data from cyclic double-cantilevered beam tests was used with the peak calculated G values to generate a curve predicting fatigue failure by unstable delamination as a function of the number of loading cycles. The calculated fatigue lives compared well with the test data.

Steeves C.A., Fleck N.A. [53], conducted experiments on tapered laminates loaded in axial compression. Failures by microbuckling or delamination were observed to nucleate near the dropped plies. The results indicate that existing model of micro buckling give adequate predictions of strength in specimens with mild geometric defects, while a new model given here for

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delamination provides satisfactory predictions of strength for more severe defects. Progressive failure and post buckling response of tapered composite plates is analyzed by Liu D. [54], a nonlinear finite element formulation is developed based on the first-order shear deformation theory and the geometric nonlinearity in the von karman sense for the analysis of tapered laminated plates. The tensor polynomial form of failure criterion is used in conjunction with a progressive failure simulation methodology to study the progressive failure. A detailed parametric study of the deterministic progressive failure of tapered laminates under the action of uni-axial and bi-axial compression, in-plane positive shear, and in-plane negative shear loadings is conducted. The first-ply failure load, the ultimate failure load, the buckling load, the associated maximum transverse displacements, and locations and modes of failure of tapered laminated plates are determined for different tapered composite plates of 5208 epoxy materials. Stochastic progressive failure under the action of bi-axial compressive combined with in-plane positive shear loading is also investigated.

Table-1. Lay-up configurations.

Lay-up configurations	**Lay-up at the left** (thick) **end**	**Lay-up at the left** (thick) **end**	
	Taper configurations A, B, C, D and F	**Taper configurations A, B, C and F**	**Taper configurations D**
LC(1)	$(0/90)_{4s}$	$(0/90)_{2s}$	$(0)_8$
LC(2)	$(\pm45/0/90)_{2s}$	$(\pm45/0/90)_s$	$(45/0)_s$
LC(3)	$(\pm45)_{2s}$	$(\pm45)_{2s}$	$(45)_8$
LC(4)	$(0/90)_8$	$(0/90)_4$	$(0_4/90_4)$
LC(5)	$(\pm45/0/90)_4$	$(\pm45/0/90)_2$	$((45/0)_2/(-45/90)_2)$
LC(6)	$(\pm45)_8$	$(\pm45)_4$	$((45)_4/(-45)_4)$

In the parametric study, five taper configurations and for each taper configuration, six layup configurations are considered defied in Table-1.

The results show the following facts:

- The most critical points corresponding to the first-ply failure lie near the loaded edge of the plate (i.e., the thin end of the plate). If the maximum deflection is downward, it happens at the outermost top layer, as in the case of the lay-up configurations LC (4) and LC (5).
- For the laminates with (±□) 4s lay-up configuration at the left end, the maximum deflections are all upward. The most critical points corresponding to the first-ply failure lie near the loaded edge of the plate (i.e., the thin end of the plate), and the failure occurs at the outermost bottom layer.
- In general, for the lay-up and taper configurations and the boundary condition considered, the ultimate failure load is about 2.0-6.3 times the buckling load, the first-ply failure load is about 2.0-4.6 times the buckling load, and the ultimate failure load is about 1.0-1.8 times the first-ply failure load. The largest difference between the buckling load and the first-ply failure load corresponds to the tapered laminate with the lay-up configuration LC (1), and the taper configuration B or C. For this case, the first-ply failure load is about 4.5 times the buckling load. As for the difference between the buckling load and the ultimate failure load, the largest difference corresponds to the tapered laminate with the lay-up configuration LC (4) and the taper configuration D. For this case, the ultimate failure load is about 6.3 times the buckling load.

Allegri G. *et al*. [55] developed an optimization tool, based on fracture mechanics for preliminary design of tapered composite laminates. The optimization method comprises two distinct stages; the first is purely deterministic and it allows identifying which plies need to be dropped in order to obtain a prescribed laminate stacking sequence at the thin end. The second optimization stage defines which is the optimal dropping sequence and it is based on a probabilistic meta-heuristic algorithm, namely simulated annealing with stochastic tunneling. Comparisons with high fidelity FE results for a simple tapered beam configuration under pure bending have demonstrated that the proposed optimization method is robust and reliable; the optimized solution is significantly stronger than an alternative configuration defined by using standard design guidelines from the literature. The predicted strength values for the tapered laminate configurations considered here were in reasonably good agreement with the FE results. Anyway the optimization tool presented here is not meant to be employed as a means for accurately assessing the strength of tapered laminates; the main aim is providing a tool for performing quick comparative assessments among various laminate lay-ups in a preliminary design stage, i.e., when the requirements for modest computational times are more important than ultimate accuracy. In order to provide a comparison in terms of computational efficiency, it is worth stressing that the analytical optimization method takes a few minutes to run on a standard desktop computer, while the LS-DYNA models require several hours to run.

FUTURE RESEARCH

Tapered laminated structures, which are formed by dropping off some of the plies at discrete positions over the laminate, have received much attention from researchers because of their structural tailoring capabilities, damage tolerance, and their potential for creating significant weight savings in engineering applications. A review of different analysis of tapered laminated composite structures with emphases on static and dynamics analysis, buckling analysis, Vibration analysis, delamination, Interlaminar stress are reviewed in this study. Based on the author's investigation, it has been found that the research on the following aspects of the tapered laminated composite structures is relatively limited and may attract more interests in the future research.

- Material and geometric nonlinearity effects of tapered laminates structure
- Micromechanical approach using damage analysis
- Failure and damage analysis under cyclic loadings and viscoelastic effects (thermal and creep effects)
- Failure and damage analysis under Failure induced at geometric and material discontinuities
- Buckling, Vibration, Progressive failure analysis, Interlaminar stress and delamination analysis, Nonlinear analysis of composite laminated skew plates with ply drop-off (tapered laminated skew plates) have not been studied yet.

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