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FUZZY COMPROMISE APPROACH FOR SOLVING BI-LEVEL LINEAR PROGRAMMING PROBLEM WITH FUZZY PARAMETERS

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ABSTRACT

In this paper, we have proposed an algorithm to solve bi-level linear programming problems with fuzzy parameters without converting it to crisp equivalent problem and obtained a fuzzy compromise solution. Since the decision variables and objective functions are represented as fuzzy membership functions, the proposed fuzzy compromise solution will be more practical and reasonable. A numerical example is provided to illustrate the solution method proposed in this paper.

Keywords: bi-level linear programming problem, triangular fuzzy numbers, fuzzy parameters.

1. INTRODUCTION

Decision making in many planning problem involves hierarchical administrative structure, which involves decisions of several individuals with independent and conflicting objectives. In a bi-level decision making problem, the decision maker at the upper level is termed as the leader, and the lower level is termed as the follower. When the leader attempts to optimize his objective, the follower tries to find an optimal solution according to each of possible decisions made by the leader [1] (1998). That is the leader sets its goal or decisions and then asks the follower for their optima which are calculated in isolation; the lower-level decision makers decisions are then submitted and modified by the leader with consideration of the overall benefit for the organization and the process is continued until a satisfactory solution is reached. This decision - making process is extremely practical to decentralized systems such as agriculture, government policy, economic systems, finance, warfare, transportation, network designs, and is especially for conflict resolution.

The decision making procedure is more complex to implement since the follower has to coordinate with the leader with some restrictions over his decision parameters. In literature many approaches such as Kuhn-tucker approach [2], K-th best approach [3], fuzzy approach [4] and branch and bound algorithm [2, 5, 14] have been developed to solve classical bi-level linear programming problem whose parameters are known precisely. But in real-life situations these parameters may not be known precisely due to their uncertainty and inexactness. These uncertainty and inexactness are caused because of measurement inaccuracy, insufficient information, simplification of physical models, variations of the parameters of the system, computational errors etc. Consequently, uncertainties cannot be successfully handled using traditional mathematical tools but may be dealt with using a wide range of existing theory of fuzzy sets.

If the decision parameters involved in a bi-level linear programming problem are represented by fuzzy numbers, then the resulting problem is called fuzzy bi-level linear programming problem.

In literature many authors such as Dempe [6], Zhang [7, 8, 9, 10], G. Zhang, J. Lu and T. Dillon [15] have studied the solution approaches for fuzzy bi-level decision making problems. Dempe has formulated the fuzzy bi-level programming problem and described one possible approach by formulating a crisp optimization problem. Zhang studied fuzzy bi-level decision making problems using Extended Kuhn-Tucker Approach, an approximation branch-and-bound algorithm. He also provided a solution procedure to fuzzy bi-level linear programming problem with multiple objectives and an application of fuzzy bi-level linear programming problem in logistics planning problem. In most of the approaches, the authors have converted the given bi-level linear programming problem with fuzzy parameters to one or more equivalent classical problems and obtained the optimal solution

But our aim is to find a fuzzy compromise solution to the given fuzzy bi-level linear programming problem without converting it into classical problems. The decision made by the leader independently may be affected by the action and reactions of the follower. The external effect on a decision maker's problem can be reflected in both his objective function and his set of feasible decisions.

In this paper, we have proposed an algorithm to solve bi-level linear programming problem with fuzzy parameters without converting it to crisp equivalent problem and obtained a fuzzy compromise solution. In section 2, we recall the basic concepts and the results of triangular fuzzy number and their arithmetic operations. In section 3 we introduce the bi-level linear programming problem with fuzzy parameters and related results. In section 4, we proposed a new algorithm to find the fuzzy compromise solution for the given problem. A numerical example is also provided to illustrate the theory developed in this paper.

2. PRELIMINARIES

We recall some basic concepts and related results of triangular fuzzy numbers discussed in [11].

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Definition 2.1

A fuzzy set $\tilde{a}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function $\tilde{a}: R \to [0,1]$ has the following characteristics:

(i) $\tilde{a}$ is convex

i.e. $\tilde{a}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\tilde{a}(x_1), \tilde{a}(x_2)\}$,

for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$.

(ii) $\tilde{a}$ is normal i.e., there exists an $x \in R$ such that $\tilde{a}(x) = 1$

(iii) $\tilde{a}$ is Piecewise continuous.

Definition 2.2

A fuzzy number $\tilde{a}$ on R is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function $\tilde{a}: R \to [0,1]$ has the following characteristics:

$$\tilde{a}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & \text{for } a_2 \leq x \leq a_3 \\ 0, & \text{elsewhere} \end{cases}$$

We denote this triangular fuzzy number by $\tilde{a} = (a_1, a_2, a_3)$. We use F(R) to denote the set of all triangular fuzzy numbers. Also if $m = a_2$ represents the modal value or midpoint, $\alpha = (a_2 - a_1)$ represents the left spread and $\beta = (a_3 - a_2)$ represents the right spread of the triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$, then the triangular fuzzy number $\tilde{a}$ can be represented by the triplet $\tilde{a} = (\alpha, m, \beta)$. i.e., $\tilde{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$.

Definition 2.3

A triangular fuzzy number $\tilde{a} \in F(R)$ can also be represented as a pair $\tilde{a} = (\underline{a}, \overline{a})$ of functions $\underline{a}(r)$ and $\overline{a}(r)$ for $0 \leq r \leq 1$ which satisfies the following requirements:

(i) $\underline{a}(r)$ is a bounded monotonic increasing left continuous function.

(ii) $\overline{a}(r)$ is a bounded monotonic decreasing left continuous function.

(iii) $\underline{a}(r) \leq \overline{a}(r)$, $0 \leq r \leq 1$

Definition 2.4

For an arbitrary triangular fuzzy number $\tilde{a} = (\underline{a}, \overline{a})$, the number $a_0 = \left(\dfrac{\underline{a}(1) + \overline{a}(1)}{2}\right)$ is said to be a location index number of $\tilde{a}$. The two non-decreasing left continuous functions $a_* = (a_0 - \underline{a})$, $a^* = (\overline{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index function respectively. Hence every triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ can also be represented by $\tilde{a} = (a_0, a_*, a^*)$.

2.1. Ranking of triangular fuzzy numbers

Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. For an arbitrary triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*)$ with parametric form $\tilde{a} = (\underline{a}(r), \overline{a}(r))$, we define the magnitude of the triangular fuzzy number $\tilde{a}$ by:

$$\text{Mag}(\tilde{a}) = \frac{1}{2}\left(\int_0^1 (\underline{a} + \overline{a} + a_0)\ f(r)\ dr\right)$$
$$= \frac{1}{2}\left(\int_0^1 (a^* + 4a_0 - a_*)\ f(r)\ dr\right).$$

where the function $f(r)$ is a non-negative and increasing function on $[0,1]$ with $f(0) = 0$, $f(1) = 1$ and $\int_0^1 f(r)\ dr = \frac{1}{2}$. The function $f(r)$ can be considered as a weighting function. In real life applications, $f(r)$ can be chosen by the decision maker according to the situation. In this paper, for convenience we use $f(r) = r$.

The magnitude of a triangular fuzzy number $\tilde{a}$ synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. $\text{Mag}(\tilde{a})$ is used to rank fuzzy numbers. The larger $\text{Mag}(\tilde{a})$, the larger fuzzy number.

For any two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ in F(R), we define the ranking of $\tilde{a}$ and $\tilde{b}$ by comparing the $\text{Mag}(\tilde{a})$ and $\text{Mag}(\tilde{b})$ on R as follows:

(i) $\tilde{a} \succeq \tilde{b}$ if and only if $\text{Mag}(\tilde{a}) \geq \text{Mag}(\tilde{b})$

(ii) $\tilde{a} \preceq \tilde{b}$ if and only if $\text{Mag}(\tilde{a}) \leq \text{Mag}(\tilde{b})$

(iii) $\tilde{a} \approx \tilde{b}$ if and only if $\text{Mag}(\tilde{a}) = \text{Mag}(\tilde{b})$

Definition 2.5

A triangular fuzzy number $\tilde{a} = (a_0, a_*, a^*)$ is said to be symmetric if and only if $a_* = a^*$.

Definition 2.6

A triangular fuzzy number $\tilde{a} = (a_0, a_*, a^*)$ is said to be non-negative if and only if $\text{Mag}(\tilde{a}) \geq 0$ and is denoted by $\tilde{a} \succeq \tilde{0}$. Further if $\text{Mag}(\tilde{a}) > 0$, then

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$\tilde{a} = \left(a_0, a_*, a^*\right)$ is said to be a positive fuzzy number and is denoted by $\tilde{a} \succ \tilde{0}$.

Definition 2.7

Two triangular fuzzy numbers $\tilde{a} = \left(a_0, a_*, a^*\right)$ and $\tilde{b} = \left(b_0, b_*, b^*\right)$ in $F(R)$ are said to be equivalent if and only if $Mag(\tilde{a}) = Mag(\tilde{b})$. That is $\tilde{a} \approx \tilde{b}$ if and only if $Mag(\tilde{a}) = Mag(\tilde{b})$. Two triangular fuzzy numbers $\tilde{a} = \left(a_0, a_*, a^*\right)$ and $\tilde{b} = \left(b_0, b_*, b^*\right)$ in $F(R)$ are said to be equal if and only if $a_0 = b_0, a_* = b_*, a^* = b^*$. that is $\tilde{a} = \tilde{b}$ if and only if $a_0 = b_0, a_* = b_*, a^* = b^*$.

2.2. Arithmetic operation on triangular fuzzy numbers

Ming Ma *et al*. [12] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice L. That is for a, b∈L we define $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$.

For arbitrary triangular fuzzy numbers $\tilde{a} = \left(a_0, a_*, a^*\right)$ and $\tilde{b} = \left(b_0, b_*, b^*\right)$ and $* = \{+, -, \times, \div\}$, the arithmetic operations on the triangular fuzzy numbers are defined by $\tilde{a} * \tilde{b} = \left(a_0 * b_0,\ a_* \vee b_*,\ a^* \vee b^*\right)$.

In particular for any two triangular fuzzy numbers $\tilde{a} = \left(a_0, a_*, a^*\right)$ and $\tilde{b} = \left(b_0, b_*, b^*\right)$, we define

a) **Addition:**

$$\tilde{a} + \tilde{b} = \left(a_0, a_*, a^*\right) + \left(b_0, b_*, b^*\right) = \left(a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}\right)$$

b) **Subtraction:**

$$\tilde{a} - \tilde{b} = \left(a_0, a_*, a^*\right) - \left(b_0, b_*, b^*\right) = \left(a_0 - b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}\right).$$

c) **Multiplication:**

$$\tilde{a} \times \tilde{b} = \left(a_0, a_*, a^*\right) \times \left(b_0, b_*, b^*\right) = \left(a_0 \times b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}\right).$$

d) **Division:**

$$\tilde{a} \div \tilde{b} = \left(a_0, a_*, a^*\right) \div \left(b_0, b_*, b^*\right) = \left(a_0 \div b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}\right).$$

Definition 2.8 [13]

Type-2 fuzzy sets are fuzzy sets whose membership values are fuzzy sets on the interval $[0,1]$. This concept was proposed by Zadeh, as an extension of fuzzy sets. Their membership function has the form $\tilde{\tilde{A}} : X \to \Im([0,1])$ where $\Im([0,1])$ denotes the set of all ordinary fuzzy sets defined on $[0,1]$.

Definition 2.9

A different generalization of ordinary fuzzy sets involves fuzzy sets defined within a universal set whose elements are ordinary fuzzy set. These fuzzy sets are known as level-2 fuzzy sets. Their membership function has the form $\tilde{\tilde{a}} : \Im(X) \to [0,1]$ where $\Im(X)$ denotes the set of all ordinary fuzzy sets of X. Level -2 fuzzy sets allow us to deal with situations in which elements of the universal set cannot be specified precisely, but only approximately.

Definition 2.10

We can also formulate fuzzy sets that are of type-2 and also of level 2 whose membership functions have the form $\tilde{\tilde{A}} : \Im(X) \to \Im([0,1])$ where $\Im(X)$ denotes the set of all ordinary fuzzy sets of X and $\Im([0,1])$ denotes the set of all ordinary fuzzy sets defined on $[0,1]$.

3. BILEVEL LINEAR PROGRAMMING PROBLEM WITH FUZZY PARAMETERS

Bi-level linear programming problem with fuzzy parameters involves two optimization problems where the constraint region of the leader problem is implicitly determined by another optimization problem on the follower level. A bi-level linear programming problem with triangular fuzzy numbers is defined as follows:

$$\max_{\tilde{x}_1} \tilde{Z}_1 \approx \tilde{c}_{11}\tilde{x}_1 + \tilde{c}_{12}\tilde{x}_2$$

where $\tilde{x}_2$ solves

$$\max_{\tilde{x}_2} \tilde{Z}_2 \approx \tilde{c}_{21}\tilde{x}_1 + \tilde{c}_{22}\tilde{x}_2$$

$$\text{s.t } \tilde{A}_{i1}\tilde{x}_1 + \tilde{A}_{i2}\tilde{x}_2 \le \tilde{B}_i$$

$$i = 1, 2, ...p$$

where $\tilde{c}_{11}, \tilde{c}_{12} \in F(R^n), \tilde{c}_{21}, \tilde{c}_{22} \in F(R^m)$, $\tilde{A}_{i1} = (\tilde{a}_{i1})_{p\times n}$ and $\tilde{A}_{i2} = (\tilde{a}_{i2})_{p\times m}$.

If $\tilde{x}_1$, $\tilde{x}_2$, $\tilde{c}_{11}$, $\tilde{c}_{12}$, $\tilde{c}_{21}$, $\tilde{c}_{22}$, $\tilde{A}_{i1}$ and $\tilde{A}_{i2}$ are represented by location index number, left fuzziness index function and right fuzziness index function respectively, then the above problem can be rewritten as follows:

$$\max_{\tilde{x}_1} \tilde{Z}_1 \approx \left\langle (c_{11})_0,\ (c_{11})_*, (c_{11})^* \right\rangle \times \left\langle (x_1)_0, (x_1)_*, (x_1)^* \right\rangle + \left\langle (c_{12})_0,\ (c_{12})_*, (c_{12})^* \right\rangle \times \left\langle (x_2)_0, (x_2)_*, (x_2)^* \right\rangle$$

where $\tilde{x}_2$ solves

$$\max_{\tilde{x}_2} \tilde{Z}_2 \approx \left\langle (c_{21})_0,\ (c_{21})_*, (c_{21})^* \right\rangle \times \left\langle (x_1)_0, (x_1)_*, (x_1)^* \right\rangle + \left\langle (c_{22})_0,\ (c_{22})_*, (c_{22})^* \right\rangle \times \left\langle (x_2)_0, (x_2)_*, (x_2)^* \right\rangle$$

$$\text{subject to } \left\langle (A_{i1})_0, (A_{i1})_*, (A_{i1})^* \right\rangle \times \left\langle (x_1)_0, (x_1)_*, (x_1)^* \right\rangle$$
$$+ \left\langle (A_{i2})_0, (A_{i2})_*, (A_{i2})^* \right\rangle \times \left\langle (x_2)_0, (x_2)_*, (x_2)^* \right\rangle$$
$$\leq \left\langle (B_i)_0, (B_i)_*, (B_i)^* \right\rangle, \; i = 1, 2, \ldots\ldots p$$

Definition 3.1

Let $\tilde{S} = \left\{ (\tilde{x}_1, \tilde{x}_2) : \tilde{A}_{i1}\tilde{x}_1 + \tilde{A}_{i2}\tilde{x}_2 \leq \tilde{B}_i, \; i = 1, 2, \ldots, p \right\}$ be the fuzzy feasible set of the Leader's problem.

For any fixed choice of $\tilde{x}_1$, follower will select a value of $\tilde{x}_2$ to maximize his objective function $\max \tilde{Z}_2$. Hence, for each value of $\tilde{x}_1$, the follower will react with a corresponding value of $\tilde{x}_2$. This induces a functional relationship between the decisions of leader and the reactions of follower. We will assume that the reaction function $\tilde{x}_2 = \psi(\tilde{x}_1)$ is completely known by the leader.

Definition 3.2

We define

$$\psi_{\tilde{Z}_2}(\tilde{S}) = \tilde{S}_1 = \left\{ (\tilde{x}_1^*, \tilde{x}_2^*) \in \tilde{S} \,/\, \tilde{Z}_2(\tilde{x}_1^*, \tilde{x}_2^*) = \max \tilde{Z}_2(\tilde{x}_1^*, \tilde{x}_2^*) \right\}$$

as the follower's decision space or the set of rational reactions of $\tilde{Z}_2$.

Definition 3.3

A point $(\tilde{x}_1, \tilde{x}_2)$ is said to be semi fuzzy feasible if and only if $(\tilde{x}_1, \tilde{x}_2) \in \tilde{S}$. A point $(\tilde{x}_1^\circ, \tilde{x}_2^\circ)$ is said to be fuzzy feasible if and only if $(\tilde{x}_1^\circ, \tilde{x}_2^\circ) \in \tilde{S}_1$.

Definition 3.4

A feasible point $(\tilde{x}_1^{\square}, \tilde{x}_2^{\square})$ is said to be fuzzy optimal if and only if $\tilde{Z}_1(\tilde{x}_1^{\square}, \tilde{x}_2^{\square})$ is unique for all $(\tilde{x}_1^{\square}, \tilde{x}_2^{\square}) \in \tilde{S}_1$ and $\tilde{Z}_1(\tilde{x}_1^{\square}, \tilde{x}_2^{\square}) \succ \tilde{Z}_1(\tilde{x}_1, \tilde{x}_2)$ for all feasible pair $(\tilde{x}_1, \tilde{x}_2) \in \tilde{S}_1$.

Definition 3.5

$\tilde{x}'$ is said to be a fuzzy compromise solution if there doesn't exist another $(\tilde{x}_1, \tilde{x}_2) \in \tilde{S}_1$ such that $\tilde{Z}_1(\tilde{x}_1', \tilde{x}_2') \leq \tilde{Z}_1(\tilde{x}_1, \tilde{x}_2)$.

Although the set of rational reactions for the follower problem may be non convex, it does possess some of the important properties of convex sets. The following theorem and corollaries help to characterize both $\tilde{S}_1$ and the optimal solution of the follower's problem.

Theorem: 3.1

Suppose $\tilde{S} = \left\{ (\tilde{x}_1, \tilde{x}_2) : \tilde{A}_{i1}\tilde{x}_1 + \tilde{A}_{i2}\tilde{x}_2 \leq \tilde{B}_i \;\; , i = 1, 2, \ldots\ldots p \right\}$ is bounded. Let $\tilde{S}_1 = \psi_{\tilde{Z}_2}(\tilde{S})$. Let $\tilde{y}_1, \tilde{y}_2, \ldots\ldots\ldots\tilde{y}_r$ be any r points of $\tilde{S}$ and $\lambda_1, \lambda_2, \ldots\ldots\ldots\ldots, \lambda_r > 0$ be scalars with $\sum_{j=1}^{r} \lambda_j \tilde{y}_j \in \tilde{S}_1$. Then $\tilde{y}_j \in \tilde{S}_1$ for all $j = 1, 2, \ldots\ldots r$.

Theorem states that the set $\tilde{S}_1$, possesses a weak convex-like property with respect to the set $\tilde{S}$. Any point in $\tilde{S}$ which strictly contributes in any convex combination of points in $\tilde{S}$ to form a point in $\tilde{S}_1$ must also be in $\tilde{S}_1$.

Definition 3.6 [5]

An fuzzy ideal solution of leader's and follower's problem is defined as the vector, whose components are composed by maximum value of each fuzzy objective functions under the given constraints, $\tilde{Z}^* = [\tilde{Z}_1^*, \tilde{Z}_2^*] = [\max \tilde{Z}_1, \max \tilde{Z}_2]$.

Definition 3.7

A fuzzy negative ideal solution of leader's and follower's problem is defined as the vector, whose components are composed by minimum value of each fuzzy objective functions under the given constraints, $\tilde{Z}^- = [\tilde{Z}_1^-, \tilde{Z}_2^-] = [\min \tilde{Z}_1, \min \tilde{Z}_2]$

Definition 3.8

The membership of each fuzzy objective function's satisfaction degree is defined as follows:

$$\mu_{\tilde{Z}_k}(x) = \begin{cases} 1 & \text{if } \tilde{Z}_k(x) \geq \tilde{Z}_k^* \\ 1 - \dfrac{\tilde{Z}_k^* - \tilde{Z}_k(x)}{\tilde{Z}_k^* - \tilde{Z}_k^-} & \text{if } \tilde{Z}_k^- < Z^k \leq \tilde{Z}_k^* \\ 0 & \text{if } \tilde{Z}_k(x) \leq \tilde{Z}_k^- \end{cases} \qquad k = 1, 2.$$

Definition 3.9

The membership function of the decision variable $\tilde{x}_1$ of the leader's problem satisfaction degree is defined as

$$\mu_{\tilde{x}_1}(\tilde{x}_1^*) = \begin{cases} \dfrac{\tilde{x}_1 - (\tilde{x}_1^* - p_1^l)}{p_1^l}, \text{ for } & \tilde{x}_1^* - p_1^l \leq \tilde{x}_1 \leq \tilde{x}_1^* \\ \dfrac{(\tilde{x}_1^* + p_1^r) - \tilde{x}_1}{p_1^r}, \text{ for } & \tilde{x}_1^* \leq \tilde{x}_1 \leq \tilde{x}_1^* + p_1^r \end{cases}$$

It is important that leader should specify his goal with his tolerance to follower in order to direct him to search for solution in the right direction. Hence the most preferred decision is $\tilde{x}_1^*$, whereas the acceptable decision is at $\tilde{x}_1^* - p_1^l$ and $\tilde{x}_1^* + p_1^r$.

4. A NEW ALGORITHM TO SOLVE FUZZY BI LINEAR PROGRAMMING PROBLEM

Step-1: Represent each fuzzy parameter $\tilde{a} = (a_1, a_2, a_3)$ in terms of $\tilde{a} = \left(a_0, a_*, a^*\right)$.

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Step-2: Find the fuzzy ideal and fuzzy negative ideal solution for both leader and follower's fuzzy objective function subject to the given constraints using fuzzy version of simplex algorithm for the fully fuzzy linear programming problem proposed by Mohanaselvi and Ganesan [11].

Step-3: Define the membership functions corresponding to each fuzzy objective function of leader and follower $\mu_{\tilde{Z}_1}(x)$ and $\mu_{\tilde{Z}_2}(x)$ respectively.

Step-4: Define the membership function for the decision variable $\tilde{x}_1^*$, $\mu_{\tilde{x}_1}(\tilde{x}_1^*)$ with its tolerance level specified by the leader.

Step-5: Formulate the fuzzy model of the given fuzzy bi-level linear programming problem

$$
\begin{aligned}
&\max \lambda \\
&\text{s.t } \tilde{A}_{i1}\tilde{x}_1 + \tilde{A}_{i2}\tilde{x}_2 \le \tilde{B}_i \qquad i = 1,2,......p \\
&\mu_{\tilde{x}_1}(\tilde{x}_1) \ge \lambda I \\
&\mu_{\tilde{Z}_1}(\tilde{Z}_1) \ge \lambda \\
&\mu_{\tilde{Z}_2}(\tilde{Z}_2) \ge \lambda \\
&\lambda \in [0,1] \text{and } \tilde{x}_1, \tilde{x}_2 \succeq \tilde{0} .
\end{aligned}
$$

where $I = (1,1,...,1)^T$ and is same dimension as $\tilde{x}_1$ and $\lambda = \min\{\mu_{\tilde{Z}_1}(\tilde{Z}_1), \mu_{\tilde{Z}_2}(\tilde{Z}_2), \mu_{\tilde{x}_1}(\tilde{x}_1)\}$.

Step-6: Solve the above bi-level linear programming problem with fuzzy parameters without converting to equivalent crisp problem using fuzzy version of simplex algorithm for the fully fuzzy linear programming problem to obtain the fuzzy compromise solution.

5. NUMERICAL EXAMPLE

Consider the following bi-level linear programming problem with fuzzy parameters

$$\max_{\tilde{x}_1} \tilde{Z}_1 \approx (1,3,4)\tilde{x}_1 + (1,2,5)\tilde{x}_2$$

where $\tilde{x}_2$ solves

$$
\begin{aligned}
&\max_{\tilde{x}_2} \tilde{Z}_2 \approx -(0,1,2)\tilde{x}_1 + (1,2,5)\tilde{x}_2 \\
&\text{s.t } -(1,2,5)\tilde{x}_1 + (0,1,2)\tilde{x}_2 \le (0,1,2) \\
&(0,1,2)\tilde{x}_1 \le (1,2,5) \\
&(0,1,2)\tilde{x}_1 + (0,1,2)\tilde{x}_2 \le (1,3,4) \\
&\tilde{x}_1, \tilde{x}_2 \ge \tilde{0}
\end{aligned}
$$

Step-1: Representing the triangular fuzzy numbers in terms of left and right index functions, we have

$$\max_{\tilde{x}_1} Z_1 = (3, 2-2r, 1-r)\tilde{x}_1 + (2, 1-r, 3-3r)\tilde{x}_2$$

where $\tilde{x}_2$ solves

$$\max_{\tilde{x}_2} Z_2 = -(1, 1-r, 1-r)\tilde{x}_1 + (2, 1-r, 3-3r)\tilde{x}_2$$

such that

$$
\begin{aligned}
&-(2, 1-r, 3-3r)\tilde{x}_1 + (1, 1-r, 1-r)\tilde{x}_2 \le (1, 1-r, 1-r) \\
&(1, 1-r, 1-r)\tilde{x}_1 \le (2, 1-r, 3-3r) \\
&(1, 1-r, 1-r)\tilde{x}_1 + (1, 1-r, 1-r)\tilde{x}_2 \le (3, 2-2r, 1-r) \\
&\text{and } \quad \tilde{x}_1, \tilde{x}_2 \ge \tilde{0} \text{ ; } r \in [0,1]
\end{aligned}
$$

Step-2: Using fuzzy version of simplex algorithm proposed by Mohanaselvi and Ganesan [11], the fuzzy ideal and fuzzy negative ideal solution of leaders and followers fuzzy objective function subject to the given constraints is given by:

Table-1. Fuzzy ideal and fuzzy negative ideal solution.

	max	min
$\tilde{Z}_1$	$(8, 2-2r, 3-3r)$	$(0,0,0)$
$\tilde{Z}_2$	$(4, 2-2r, 3-3r)$	$(-2, 1-r, 3-3r)$

Step-3: Define the membership of each objective function's $\tilde{Z}_1$ and $\tilde{Z}_2$ satisfaction degree using definition 3.6.

Step-4: Define the membership of the decision variable $\tilde{x}_1$ of the leader's problem satisfaction degree and the tolerance level.

Step-5: Now, solve the following problem.

$$
\begin{aligned}
&\max \lambda \\
&\text{subject to } (3, 2-2r, 1-r)\tilde{x}_1 + (2, 1-r, 3-3r)\tilde{x}_2 \\
&\qquad -(8, 2-2r, 3-3r)\lambda \ge (0,0,0) \\
&(1, 1-r, 1-r)\tilde{x}_1 - (2, 1-r, 3-3r)\tilde{x}_2 \\
&\qquad +(6, 2-2r, 3-3r)\lambda \le (2, 1-r, 3-3r) \\
&\tilde{x}_1 - 0.8\lambda \ge (1.2, 1-r, 3-3r) \\
&\tilde{x}_1 + 0.1\lambda \le (2.1, 1-r, 3-3r) \\
&-(2, 1-r, 1-r)\tilde{x}_1 + (1, 1-r, 1-r)\tilde{x}_2 \le (1, 1-r, 1-r) \\
&(1, 1-r, 1-r)\tilde{x}_1 \le (2, 1-r, 3-3r) \\
&(1, 1-r, 1-r)\tilde{x}_1 + (1, 1-r, 1-r)\tilde{x}_2 \le (3, 2-2r, 1-r) \\
&\lambda \in [0,1],\ r \in [0,1] \quad \text{and } \tilde{x}_1, \tilde{x}_2 \succeq \tilde{0}
\end{aligned}
$$

Solving the above single objective fuzzy linear programming problem without converting to equivalent crisp problem using fuzzy version of simplex algorithm, we have the fuzzy compromise solution,

$\tilde{x}_1' = (1.62, 2-2r, 3-3r)$ and $\tilde{x}_2' = (1.38, 2-2r, 3-3r)$ for $\lambda = 0.52$.

To take account of overall satisfactory balance between both levels, leader needs to compromise with the follower on leader's minimal satisfactory level.

6. CONCLUSION

In a fuzzy bilevel optimization problem the leader moves first and attempts to maximize his objective function whereas the follower observes the leader's action and moves in a way that is optimal to him which may not be practical. Hence we have proposed a new algorithm to find the fuzzy compromise solution without converting the fuzzy bilevel linear programming problem to an equivalent crisp problem.

Since the decision variables and objective functions are represented as fuzzy membership functions, the proposed fuzzy compromise solution will be more practical and reasonable. Different tolerance levels of the leader will provide the follower to obtain his fuzzy compromise solution. It is important to note that the follower have the flexibility of having the control over the decision variable of the leader whose tolerance level will be specified by the leader.

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