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EFFECT OF WALL THICKNESS ON THE TORSIONAL -DISTORTIONAL RESPONSE OF THIN-WALLED BOX GIRDER STRUCTURES

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ABSTRACT

Trapezoidal box girders are often used in straight and curved bridges due to the large torsional stiffness that results from the closed cross section. However, the torsional loading on the box girder can cause the cross section to distort from its original shape, which results in distortional stresses in the walls of the box section. This paper presents an analytical study on variation of these distortional stresses with the thinness of the walls of trapezoidal box girder sections. Typical torsional loads on box girder bridges were used to obtain the distortional components of these applied torsional loads. Expressions for the generalized governing equations of torsional and distortional equilibrium were obtained on the basis of Vlasov's theory of thin-walled structures. Using the principle of minimum potential energy of a structure, the potential energy of the frame at equilibrium was obtained and minimized with respect to its functional variables to obtain the differential equations of equilibrium for torsional-distortional analysis of trapezoidal box girders structures. Strain modes diagram were generated for various cross sectional member thicknesses and used to obtain the coefficients for the differential equations of equilibrium. By keeping the wall thicknesses of various cross sectional members (deck slab, web, and bottom flange slab) uniform, a practical range of wall thickness for such girders was obtained. By keeping the thickness of the web and bottom flange slab constant, within the practical range of wall thickness, and varying the thickness of the deck slab alone, the effect of slab thickness on the torsional-distortional response of thin-walled mono-symmetric box girder bridges was studied.

Keywords: box girder, deck slab, distortion, thin-walled, torsion, wall thickness.

1. INTRODUCTION

Torsional effects created by eccentric loads (on straight bridge girders or by centrifugal forces on curved bridge girders tend to deflect and rotate the box girder structure and also distort the cross section in terms of displacement. These may give rise to significant transverse bending and longitudinal warping stresses which are usually not easy to evaluate.

The torsional-distortional behaviour of such box girder structures may be exactly predicted by a closed-form solution of the governing differential equations for the appropriate boundary restraints, interface conditions and torsional loadings. Alternately, the box girder may be descretized and a numerical or matrix method of analysis may be applied for an accurate approximation of the response. Finite element (matrix) formulations of the problem are numerous, Lonkar (1968), whereas closed-form and numerical technique solutions are scarce, Lee and Szoba (1967), Osadebe and Mbajiogu (2006).

In this work, a general procedure for obtaining and applying the governing differential equations of equilibrium for torsional-distortional analysis of mono-symmetric box girder bridges based on Vlasov's theory of thin-walled structures, Vlasov (1958), is presented.

Eleven specific cross section examples involving various wall thicknesses and cross-sectional arrangements were considered. The results obtained were used to predict the effect of deck slab thickness on the torsional-distortional behaviour of reinforced concrete box girder bridges.

2. THEORY

2.1. Background principles

The elastic strains and stresses as well as their corresponding strain relations for a thin walled box girder can be obtained after the formulation of Vlasov's displacement functions. In this study, generalized strain fields (modes) were employed in order to reduce the number of unknown displacement functions in the equations of equilibrium. Torsional- distortional analysis was accomplished by using energy principles on the basis of minimum potential energy to obtain the energy functional of thin walled box girder structures with arbitrary deformable cross sections. Using the principle of variational calculus the energy functional was minimized with respect to its functional variables which are displacement functionals and their first and second order derivatives, to obtain the equations of equilibrium in the form of a system of linear differential equations in displacement quantities.

After obtaining the strain modes diagrams, the coefficients of these equations were determined using Morh's integral for displacement computations. By solving these linear differential equations the displacement functions (longitudinal and transverse displacements) were obtained and the distortional bending moments evaluated.

2.2. Potential energy of a thin-walled box structure

The potential energy of a box structure in terms of the strain energy and the work done by external loads is as follows, Osadebe and Chidolue (2012):

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$$\Pi = \frac{E}{2}\sum a_{ij}U_i\,'(x)U_j\,'(x)dx +$$
$$+\frac{G}{2}\left[\sum b_{ij}U_i(x)U_j(x) + \sum c_{kj}U_k(x)V_j\,'(x)\right]dx +$$
$$+\frac{G}{2}\left[\sum c_{ih}U_i(x)V_h\,'(x) + \sum r_{kh}V_k\,'(x)V_h\,'(x)\right]dx +$$
$$+\frac{E}{2}\sum s_{hk}V_k(x)V_h(x)dx - \sum q_h V_h dx \quad (1)$$

where Π = the total potential energy of the box structure, $U_i(x)$ and $V_k(x)$ are unknown functions which express the laws governing the variation of the displacements along the length of the box girder frame.

q_h = Line load per unit area applied in the plane of the box girder plates
E = Modulus of elasticity
G = Shear modulus

a_{ij}, b_{ij}, c_{kj}, r_{kh}, s_{hk} are Vlasov's coefficients given by the following expressions, Osadebe and Chidolue (2012).

$$a_{ij} = a_{ji} = \int \varphi_i(s)\varphi_j(s)dA \quad \text{(a)}$$

$$b_{ij} = b_{ji} = \int \varphi_i^{'}(s)\varphi_j^{'}(s)dA \quad \text{(b)}$$

$$c_{kj} = c_{jk} = \int \varphi_k^{'}(s)\psi_j(s)dA \quad \text{(c)}$$

$$c_{ih} = c_{hi} = \int \varphi_i^{'}(s)\psi_k(s)dA \quad \text{(d)}$$

$$r_{kh} = r_{hk} = \int \psi_k(s)\psi_h(s)dA; \quad \text{(e)}$$

$$s_{kh} = s_{hk} = \frac{1}{E}\int \frac{M_k(s)M_h(s)}{EI_{(s)}}ds \quad \text{(f)}$$

$$q_h = \int q\psi_h ds \quad \text{(g) (2)}$$

These coefficients depend on a combination of elementary displacements or strain fields; three in the longitudinal direction and four in the transverse direction. The strain fields are:

φ_1 = out of plane displacement strain mode, due to vertical load normal to bridge longitudinal / horizontal plane

φ_2 = out of plane displacement strain mode, due to horizontal load normal to bridge longitudinal/vertical plane

φ_3 = out of plane displacement strain mode, due to warping of the cross section

ψ_1 = in-plane displacement strain mode, due to vertical load, normal to bridge longitudinal / horizontal plane

ψ_2 = in-plane displacement strain mode, due to horizontal load normal to bridge longitudinal / vertical plane

ψ_3 = in-plane displacement strain mode, due to distortion of the cross section

ψ_4 = in-plane displacement strain mode, due to pure rotation of the cross section.

$\varphi_i\,'$ and $\varphi_j\,'$ are first order derivatives of φ_i and φ_j, respectively.

Some or all of these strain modes may be present in a given frame depending on the geometry of the cross section and the nature of loading.

2.3. Governing equations of equilibrium

The governing equations of torsional-distortional equilibrium are obtained by minimizing the energy functional equation (1), with respect to its functional variables U(x) and V(x) using Euler Lagrange technique, Elsgolt (1980).

Minimizing with respect to U(x) we obtain;

$$k\sum_{i=1}^{m} a_{ij}U_i\,''(x) - \sum_{i=1}^{m} b_{ij}U_i(x) - \sum_{k=1}^{n} c_{kj}V_k\,'(x) = 0 \quad (3)$$

Minimizing with respect to V(x) we have;

$$\sum c_{ih}U_i\,'(x) - \kappa\sum s_{hk}V_k(x)$$
$$+\sum r_{kh}V_k\,''(x) + \frac{1}{G}\sum q_h = 0 \quad (4)$$

$$\text{where, } \kappa = \frac{E}{G} = 2(1+\nu) \quad (5)$$

ν = poisson ratio

Equations (3) and (4) are Vlasov's generalized differential equations of torsional-distortional equilibrium for a box girder. They are presented in matrix form as follows:

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$$\kappa\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{Bmatrix} U_1'' \\ U_2'' \\ U_3''\end{Bmatrix} - \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33}\end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \\ U_3\end{Bmatrix} -$$

$$-\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34}\end{bmatrix}\begin{Bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4'\end{Bmatrix} = 0 \qquad (6)$$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43}\end{bmatrix}\begin{Bmatrix} U_1' \\ U_2' \\ U_3'\end{Bmatrix} - \kappa\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{41} & s_{43} & s_{44}\end{bmatrix}\begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4\end{Bmatrix} +$$

$$+\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44}\end{bmatrix}\begin{Bmatrix} V_1'' \\ V_2'' \\ V_3'' \\ V_4''\end{Bmatrix} + \frac{1}{G}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4\end{bmatrix} = 0 \qquad (7)$$

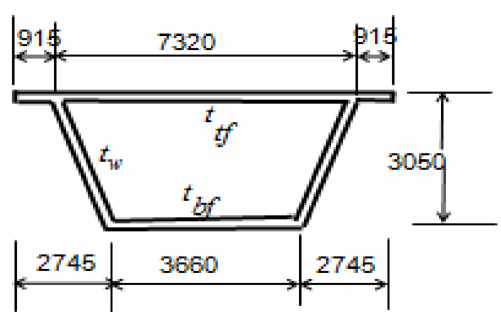
The a_{ij} coefficients depend on the interaction between strain modes φ_1, φ_2 and φ_3, giving rise to the elements in the first square bracket of equation (6). The b_{ij} coefficients depend on the interaction between the derivatives of strain modes φ_1, φ_2 and φ_3, i.e., φ_1', φ_2', and φ_3', giving rise to the elements in the second square bracket in equation (6). The c_{kj} and c_{ih} coefficients depend on the interactions between the derivatives of strain modes φ_i and strain modes ψ_k, i.e., between $\varphi_1', \varphi_2', \varphi_3'$ and $\psi_1, \psi_2, \psi_3, \psi_4$, from where we obtain the elements in the third square bracket in equation (6) and the first square bracket in equation (7). The r_{kh} coefficients depend on the interaction between strain modes $\psi_1, \psi_2, \psi_3, \psi_4$, giving rise to the elements in the third square bracket in equation (7). The s_{hk} coefficients given by equation 2(f) depend on the bending deformation of the strip, characterized by the distortional bending moments M_1, M_2, M_3, and M_4 due to ψ_1, ψ_2, ψ_3, and ψ_4 strain modes. To compute the coefficients we need to construct the diagram of the bending moments due to the strain modes, ψ_1, ψ_2, ψ_3 *and* ψ_4. Incidentally, ψ_1, ψ_2 and ψ_4 strain modes do not generate distortional bending moments as they involve pure bending and pure rotation. Only ψ_3 strain mode generates distortional bending moment which can be evaluated using the distortion diagram for the relevant cross section, Rekach (1978). Consequently the relevant expression for the s_{hk} coefficients becomes;

$$s_{hk} = s_{kh} = \frac{1}{E}\int_s \frac{M_3(s)M_3(s)}{EI_s},$$

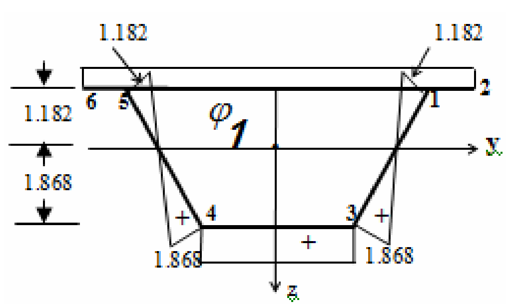
where $M_3(s)$ is the distortional bending moment of the relevant cross section.

3. STRAIN MODE DIAGRAMS AND EVALUATION OF VLASOV'S COEFFICIENTS

Figure-1(a) shows the cross section of a single cell mono symmetric thin-walled box girder structure (regarded as a frame) for which the strain modes diagrams were obtained for various combinations of top flange, bottom flange and web thicknesses, ranging from 100mm to 400mm. For each combination of cross section dimensions the strain modes diagrams were obtained and Vlasov's coefficients computed for numerical analysis of the box girder structures. The procedures for evaluation of Vlasov's coefficients are given in literatures, Chidolue (2012), Osadebe and Chidolue (2012), Rekach (1978). Figures 1(b) to 1(h) show typical generalized strain modes diagrams for a single cell mono-symmetric thin-walled box girder structure with wall thickness of 300mm for top flange and 350mm for web and bottom flange. The computed relevant Vlasov's coefficients for torsional-distortional analysis of this box girder structure and other box girder structures with various combinations of wall thicknesses are given in Tables 1(a) and 1(b).



(a) Single cell box girder section for numerical analysis.



(b) Longitudinal strain mode diagram: y-y axis bending

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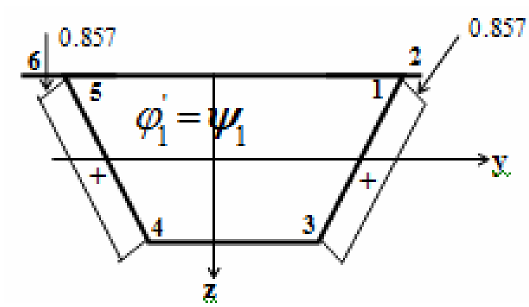
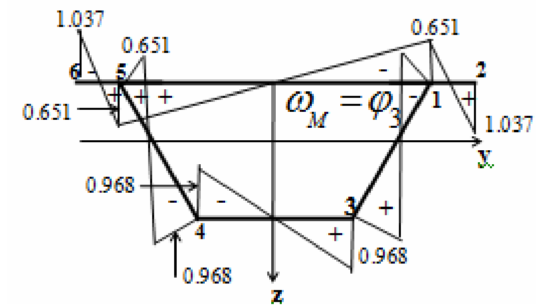
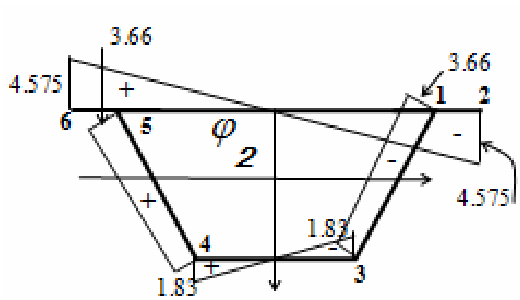


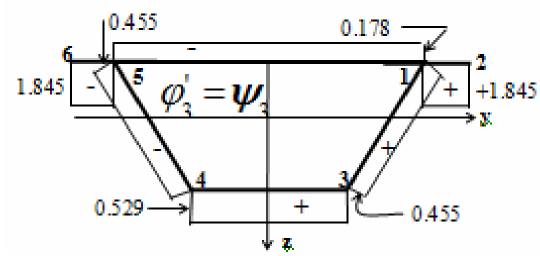
(c) Transverse strain mode in y-direction.



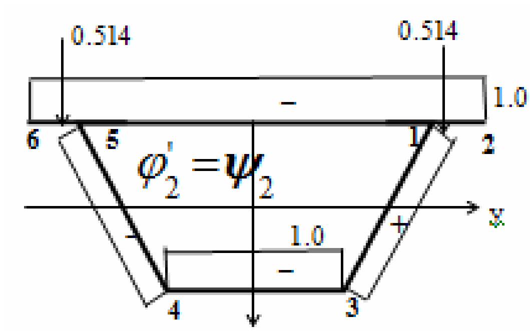
(f) Warping function diagram.



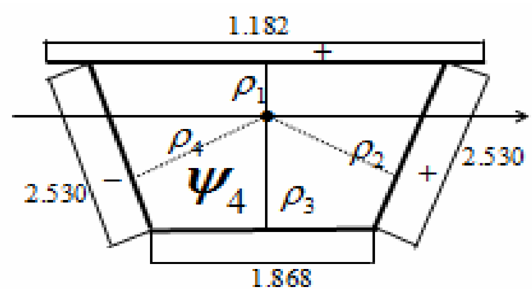
(d) Longitudinal strain mode diagram
(Bending about z-z axis).



(g) Distortion diagram.



(e) Transverse strain mode in z-direction.



(h) Pure Rotation diagram.

Figure-1. Generalized strain modes for single cell mono-symmetric box girder frame with $t_{tf} = 300mm,\ t_w = t_{bf} = 350mm$.

Table-1(a). Values of relevant vlasov's coefficients.

Uniform thickness of cross section members $t_{tf} = t_{bf} = t_w$ (mm)	a_{33}	b_{33}, c_{33}, r_{33}	$c_{34}, c_{43}, r_{34}\, r_{43}$	r_{44}	$s_{33} \times 10^{-4}$
100	0.373	0.693	0.623	7.20	0.217
150	0.560	1.040	0.935	10.80	0.734
200	0.746	1.386	1.246	14.40	1.740
250	0.933	1.733	1.558	18.00	3.398
300	1.119	2.079	1.869	21.60	5.872
350	1.306	2.246	2.181	25.20	9.325
400	1.492	2.772	2.492	28.80	13.92

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Table-1(b). Values of relevant vlasov's coefficients for varying wall thickness.

t_{bf}, t_w = 350mm, varying t_{tf} (mm) see Figure-1(a)	a_{33}	b_{33}, c_{33}, r_{33}	c_{34}, c_{43}, r_{34} r_{43}	r_{44}	s_{33} x 10^{-3}
300	1.468	2.812	2.335	24.243	0.888
350	1.306	2.246	2.181	25.20	0.932
400	3.351	3.820	2.558	26.765	3.065
500	10.603	7.654	2.964	27.35	10.610
700	23.653	14.295	3.105	29.222	27.89

4. FORMULATION OF DIFFERENTIAL EQUATIONS OF EQUILIBRIUM FOR MONO SYMMETRIC BOX GIRDER SECTIONS

The relevant coefficients for torsional-distortional equilibrium (strain modes 3 and 4), are a_{33}, b_{33}, c_{33}, c_{34}, r_{33}, r_{34}, r_{44} and s_{33}. All other coefficients are taken as zero. Substituting these into the matrix notation equations (6) and (7) and multiplying out, we obtain:

$$ka_{33}U_3'' - b_{33}U_3 - c_{33}V_3' - c_{34}V_4' = 0 \quad (8)$$

$$c_{33}U_3' - ks_{33}V_3 + r_{33}V_3'' + r_{34}V_4'' = -\frac{q_3}{G} \quad (9)$$

$$c_{43}U_3' + r_{43}V_3'' + r_{44}V_4'' = -\frac{q_4}{G} \quad (10)$$

Simplifying further by eliminating U_3, U_3' and U_3'' we obtain the coupled differential equations of torsional-distortional equilibrium for the analysis of mono symmetric box girder sections as follow:

$$\begin{aligned} &\beta_1 V_4'' - \gamma_1 V_3 = K_1 \quad (a) \\ &\alpha_1 V_3^{iv} + \alpha_2 V_4^{iv} - \beta_2 V_4'' = K_2 \quad (b) \end{aligned} \quad (11)$$

where, V_3 and V_4 are the distortional displacement and the torsional displacement, respectively of the box girder structure.

$$\alpha_1 = ka_{33}c_{43}: \quad \alpha_2 = ka_{33}r_{44}: \quad \gamma_1 = c_{43}ks_{33} \quad (12)$$

$$\beta_1 = r_{34}c_{43} - c_{33}r_{44}: \quad \beta_2 = \frac{b_{33}r_{44} - c_{34}c_{43}}{ka_{33}c_{43}}, \quad (13)$$

$$K_1 = c_{33}\frac{\overline{q}_4}{G} - c_{43}\frac{\overline{q}_3}{G} \quad K_2 = \left(\frac{b_{33}}{ka_{33}c_{43}}\right)\frac{\overline{q}_4}{G} \quad (14)$$

5. TORSIONAL-DISTORTIONAL ANALYSIS OF THIN-WALLED MONO SYMMETRIC BOX GIRDER STRUCTURES

In this section the solutions of the differential equations of equilibrium, equations 11(a) and 11(b) are obtained for various combinations of wall thicknesses of the single cell mono symmetric box girder section shown in Figure-1 (a).

Live loads were considered according to AASHTO-LRFD (1998), following the HL-93 loading: uniform lane load of 9.3N/mm distributed over a 3m width plus tandem load of two 110 KN axles. The loads were positioned at the outermost possible location to generate the maximum torsional effects on the box girder bridge. A three span simply supported bridge deck structure, 50m per span, was considered.

The obtained torsional loads are as follows; $\overline{q}_3 = 157.16KN$, $\overline{q}_4 = 1808.13KN$.

The governing equations of equilibrium are

$$\begin{aligned} &\beta_1 V_4'' - \gamma_1 V_3 = K_1 \\ &\alpha_1 V_3^{iv} + \alpha_2 V_4^{iv} - \beta_2 V_4'' = K_2 \end{aligned} \quad (15)$$

Taking, for example, the basic cross section, Figure-1a, with $t_{tf} = t_{bf} = t_w = 200$mm, the values of the relevant coefficients from Table-1(a) are;

$$a_{33} = 0.746; \; b_{33} = c_{33} = r_{33} = 1.386$$
$$c_{34} = c_{43} = r_{34} = r_{43} = 1.246;$$
$$r_{44} = 14.40, \; s_{33} = 1.740 * 10^{-4}$$

The parameters for the governing equations are,

$$\alpha_1 = ka_{33}c_{43} = 2.324$$

$$\alpha_2 = ka_{33}r_{44} = 26.857$$

$$\beta_1 = r_{34}c_{43} - c_{33}r_{44} = -18.406$$

$$\beta_2 = b_{33}r_{44} - c_{34}c_{43} = 18.406$$

$$\gamma_1 = c_{43}ks_{33} = 5.420 * 10^{-4}$$

$$K_1 = b_{33}q_{33} / G = 2.836 * 10^{-5}$$

$$K_2 = \frac{c_{33}q_4 - c_{43}q_3}{G} = 2.356*10^{-4}$$

$$E = 24*10^9 N/m^2, G = 9.6*10^9 N/m^2, k = 2.5$$

Substituting these parameters into equation (15) we obtain:

$$\begin{aligned} &-18.406V_4^{iv} - 5.420*10^{-4}V_3 = 2.836*10^{-4} \\ &2.324V_3^{iv} + 26.856V_4^{iv} - 18.406V_4'' = 1.833*10^{-3} \end{aligned} \quad (16)$$

Integrating by method of trigonometric series with accelerated convergence, we have

$$V_3(x) = 355.31*10^{-3} Sin(\pi x/50)$$

$$V_4(x) = 3.047*10^{-3} Sin(\pi x/50) \quad (17)$$

The results for similar analysis of the thin-walled mono-symmetric box girder structures whose wall thicknesses and Vlasov's coefficients are given in tables 1(a) and 1(b) respectively are given in Tables 2(a) and 2(b).

6. RESULTS AND DISCUSSIONS

The first set of results, Table-2(a) is for box girder sections with uniform wall thicknesses, while the second set of results, Table-2(b), is for varying deck thickness. In the later case, the thicknesses of the web and bottom flange materials were kept constant while varying the thickness of the deck (top flange) slab.

Table-2(a). Variation of maximum distortional and torsional displacements with wall thickness.

Uniform wall thickness (mm)	**Distortional displacement** $V_3 * Sin\frac{\pi x}{50}$ (mm)	**Torsional displacement** $V_4 * Sin\frac{\pi x}{50}$ (mm)
100	3155.45	6.652
150	923.25	4.392
200	355.33	3.047
250	207.40	2.73
300	109.32	2.095
350	69.42	1.808
400	46.74	1.586

Table-2(b). Variation of maximum (mid span) distortional and torsional displacements with deck thickness.

t_{bf}, t_w = **350mm, Varying** t_{tf} (mm)	**Distortional displacement** $V_3 * Sin\frac{\pi x}{50}$ (mm)	**Torsional displacement** $V_4 * Sin\frac{\pi x}{50}$ (mm)
300	79.32	1.892
400	29.22	1.723
500	15.00	1.694
700	10.48	1.587

6.1. Practical range of wall thickness

For a given box girder span and cross section dimensions, there are corresponding values of thickness of materials which enables the bridge structure to satisfy both strength and serviceability requirements. For the purpose of this study, it is desired to determine the practical range of material (wall) thicknesses that will satisfy both strength and deflection requirements of the box girder structure.

Whereas provision of appropriate reinforcements obtained from design calculations takes care of strength requirement of the box girder, deflection is essentially controlled by allowable values, depending on the code of practice. Taking the allowable deflection on a bridge structure as span/800, Xanthakos (1994), the practical range of thickness of material for the box girder section depends on the minimum values of the thickness of top flange (deck), bottom flange, and web materials at which the maximum (mid-span) deflection does not exceed span/800.

Taking the total deflection Δ to be made up of the bending (theoretical) deflection δ and the distortional load deformation V_3, it therefore follows that $\Delta = \delta + V_3$. For deflection requirements to be satisfied $(\delta + V_3) \le Span/180$, $\therefore\ V_3 \le |Span/180 - \delta|$. Hence limiting (uniform) thickness for top flange (deck), bottom flange and web materials are those for which

$$V_3 \le |Span/180 - \delta| \quad (18)$$

Once the practical range of the uniform thickness of materials of the box girder section is established, the influence of any member thickness (in this case the thickness of the deck slab) on the torsional-distortional behaviour of the thin-walled box girder structure can be studied further.

Table-3 shows the theoretical deflections compared with the distortional displacements for various wall thicknesses of the box girder section. The Table suggests that the minimum wall thickness of the box from practical point of view lies between 250mm and 300mm. This is further amplified by plots of the variation of maximum deflection, maximum distortion and limiting deflection with box girder wall thickness shown in Figure-2, from where it can be ascertained that equation (18) is

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satisfied at all material thicknesses greater than 275mm. Thus, the practical range of wall thickness within the limits of this study is 275mm and above. Figure-2 also reveals that within this range of uniform wall thickness, the distortional displacement is less than the limiting deflection, and the wall thicknesses of the box girder structure satisfy deflection requirements.

By keeping the thickness of the bottom flange and the web fixed at 300mm while varying that of the deck slab, the result obtained, Figure-3, shows the variation of torsional and distortional displacements with thickness of the deck slab of a reinforced concrete box girder structure. A point to note here which is not well defined in Figure-3, because of limitation of vertical axis scale is that the variation of distortional displacement with slab thickness is parabolic in shape and asymptotic to a vertical axis drawn through the minimum permissible wall thickness.

Table-3. Comparison of theoretical deflection, limiting deflection and distortional displacement.

Uniform thickness of walls	**Theoretical deflection** δ (mm)	$\left\lvert \frac{span}{800} - \delta \right\rvert$ **Limiting deflection** (mm))	**Distortional displacement** V_3 (mm)
150	290	227.5	923
200	250	187.5	355
250	224	161.5	207
300	211	148.5	109
350	197	134.5	69
400	187	124.5	47
500	172	109.5	22
600	162	99.5	12
700	151	88.5	9

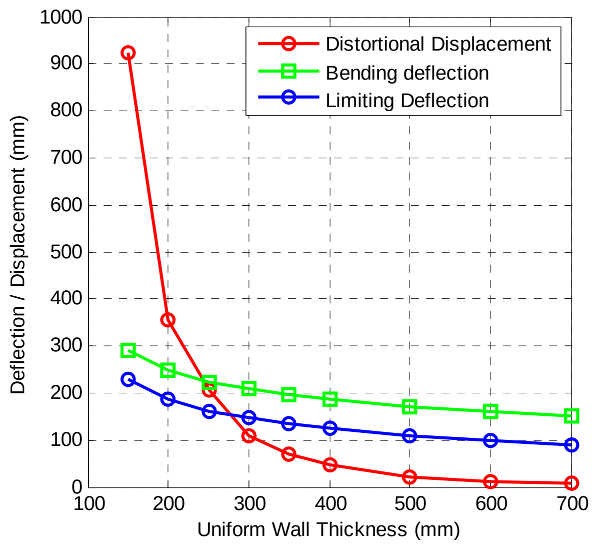

Figure-2. Variation of maximum deflection, maximum distortion and limiting deflection with uniform box girder wall thickness.

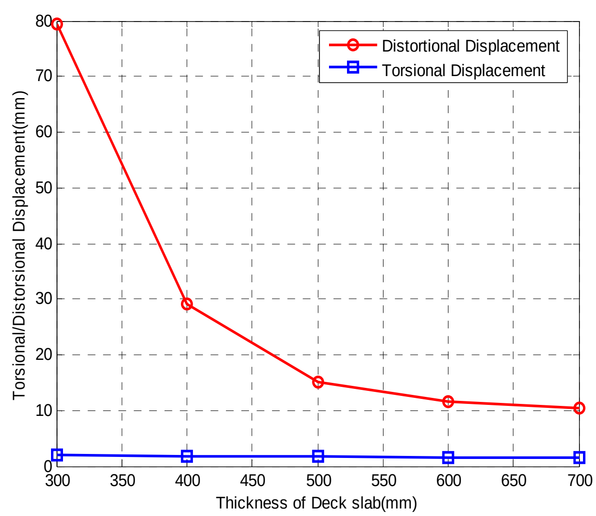

Figure-3. Variation of maximum distortional and maximum torsional displacements with thickness of deck slab.

Generally it can be stated that outside the practical range of material thicknesses the distortional displacements are far greater than the theoretical deflections. Torsional displacements were found to be negligible, $\leq 15\%$ of the corresponding distortional deformations.

CONCLUSIONS

As would be expected, increasing the thickness of the cross sectional member's increases the torsional rigidity of the box girder frame and lowers the distortional deformation. Box girders with wall thicknesses within the practical range have moderate distortional displacements (lower than the theoretical deflection of the girder. The practical range of wall thickness for trapezoidal box girders depend on parameters such as span of girder, overall dept of girder, width of top flange, and the inclination of the web members.

For every cross sectional dimensions of a mono symmetric box girder, there is a corresponding minimum thickness of the members for the structure to satisfy distortional deformation as well as bending deformation (deflection). Optimization design of trapezoidal box girder structures is necessary in order to obtain sections that simultaneously satisfy bending, deflection and torsional and distortional stresses.

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