



NONLINEAR FINITE ELEMENT ANALYSIS OF SHALLOW REINFORCED CONCRETE BEAMS USING SOLID65 ELEMENT

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ABSTRACT

Shallow reinforced concrete beams are widely used in building design and construction of waffle and ribbed slabs in Jordan and in many other countries. Previous researches included experimental based testing that has been widely used as a means to study the response of reinforced concrete flexural elements. In this regard the use of finite element analysis is gaining popularity as it is cost effective. Thus this work aims to study the behavior of shallow reinforced concrete beams when subjected to transverse loading, in terms of the resulting distribution of stresses, cracks and load deflection relationship, using finite element analysis utilizing ANSYS software. Solid65 eight noded isoparametric elements are used to model the concrete. The behavior of concrete material in compression is elasto-plastic work hardening model which is terminated at the onset of crushing. Link180 element models the reinforcement; the material model for the discrete steel reinforcement is linear elastic prior to initial yield surface, beyond that it is perfectly plastic.

Keywords: finite element analysis, flexural elements, concrete beams, flexural behavior.

INTRODUCTION

There is an immense work in the field of nonlinear finite element analysis of reinforced concrete structures [1]. The experimental work is costly and time consuming, whereas the utilization of finite element analysis to study the behavior of reinforced concrete members is cost effective. Recently the existence of commercial computer software that can model the behavior of reinforced concrete structural elements until failure made computer numerical modeling more attractive as it minimizes the cost and is much faster. Data obtained from a finite element analysis package is not useful unless the necessary steps are taken to understand what is happening within the model that is created using the software.

Faherty [2] studied a reinforced concrete beam using finite element method of analysis. The adopted beam modeling was simply supported and was loaded with two symmetrically placed concentrated transverse loads. Kachlakev *et al.*, [3] used Ansys to study concrete beam members with externally bonded Carbon Fiber Reinforced Polymer fabric.

The earliest publication on the application of the finite element method to the analysis of RC structures was presented by Ngo and Scordelis [4]. In their study, simple beams were analyzed with a model in which concrete and reinforcing steel were represented by constant strain triangular elements, and a special bond link element was used to connect the steel to the concrete and describe the bond slip effect. A linear elastic analysis was performed on beams with predefined crack patterns to determine principal stresses in concrete, stresses in steel reinforcement and bond stresses.

Nilson [5] introduced nonlinear material properties for concrete and steel and a nonlinear bond-slip relationship into the analysis and used an incremental load method of nonlinear analysis. Four constant strain

triangular elements were combined to form a quadrilateral element by condensing out the central node. Cracking was accounted for by stopping the solution when an element reached the tensile strength.

For the analysis of RC beams with material and geometric nonlinearities Rajagopal [6] developed a layered rectangular plate element with axial and bending stiffness in which concrete was treated as an orthotropic material. Reinforced concrete beams have also been treated by many other investigators [7, 8, 9] using similar methods.

Reza and Seyed [10] experimentally and theoretically investigated six under-reinforced concrete beams. Each concrete beam was reinforced with two 16mm diameter steel bars for tension. Balamuralikrishnan *et al.*, 2008 [11] investigated rectangular beams with bonded CFRP fabric. The beams were subjected to the four point bending.

The objective of this paper is to perform a nonlinear finite element analysis on a shallow reinforced concrete beam utilizing Ansys software. The work involves studying the crack development as the applied transverse load is increased and also deriving the load deflection curve. The outcomes of the Ansys software simulation is compared with computed values carried out in accordance with Building Code Requirements for Structural Concrete ACI (318M-11).

MODELING OF RC BEAM

The solid65 element models the nonlinear response of reinforced concrete. The behavior of the concrete material is based on a constitutive model for the triaxial behavior of concrete after Williams and Warnke [12]. Solid 65 is capable of plastic deformation, cracking in three orthogonal directions at each integration point. The cracking is modelled through an adjustment of the material properties that is done by changing the element



stiffness matrices. If the concrete at an integration point fails in uniaxial, biaxial, or triaxial compression, the concrete is assumed crushed at that point. Crushing is defined as the complete deterioration of the structural integrity of the concrete.

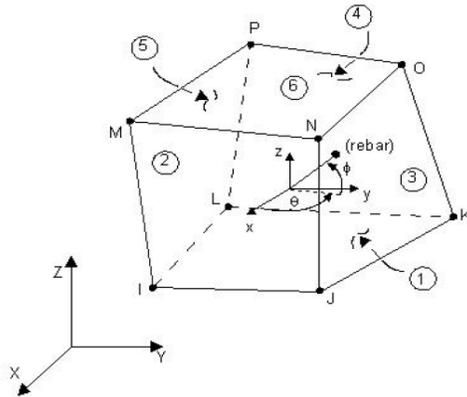


Figure-1. Solid65 element [13].

Ansys allows entering three reinforcement bars materials in the concrete, each rebar material corresponds to the x, y, and z directions of the smeared element [13]. A schematic of the element is shown in Figure-1.

MATERIAL PROPERTIES

Table-1 lists concrete properties within Solid65 element prior to initial yield surface, beyond that concrete parameters are shown in Table-2.

The solid65 element is capable of cracking in tension and crushing in compression. The multi linear isotropic concrete model uses the von Mises failure criterion along with Willam and Warnke model to define the failure of concrete.

Table-1. Concrete properties prior to initial yield surface.

Material	Material model	Modulus of elasticity MPa	Poisson's ratio
Concrete	Linear elastic	25743	0.3

Table-2. Concrete parameters beyond initial yield surface.

Open shear transfer coefficient, β_t	0.3
Closed shear transfer coefficient, β_c	0.9
Uniaxial cracking stress	3.78 Mpa
Uniaxial crushing stress f_c	50 Mpa

The compressive uniaxial stress-strain relationship for the concrete model in Figure-2 was obtained using the following equations to compute the multilinear isotropic stress-strain curve for the concrete [8].

$$f = \frac{E_c \varepsilon}{1 + \left(\frac{\varepsilon}{\varepsilon_o}\right)^2} \tag{1}$$

$$E_c = \frac{f}{\varepsilon} \tag{2}$$

$$\varepsilon_o = \frac{2f'_c}{E_c} \tag{3}$$

Where

f = stress at any strain

ε = strain at stress f

ε_o = strain at ultimate compressive strength

E_c = concrete modulus of Elasticity

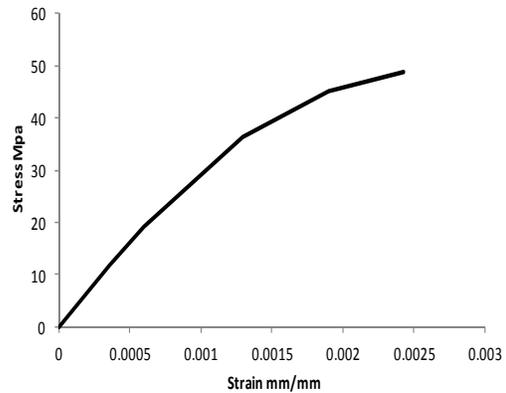


Figure-2. Concrete stress strain curve for unidirectional monotonic compressive loading.

Cracking and crushing are determined by a failure surface. Once the failure surface is surpassed, concrete cracks, if any, principal stress is tensile, while the crushing occurs if all principal stresses are compressive. The failure surface for compressive stresses is based on Willam-Warnke failure criterion which depends on five material parameters. Tensile stress consists of a maximum tensile stress criterion: a tension cutoff. Unless plastic deformation is taken into account, the material behavior is linear elastic until failure. When the failure surface is reached, stresses in that direction have a sudden drop to zero, there is no strain softening neither in compression nor in tension. Two shear transfer coefficients are presented in Table-2, one for open cracks and the other for closed ones, and are used to consider the retention of shear stiffness in cracked concrete.

As shown in Figure-3, Material Model for steel reinforcement is linear elastic prior to initial yield surface, beyond the initial yield surface it is perfectly plastic, in tension and compression loading.

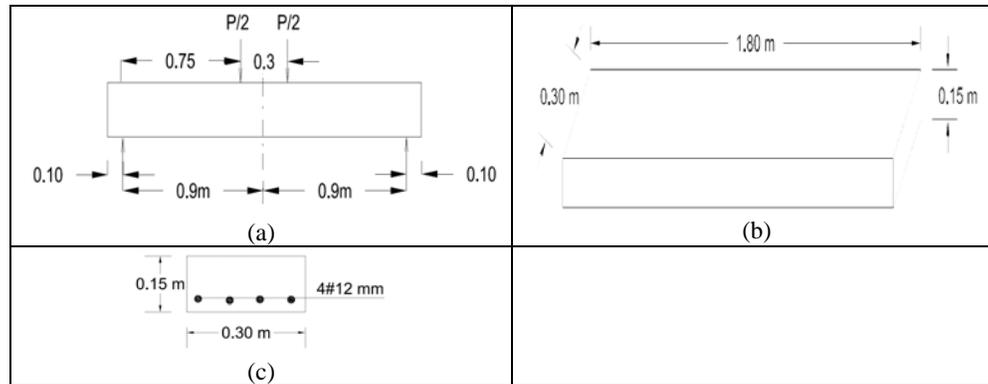
**Table-3.** Properties for the steel reinforcement.

Material model prior to initial yield surface	Linear elastic
Elastic modulus, E_s	200 GPa
Poisson's ratio	$\nu=0.3$
Yield stress, f_y	412 MPa
Material model beyond initial yield surface and up to failure	perfect plastic

Table-3 lists the properties of the 4 #12 mm bars steel reinforcement.

NUMERICAL EXAMPLE

As illustrated in Figure-3, the adopted reinforced concrete shallow beam is simply supported and loaded with two symmetrically placed concentrated transverse loads. Beam width = 0.30m, depth = 0.15m, beam effective span length = 1.80m.

**Figure-3.** Reinforced concrete beam (a) Loading (b) Dimensions (c) Section details

LOADS AND BOUNDARY CONDITIONS

The model is considered symmetric about a vertical section at mid span, a distance 0.9m from the left support, and also is symmetric about a vertical section at $b/2$. Thus the computer model section width is 0.15m, and section depth is 0.150 m as shown in Figure-4.

Nodes defining a vertical plane through the beam cross-section at mid span that is a distance of 0.9m from left support, define a plane of symmetry, nodes on this

plane have a degree of freedom constraint $u_x = 0$. Nodes defining a vertical plane at half the cross section width, define another plane of symmetry, nodes at this plane have a degree of freedom constraint $u_z = 0$. The boundary conditions for both planes of symmetry, supports and loadings are shown in Figure-4.

RESULTS AND DISCUSSIONS

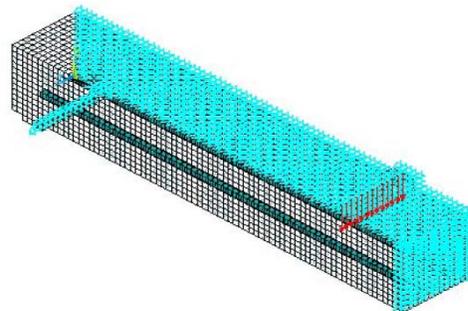
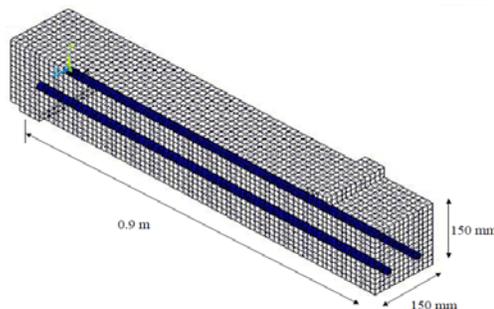
**Figure-4.** Ansys model (a) Finite Element discretization. (b) Boundary conditions.

Figure-5 illustrates the development of cracks within the shallow reinforced concrete beam structural model. As the applied load exceeds the cracking moment, bending cracks start to appear. The initial cracking of the beam appeared at a load $P = 10.1$ KN. The first crack appeared in the constant moment region, and is a vertical flexural crack. It is noticed that bending cracks development is flat and sudden, this implies that the tensile stress relaxation is not functioning properly,

leading to a sudden stress drop. At larger loading, bending shear cracks appear as shown in Figure-5 (b), (c). Subsequent cracking occurs as more load is applied to the beam. Cracking increases in the constant moment region, and the beam begins cracking out towards the supports as the load becomes larger. Significant flexural cracking occurred at $P = 17.2$ KN. Yielding of steel reinforcement occurred when a force of 20.4 KN was reached.



Beyond steel yielding, the deformations become larger, the cracked moment of inertia, yielding of steel and nonlinear concrete material define the response. Thus larger deflection takes place at mid-span, and significant flexural and diagonal cracks appear. Figure-5(c) shows successive cracking of the beam beyond yielding of steel reinforcement. At 32 KN cracking extends towards the top and the beam is at the verge of failure.

Figure-6 illustrates the deflected shape of the beam model. Initially and up to a load $P = 10.1$ KN, the linear elastic material behavior for both steel reinforcement and concrete defines the flexural rigidity.

When concrete stresses exceed modulus of rupture, cracked transformed moment of inertia in addition to linear elastic steel and concrete behavior defines the flexural rigidity of the beam, then at concrete compressive stresses beyond $0.5f_c$ nonlinear concrete behavior takes place, at this stage deflection is computed based on curvature.

The load deformation response of the model compares well with the calculated deflections in accordance with Building Code Requirements for Structural Concrete ACI [14], until about 75% of the beam nominal strength.

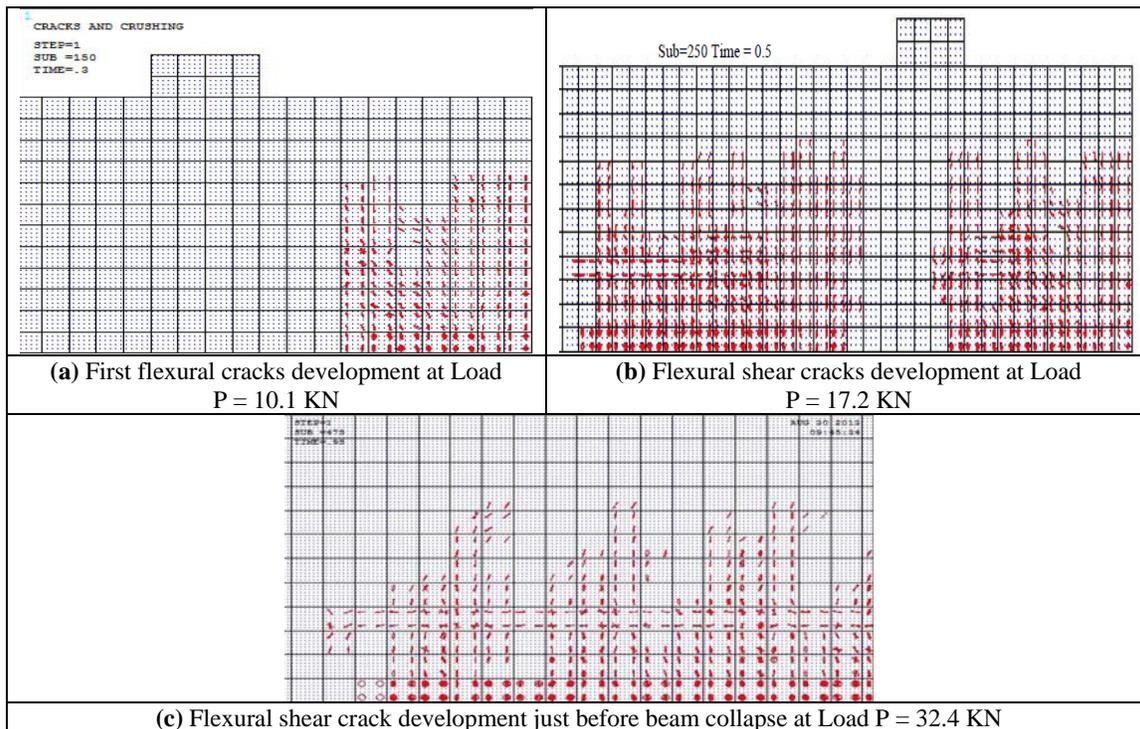


Figure-5. Bending and shear crack development at the front side of the beam. (a) Flexural cracks at time=0.3, substep=150, (b) Spread of flexural shear cracks at time=0.5, substep=250, (c) Flexural shear crack development at time=0.95, substep=479 just before beam collapse.

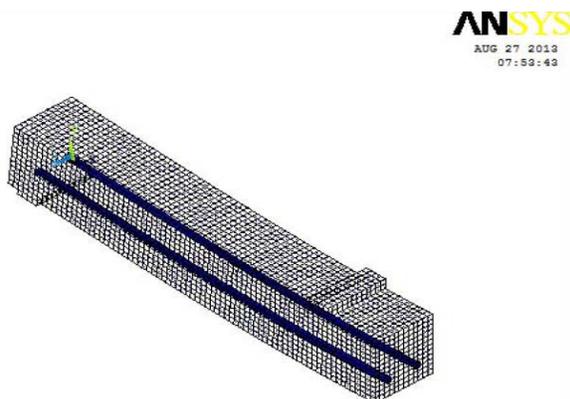


Figure-6. Deflected shape of the shallow reinforced concrete beam according to Ansys simulation.

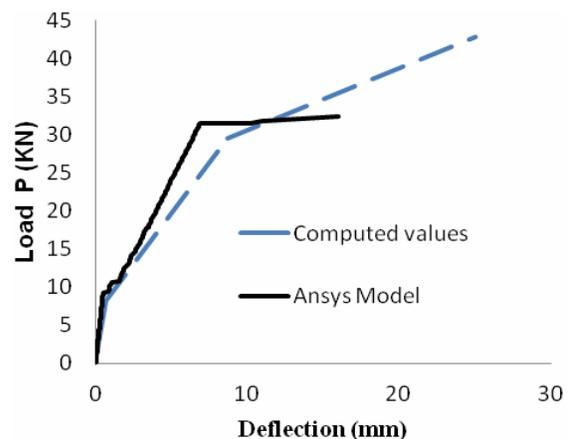


Figure-7. Load deflection relationship.



CONCLUSIONS

- The outcomes of the conducted finite element analysis, utilizing Ansys software for the shallow reinforced concrete beam, are in good agreement with the outcomes obtained from the calculations using ACI provisions regarding the load deflection relationship, and the crack initiation and propagation.
- The initial cracking in the finite element beam model took place in the form of vertical flexure cracks.
- Subsequent flexural shear cracks appeared in the model as the load was increased beyond the cracking moment strength and until steel reinforcement yielding.
- The load deflection relationship is linear elastic up to the cracking moment strength then the curve inclines more towards the horizontal. After steel reinforcement yielding the curve inclines appreciably towards the horizontal.
- The load deflection response of the model compares well with the calculated deflections until about 75% of the nominal strength of the beam. Beyond that the discrepancy in the load deflection response is attributed to the fact that the flexural cracks development in the model is flat and sudden, leading to a sudden stress drop. Spreading of cracks undermined solution convergence at higher loads.
- Structural modeling using Solid 65 finite element utilizing Ansys software may properly simulate the nonlinear behavior of shallow reinforced concrete beams.

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