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EXAMINATION OF THE LATTICE BOLTZMANN METHOD IN SIMULATION OF MANUFACTURING

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ABSTRACT

This work is concerned with the characteristics of incompressible viscous flow inside a two-sided lid-driven cavity with its two opposite walls moving with a constant velocity in parallel direction and in antiparallel direction by Lattice Boltzmann method (LBM). The model used in the present work is two-dimensional nine-velocity (*D2Q9*) square lattice as it gives more stable and accurate result when compared to two-dimensional seven-velocity (*D2Q7*) hexagonal lattice. The characteristics of flow problem are investigated for different Reynolds number and also for aspect ratio, K = 2.0 and 5.0. The formation of different vortices with the variation of Reynolds number for parallel and antiparallel motion is studied in detail. To sum up, the present study reveals many interesting features of two-sided lid-driven deep cavity flows and demonstrates the capability of the Lattice Boltzmann method to capture these features.

Keywords: lattice boltzmann method, *D2Q9* model, two-sided cavity, parallel motion, antiparallel motion.

INTRODUCTION

The fluid motion inside a closed, rectangular container with rigid walls induced by the tangential motion of a lid constitutes a classical paradigm for internal vortex flows [1]. Another classic example is the case where a flow is induced by the tangential movement of two facing cavity boundaries with uniform velocities. If the two facing walls move in the same direction, it is termed parallel wall motion and if in the opposite direction, it is termed antiparallel wall motion. The single-sided lid-driven cavity flow problem was extended to two-sided lid-driven cavity by Kuhlmann and other investigators [2-5] and they have done several experiments on two-sided lid-driven cavity with various spanwise aspect ratios.

They numerically simulated the rectangular cavity flow by parallel (or) antiparallel motion of walls. They showed that, the different vortex configurations can be generated depending on the direction of the lid motion of the walls. More recently, Perumal and Dass [6, 7] investigated the flow driven by parallel and antiparallel motion of two facing walls in a two-sided lid-driven square cavity at different Reynolds numbers using finite difference method and Lattice Boltzmann method. This two-sided lid-driven cavity problem is attractive because of its importance in industrial applications such as rolling bar in manufacturing, electronic system cooling and many others.

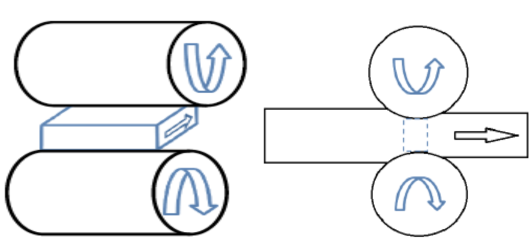

Figure-1. The rolling operation of rectangular bar.

Many physical phenomenons around us have the characteristics similar to the flow in two sided lid-driven cavities. To describe it, let us consider the manufacturing field, where the rectangular bars are rolled between two or many rollers to reduce the size of the bar to the required dimensions. Figure-1 shows the rolling operation of a rectangular bar. The flow between the two rollers can be assumed to a rectangular cavity and the flow inside the heated bar can resemble the flow inside a two sided lid driven cavity. This study can be employed to understand the effects of the roller velocity on the flow of the material of the bar.

It is known that, the lattice Boltzmann method has recently become a useful and alternative approach for computational fluid dynamics (CFD). Many researchers carried out simulations of single lid-driven cavity flow by Lattice Boltzmann method (LBM). As a computational tool, the lattice Boltzmann method differs from incompressible Navier-Stokes equations-based methods as follows [8]:

1. Navier-Stokes equations are second-order partial differential equations (PDEs); the discrete velocity model from which LBM is derived consists of a set of first-order PDEs (kinetic equations).
2. Navier-Stokes equations have nonlinear convection terms; the convection terms in LBM are linear.
3. Lattice Boltzmann Equation (LBE) is a discretized kinetic equation; Navier-Stokes equations can take integral or differential forms.
4. LBM depends on lattice structure; Navier-Stokes equations are in vector form that is independent on the coordinate and grids.
5. The Navier-Stokes solver usually employs iterative procedures to obtain a converged solution; the LBM is explicit in form and do not need iterative procedures.
6. Boundary conditions involving complicated geometries require careful treatments in both Navier-Stokes equations-based and LBM solvers. In LBM,

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the boundary condition is in the form of particle distribution functions.

Due to the kinetic nature of the Boltzmann equation, the physics associated with the molecular level interaction can be incorporated more easily in the LBE model. It is also known that, through a Chapman-Enskog analysis, one can recover the governing continuity and momentum equations in the low Mach number limit.

Furthermore, in contrast to the fairly large number of studies conducted for single-sided lid-driven cavities, only a few investigations have been carried out for flows in two-sided lid-driven cavities by continuum-based methods (FDM, FVM and FEM) and no attempt has been made to compute the flow in a two-sided lid-driven deep cavity with various Reynolds number by LBM. The present problem therefore merits careful investigation, which is attempted in this paper through the lattice Boltzmann method for various Reynolds numbers and aspect ratios for both the parallel and antiparallel motion of the walls. In this work, the aspect ratio (K) of the cavity is defined as a formula $K = D/W$, where D and W are the depth and width of the cavity respectively. A two-dimensional steady incompressible viscous flow in a two-sided lid-driven cavity with aspect ratio of 2.0 is calculated.

The present paper is organized in four sections. In Section 2, lattice Boltzmann method with two-dimensional nine-velocity square velocity model is described in some detail. In Section 3, the two-sided lid-driven deep cavity problem is described and the results with parallel and antiparallel motion of the walls are presented. Concluding remarks are made in Section 4.

NUMERICAL METHOD

Lattice Boltzmann method

The Lattice Boltzmann equation which can be linked to the Boltzmann equation in kinetic theory is written as [9].

$$f_i(\mathbf{x}+\boldsymbol{c_i}, t+1) - f_i(\mathbf{x}, t) = \Omega_i \tag{1}$$

where f_i is the particle distribution function, $\boldsymbol{c}_i$ is the particle velocity along the ith direction and Ω_i is the collision operator. The lattice BGK (LBGK) with single time relaxation model, which is a commonly used Lattice Boltzmann method, is given by [9].

$$f_i(\mathbf{x}+\boldsymbol{c}_i, t+1) - f_i(\mathbf{x}, t) = -\frac{1}{\tau}\left[f_i(\mathbf{x}, t) - f_i^{eq}(\mathbf{x}, t)\right] \tag{2}$$

Here $f_i^{eq}(\mathbf{x}, t)$ is the equilibrium particle distribution function at $\mathbf{x}, t$ and τ is the time relaxation parameter. For simulating two-dimensional flows, the two-dimensional nine-velocity model ($D2Q9$, $i = 0,1..,8$) is used in this work.

In a *D2Q9* square lattice each node has eight neighbours connected by eight links as shown in Figure-2. In the *D2Q9* LBM model have a rest particle in the discrete velocity set $\{\boldsymbol{c}_i\}$, because the LBM model with a rest particle have better computational stability and reliability. For the *D2Q9* model the discrete velocity set $\{\boldsymbol{c}_i\}$ is written as [11].

$$\boldsymbol{c}_i = \begin{cases} (0,0), & i = 0;\ \text{group } 0 \\ (\pm1,0),(0,\pm1), & i = 1,2,3,4;\ \text{group I} \\ (\pm1,\pm1), & i = 5,6,7,8;\ \text{group II} \end{cases} \tag{3}$$

In the above, group 0 indicates a rest particle, group I is for the links pointing to the nearest neighbours and group II is for the links pointing to the next-nearest neighbours.

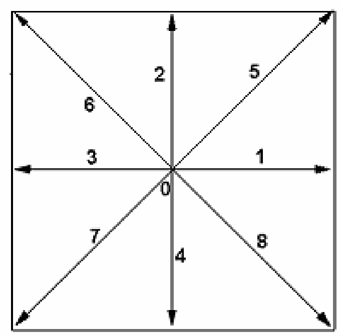

Figure-2. Two-dimensional nine-velocity square lattice model.

The equilibrium distribution functions $f_i^{eq}(\mathbf{x}, t)$, for the *D2Q9* LBE model which can be expressed in the form as [4].

$$\begin{aligned} f_i^{(0)} &= \rho w_i\left[1 - \frac{3}{2}\boldsymbol{u}^2\right],\ i = 0 \\ f_i^{(0)} &= \rho w_i\left[1 + 3(\boldsymbol{c_i}.\boldsymbol{u}) + 4.5(\boldsymbol{c_i}.\boldsymbol{u})^2 - 1.5\boldsymbol{u}^2\right],\ i = 1,2,3,4 \\ f_i^{(0)} &= \rho w_i\left[1 + 3(\boldsymbol{c_i}.\boldsymbol{u}) + 4.5(\boldsymbol{c_i}.\boldsymbol{u})^2 - 1.5\boldsymbol{u}^2\right],\ i = 5,6,7,8 \end{aligned} \tag{4}$$

where the lattice weights for *D2Q9* Lattice Boltzmann model are given by $w_0 = 4/9$, $w_1 = w_2 = w_3 = w_4 = 1/9$ and $w_5 = w_6 = w_7 = w_8 = 1/36$.

The macroscopic quantities such as density ρ and momentum density $\rho\boldsymbol{u}$ are defined in terms of the particle distribution function f_i as follows:

$$\rho = \sum_{i=0}^{N} f_i, \tag{5}$$

$$\rho \boldsymbol{u} = \sum_{i=0}^{N} f_i \boldsymbol{c}_i. \tag{6}$$

The relaxation time that fixes the rate of approach to equilibrium is related to the viscosity by [4]

$$\tau = \frac{6\upsilon + 1}{2} \tag{7}$$

where υ is the kinematic viscosity measured in lattice units. It is seen that τ = 0.5 is the critical value for ensuring a non-negative kinematic viscosity.

Boundary condition plays a crucial role in LBM simulations [8]. Implementation of boundary conditions in LBM is an important task owing to the fact that one has to translate given information from macroscopic variables to particle distribution function (f_i), since it is the only variable to be evaluated in Lattice Boltzmann Method. The bounce-back boundary condition is a popular boundary condition in LBM. It is derived from Lattice Gas Automata (LGA) and has been extensively applied in LBM simulations. In this scheme, the particle distribution function at the wall lattice node is assigned to be the particle distribution function of its opposite direction. The easy implementation of the present no-slip velocity condition supports the LBM and it is ideal for simulating fluid flows.

Code validation

First, the developed LBM code is used to compute the single sided lid-driven square cavity flow for *Re* = 1000 on a 201 × 201 lattice size. The results computed by Ghia *et al*. [10] exist for the same problem on a similar grid, which are used for the present code-validation exercise. Figures 3(a) and 3(b) shows the steady-state *x*-component of the velocity along the vertical centreline and the *y*-component of the velocity along the horizontal centreline of the cavity at *Re* = 1000. The agreement between present results and those of Ghia *et al*. [10] is excellent. The close agreement gives credibility to the result of present LBM code and it stands validated.

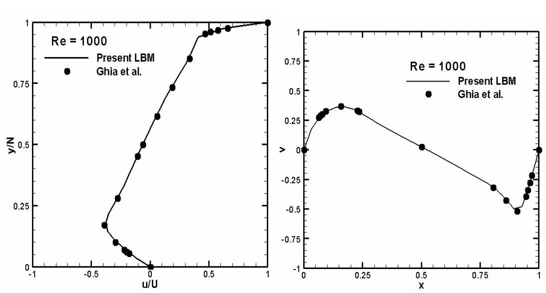

Figure-3. Code validation: (a) *u*-velocity along vertical centreline and (b) *v*-velocity along horizontal centreline for single lid-driven square-cavity (*Re* = 1000).

RESULTS AND DISCUSSIONS

Problem definition

An incompressible viscous flow in a two-sided deep cavity whose top and bottom walls move in the same (parallel motion) or opposite (antiparallel motion) direction with a uniform velocity is the problem investigated in the present work. The boundary conditions for two-sided lid-driven parallel and antiparallel wall motion cases are shown in the Figure-4(a) and Figure-4 (b).

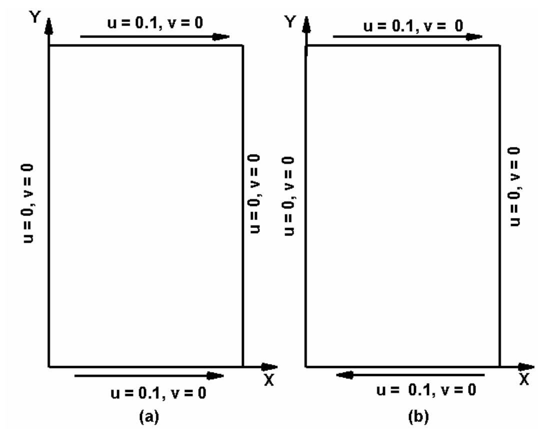

Figure-4. Two-Sided Lid-Driven Cavity for (a) parallel wall motion (b) antiparallel wall motion with boundary conditions.

Simulation procedure

The configuration of the cavity flow consists of a two-dimensional deep cavity whose top and bottom plate moves with a uniform velocity, while the other walls are fixed. The velocity components u and v are in x and y directions. Initially the velocities at all nodes, except at the top nodes, are set to zero. The *x*-velocity of the top and bottom plates is *U* = 0.1 and the y-velocity is zero. Uniform fluid density ρ = 1.0 is imposed initially. The equilibrium distribution function $f_i^{eq}(x,t)$ is calculated using Equation (4) and f_i is set to equal to $f_i^{(0)}$ for all node at $t = 0$. The distribution function can be found by a succession of propagation and collision processes. At the end of each process distribution function is set to the equilibrium state. Here, the top lid-velocity of ***U*** = 0.1 is considered.

Parallel wall motion

Case-I: Aspect ratio 2.0

An incompressible viscous flow in a two-sided cavity, whose top and bottom wall moves to the right with a uniform velocity is computed through lattice Boltzmann method. The case of a two-sided lid-driven flow in a cavity with aspect ratio of 2.0 is considered. The range of Reynolds numbers 10 to 1500 is investigated here.

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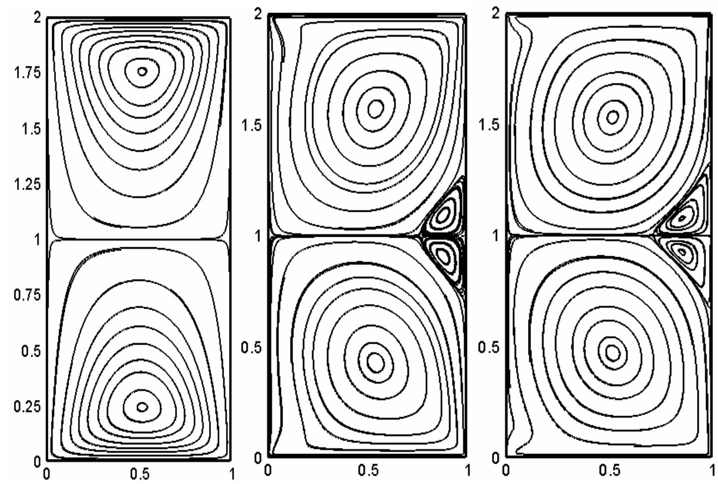
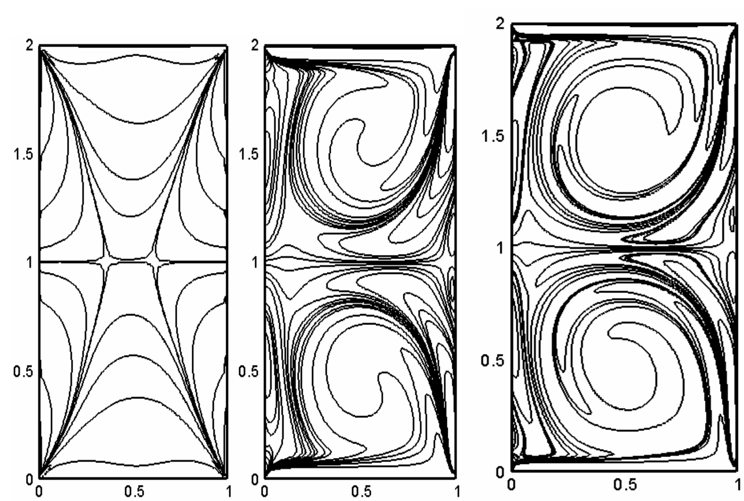


(a) Re = 10 (b) Re = 500 (c) Re = 1500

Figure-5. Streamline pattern for parallel wall motion at (a) Re = 10, (b) Re = 500 and (c) Re = 1500 by lattice Boltzmann method with aspect ratio 2.0.

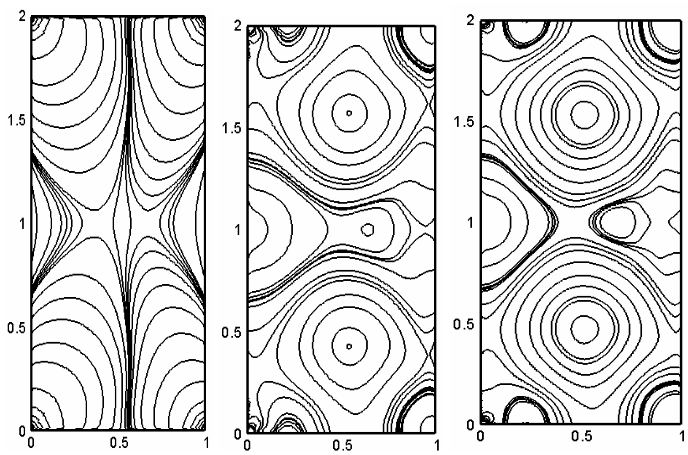
Figure-5 shows the streamline patterns for Re = 10, 500 and 1500 by lattice Boltzmann method. It is known that, both walls move in same direction, it can generate their own primary vortex. At Re = 100 (Figure-5a), two rotating primary vortices symmetrical to each other are seen to form with a `free' shear layer in between. At Re = 500 (Figure-5b), a pair of counter-rotating secondary vortices are symmetrically placed about the horizontal centreline near the centre of the right wall. As the Reynolds number increases to 1500 (Figure-5c), the secondary vortices are seen to grow in size. It may be noted that the corresponding secondary vortices for a single-sided lid-driven square cavity does not appear at a Reynolds number as low as 1500. Expectedly, the streamlines are found to be symmetrical with respect to the horizontal centreline for all Reynolds numbers.



(a) Re = 10 (b) Re = 500 (c) Re = 1500

Figure-6. Vorticity contours for parallel wall motion at (a) Re = 10, (b) Re = 500 and (c) Re = 1500 by lattice Boltzmann method with aspect ratio 2.0.

The vorticity contours for various Reynolds numbers are shown in Figure-6. It is seen that several regions of high vorticity gradients indicated by the concentration of the vorticity contours appear within the cavity. Figure-7 presents the LBM isobars at Reynolds number ranging from 10 to 1500. By examining the closed contours it is seen that the inviscid core grows with increasing values of Reynolds number. These results are well known in the literature [6, 7] and exhibit no surprises thereby confirming the fact that our Lattice Boltzmann Method results yield qualitatively accurate solutions.

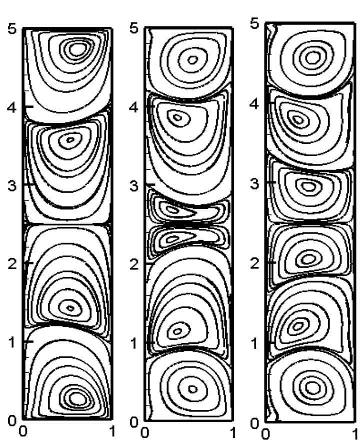


(a) Re = 10 (b) Re = 500 (c) Re = 1500

Figure-7. Pressure contours for parallel wall motion at (a) Re = 10, (b) Re = 500 and (c) Re = 1500 by lattice Boltzmann method with aspect ratio 2.0.

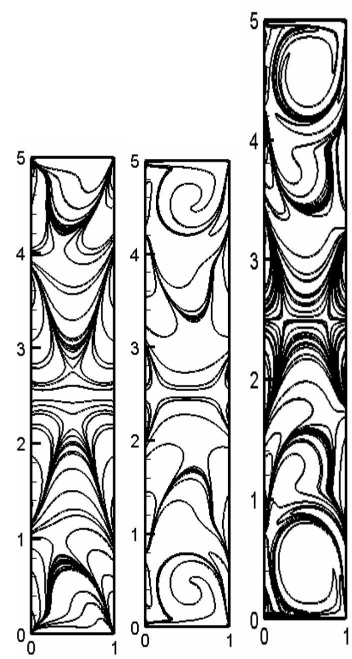
Case-II: Aspect ratio 5.0

Next, the case of a parallel motion two-sided lid-driven flow with aspect ratio of 5.0 is considered. Figures 8 and 9 shows the streamline patterns and vorticity contours for Re = 100, 700 and 2000. It is known that as the aspect ratio increases, the number of primary vortices increases. From Figure-8 it is observed that, the induced primary and secondary vortices and the streamlines are symmetric with respect to the horizontal centreline of the cavity. At Re = 100 (K = 5), four primary vortices are generated due to motion of two facing walls. As the Reynolds number increases to 700, pairs of secondary vortices which is significantly different from those primary vortices appeared in the cavity. The sizes of the secondary vortices also grow in size as Re increases. It is observed that even numbers of primary vortices are formed at all Reynolds numbers. It is seen that, the direct effect of the moving lids does not percolate too much when the aspect ratio is high.



(a) (b) (c)

Figure-8. Streamline pattern for parallel wall motion at (a) Re = 100, (b) Re = 700 and (c) Re = 2000 by lattice Boltzmann method with aspect ratio 5.0



(a) (b) (c)

Figure-9. Vorticity contours for parallel wall motion at (a) Re = 100, (b) Re = 700 and (c) Re = 2000 by lattice Boltzmann method with aspect ratio 5.0.

Antiparallel wall motion

Case-I: Aspect ratio 2.0

An incompressible viscous flow in a two-sided cavity, whose top wall moves to the right and bottom wall moves to the left with a uniform velocity is now computed through lattice Boltzmann method. The case of a two-sided lid-driven flow in a deep cavity with aspect ratio of 2.0 is considered. The range of Reynolds numbers 10 to 1500 is investigated here.

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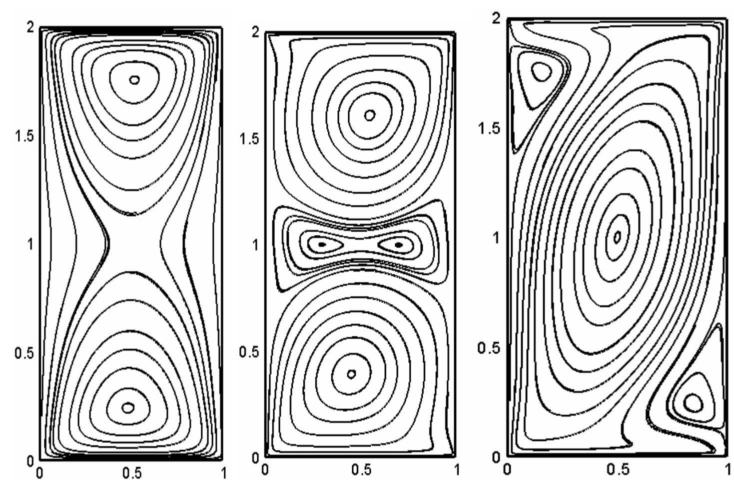
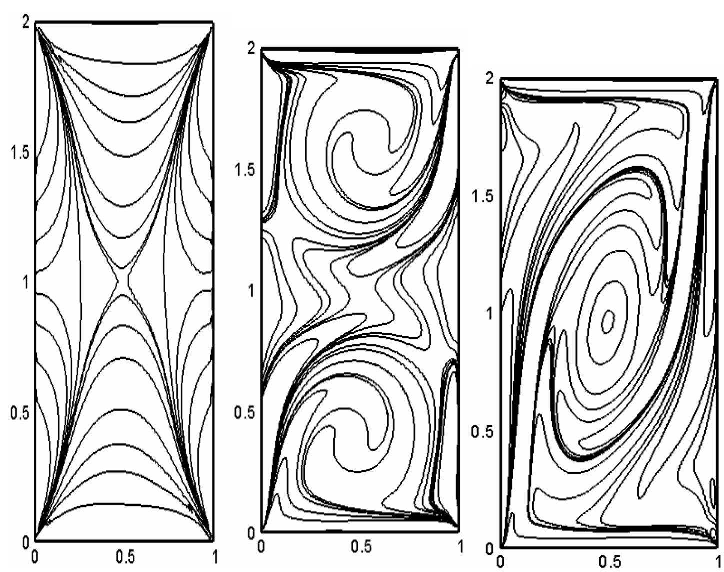
Figure-10 shows the streamline patterns for *Re* = 10, 500 and 1500. It is seen that as the aspect ratio increases, the number of primary vortices increases. At *Re* = 10 (Figure-10a), the appearence of two primary vortices and the centre of the two primary vortex cores are seen to be somewhat away from the centres of the top and bottom halves of the cavity towards the righthand top and lefthand bottom corners respectively. Because of symmetry, this pair of primary vortices has similar shapes. It may be noted that the corresponding primary vortices for a two-sided lid-driven square cavity does not appear for Reynolds number as low as 100.

At *Re* = 500 (Figure-10b), the appearence of two secondary vortices in the middle of primary vortices are symmetric in horizontal direction. As the Reynolds number increases, the primary vortex cores moves towards the centres of top and bottom halves of the cavity and are well-separated. It may be noted that in two-sided square cavity (*Re* = 500), the secondary vortices appeared near the top left and bottom right corners only. It is also observed that, the vortices and the streamlines are point symmetric with respect to the geometric centre of the cavity.



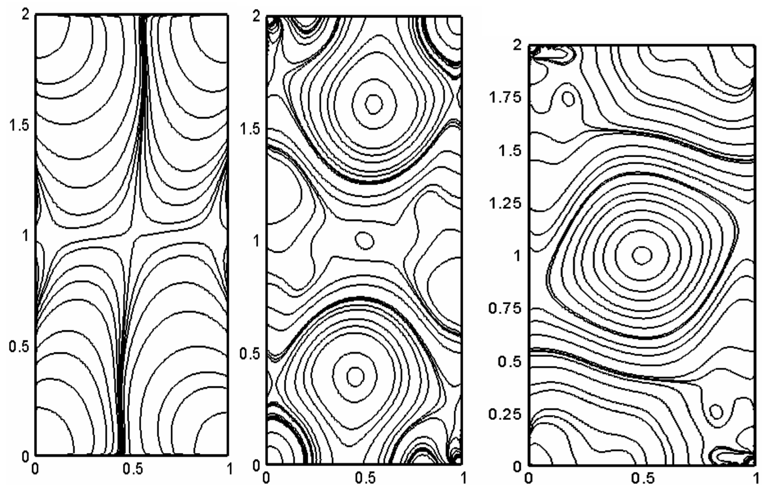
(a) *Re* = 10 (b) *Re* = 500 (c) *Re* = 1500

Figure-10. Streamline pattern for antiparallel wall motion at (a) *Re* = 10, (b) *Re* = 500 and (c) *Re* = 1500 by lattice Boltzmann method with aspect ratio 2.0.



(a) *Re* = 10 (b) *Re* = 500 (c) *Re* = 1500

Figure-11. Vorticity contours for antiparallel wall motion at (a) *Re* = 10, (b) *Re* = 500 and (c) *Re* = 1500 by lattice Boltzmann method with aspect ratio 2.0.



(a) *Re* = 10 (b) *Re* = 500 (c) *Re* = 1500

Figure-12. Pressure contours for antiparallel wall motion at (a) *Re* = 10, (b) *Re* = 500 and (c) *Re* = 1500 by lattice Boltzmann method with aspect ratio 2.0.

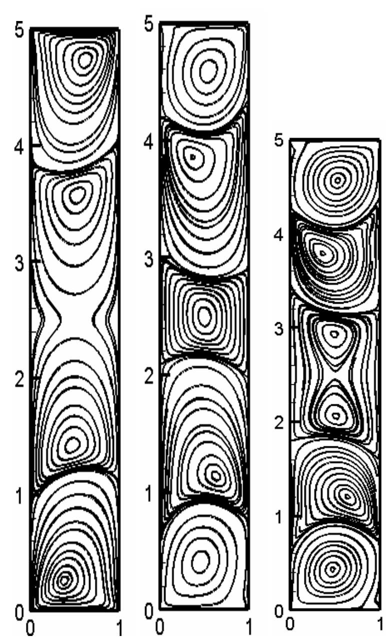
As the Reynolds number increases to 1500 (Figure-10c), this pair of secondary vortices are merged into single vertex and two secondary vortices are formed in the top-left and bottom-right end of the cavity. The vorticity and pressure contours for various Reynolds numbers using lattice Boltzmann method are shown in Figure-11 and Figure-12. It is seen that several regions of high vorticity gradients indicated by the concentration of the vorticity contours appear within the cavity. The thinning of the wall boundary layers with increasing Reynolds number is evident from these plots.

The vorticity and pressure contours for various Reynolds numbers using lattice Boltzmann method are shown in Figure-11 and Figure-12. It is seen that several regions of high vorticity gradients indicated by the concentration of the vorticity contours appear within the cavity. The thinning of the wall boundary layers with increasing Reynolds number is evident from these plots.

Case-II: Aspect ratio 5.0

Next, the case of an antiparallel motion two-sided lid-driven flow with aspect ratio of 5.0 is considered. Figures 13 and 14 shows the streamline patterns and vorticity contours for *Re* = 100, 700 and 2000. From Figure-13, it is found that the flow structure inside the cavity changes considerably with the aspect ratio. Here, the near-wall primary vortices have the same sense of rotation and are well-separated as the aspect ratio is large. At *Re* = 100 (*K* = 5), four primary vortices are generated due to antiparallel motion of two facing walls.

As the Reynolds number increases to 700, the secondary vortex formed in the middle of the primary vortices. The weaker secondary vortex splits into two separate vortices as the Reynolds number increases to 2000. The effect of aspect ratio and Reynolds number on the vortex structure is clearly seen.



(a) (b) (c)

Figure-13. Streamline pattern for antiparallel wall motion at (a) *Re* = 100, (b) *Re* = 700 and (c) *Re* = 2000 by lattice Boltzmann method with aspect ratio 5.0.

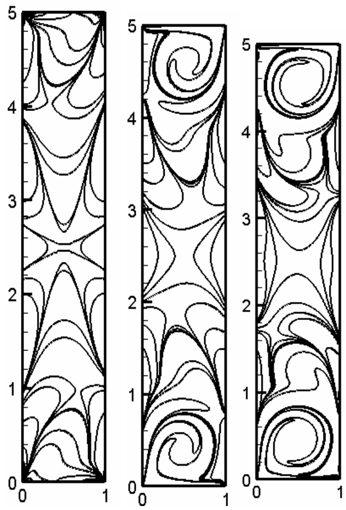


Figure-14. Vorticity contours for antiparallel wall motion at (a) *Re* = 100, (b) *Re* = 700 and (c) *Re* = 2000 by lattice Boltzmann method with aspect ratio 5.0.

CONCLUSIONS

In the present work, the two-sided lid-driven cavity is computed with the lattice Boltzmann method. The flow is investigated for both parallel and antiparallel motion of the two facing walls. The present code is validated through a careful comparison exercise with established results so that the results for the present configuration enjoy credibility. The present computations not only confirms the flow features of the problem, but also reveals the effects of Reynolds number and the aspect ratio on the flow structure in the two-sided lid-driven deep cavity in a systematic way. Consequently these results, like those of the single lid-driven deep cavity flow, may be used for validating the algorithms for computing steady flows governed by the two-dimensional incompressible Navier-Stokes equations.

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