



IMPACT RESISTANCE AND STRENGTH RELIABILITY OF FIBER REINFORCED CONCRETE USING TWO PARAMETER WEIBULL DISTRIBUTION

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ABSTRACT

In this study, an attempt has been made to investigate the impact resistance of fiber reinforced concrete (FRC), subjected to drop weight test in accordance with the procedure suggested by ACI committee 544. For this, four samples were prepared from each series of mix containing crimped and hooked end steel fibers of 1 mm diameter and 50 mm length in various proportions viz., 0%, 0.5%, 1.0% and 1.5%, with a water cement ratio of 0.42. In the view of variations of test results, two parameter weibull distributions were performed to analyze the experimental data and the impact failure strength was presented in terms of reliability function. The results indicated that incorporating steel fiber to concrete increased the impact resistance and changed the failure pattern from brittle to ductile mode. Also, results indicated that the weibull distribution allows the researchers to describe the impact failure strength of FRC in terms of reliability and safety limits. This provides a greater ease for designers by eliminating the number of experiments.

Keywords: fiber, weibull distribution, reliability, failure, impact energy.

INTRODUCTION

For the past three decades, numerous studies are being carried out on the fiber reinforced concrete (FRC), which plays a vital role in structural engineering applications (Naaman and Gopalaratnam, 1983; Nataraja *et al.*, 1999; Holschemacher, 2010; Andrea *et al.*, 2010; Yusa Sahin and Fuat Koksall, 2011; Angela *et al.*, 2012). It is a well established fact that, the thermal shock strength, ductility, fracture toughness and resistance under fatigue, dynamic and impact load can be enhanced by adding steel fiber to concrete mixtures (Banthia *et al.*, 1998; Paulo *et al.*, 2002; Bencardino *et al.*, 2010; Xu *et al.*, 2012). In the recent times, impact resistance of concrete is recognized as an important property in infrastructure construction. Several methods have been suggested by different guidelines that evaluate the impact resistance of FRC (ACI Committee 544) such as charpy test, projectile impact test, explosive test and drop weight test. Among them, drop weight is the simplest, popular and attractive method suggested by the ACI committee 544. However, a greater deviation can be observed in the drop weight test results (Nataraja *et al.*, 1999; Song *et al.*, 2004; Song *et al.*, 2005; Atef *et al.*, 2006) and it may be due to the following reasons (i) The test results are interpreted based on the recognition of first crack by visual means and this crack may occur in any direction. (ii) It is difficult to control the height of fall of drop hammer exactly, as it is being done manually. (iii) The impact resistance of concrete is determined by the impact occurring at a single point, which may be either, on a tough coarse aggregate, or fiber or matrix and (iv) Concrete is heterogeneous material. The variation of mix design may cause the change in impact resistance, including shape of aggregate, fiber geometry and distribution of fibers, etc. In the view of impact experimental test results, statistical analysis has emerged as a best choice for resolving the impact experimental test results and the significance of steel fiber in concrete.

In this study, the impact resistance of fiber reinforced concrete was investigated in view of variations of impact experimental test results and statistical analysis was performed by using the two parameter weibull distribution. The impact failure energy has been presented in terms of reliability function. It will be helpful to extend the use of FRC and further clarify the nature of impact behavior of FRC.

EXPERIMENTAL PROGRAM

Material properties

Ordinary Portland cement of 53 grade (ASTM type I) with specific gravity of 3.25 was used for preparing the concrete mix. Crushed granite gravel having the size of 12mm and 20mm were chosen as the coarse aggregates. Fine aggregate used in the concrete mix was, locally available river sand. Polycarboxylic ether based superplasticizer was used as an admixture in 0.3% to 1.0% by weight of cement. The fiber incorporated in the concrete was crimped and hooked end steel fiber, of length 50 mm, aspect ratio 50 and an equivalent diameter 1mm. The density and tensile strength of the crimped and hooked end steel fiber was 7.8 g/cm³, 1000 MPa and 1050 MPa, respectively.

Mixing proportion

Mixture design was made in accordance with the Indian standard code 10262–2009 for M30 grade of concrete. Concrete containing crimped and hooked end steel fibers were added to the mix in various proportions viz., 0%, 0.5%, 1% and 1.5%, respectively. Water binder ratio of 0.42 was adopted in this study and seven series of mixtures were prepared. The materials and code specifications are indexed in Table-1.



Impact test

Each series of freshly mixed FRC was placed in the cylindrical moulds of dimension 100 X 200mm for casting the specimens. From these cylindrical specimens, twelve discs of size 100 x 64mm were cut using a diamond cutter. The discs were then subjected to drop weight test following the guidelines of ACI committee 544.2R-89. The test consisted of repeated application of impact load in the form of blows, using a 44.5 N hammer falling from 457 mm height on the steel ball of 63.5 mm diameter, placed at the center of the top surface of disc. Number of

blows (N_1) and (N_2) that caused the first visible crack and failure respectively was noted as first crack strength and the failure strength of the sample. The schematic diagram of drop weight test machine is shown in Figure-1.

The impact energy was calculated for each concrete specimen using Equation (1):

$$\text{Impact energy } U = \left(\frac{n \cdot m \cdot V^2}{2} \right) \quad (1)$$

Table-1. Mixing proportions for 1m^3 .

Mix No.	Mixture Id	W/B	Water (Kg/m ³)	Cement (Kg/m ³)	Fine Agg. (Kg/m ³)	Coarse Agg. (Kg/m ³)	Volume fraction V_f	Fiber (Kg/m ³)	Sp (%)
1	F0	0.42	140	333	901	1162	-	-	0.3
2	FC0.5	0.42	140	333	903	1149	0.5	39	0.5
3	FC1.0	0.42	140	333	892	1135	1.0	78	0.8
4	FC1.5	0.42	140	333	885	1126	1.5	117	1.0
5	FH0.5	0.42	140	333	903	1149	0.5	39	0.5
6	FH1.0	0.42	140	333	892	1135	1.0	78	0.8
7	FH1.5	0.42	140	333	885	1126	1.5	117	1.0

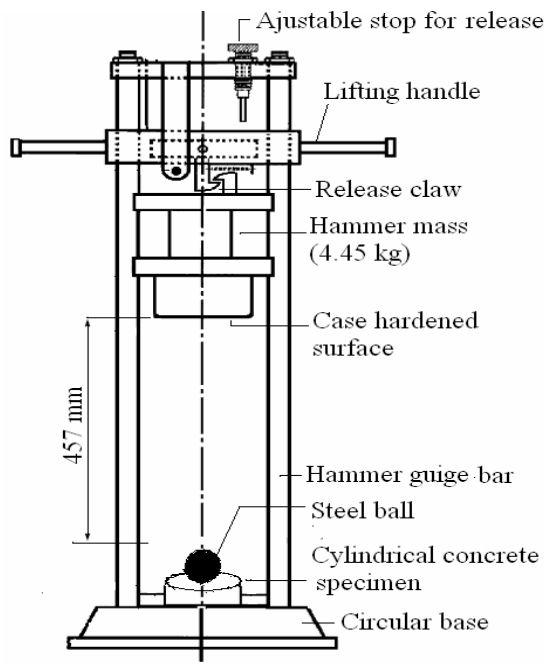


Figure-1. Schematic diagram of drop weight impact testing machine.

$$H = \left(\frac{gt^2}{2} \right) \quad (2)$$

$$V = g \cdot t \quad (3)$$

$$m = \frac{W}{g} \quad (4)$$

Where, H is the falling height of hammer, V is the velocity of the hammer at impact, W is the hammer weight, m is mass of the hammer, g is acceleration due to gravity, t is the time required for the hammer to fall from a height of 457 mm, n is the number of blows and m is the drop mass.

TEST RESULTS AND DISCUSSIONS

The number of blows required to cause the first visible crack (N_1) and final failure (N_2) of concrete specimens are indexed in Table-2 and the impact energy corresponding to number of blows are shown in Figure-2. The impact energy of specimens during every blow can be calculated as follows.

Substituting the corresponding values in Equation (2-4):

$$457 = \frac{9810t^2}{2}$$

$$t = 0.3052 \text{ s and } V = 9810 \times 0.3052 = 2994.01 \frac{\text{mm}}{\text{s}}$$

The impact energy delivered by hammer per blow can be obtained by substituting the values in Equation (1)

$$U = \frac{44.3 \times 2994.01^2}{2 \times 9810} = 20.345 \text{ kN mm}$$



By adding 0.5%, 1.0% and 1.5% dosage of crimped steel fiber (FC0.5, FC1.0 and FC1.5) the energy input necessary to cause the visibility of first crack was increased by 139%, 268% and 366%, respectively and the energy necessary to cause failure of concrete specimen was increased by 129%, 238% and 321% over the plain concrete specimen (F0). Similarly for 0.5%, 1.0% and 1.5% dosage of hooked end steel fiber (FH0.5, FH1.0, and FH1.5), the energy required to cause the initiation of first crack was increased by 180%, 300% and 381% respectively, and the energy required to cause failure of concrete specimen was increased by 151%, 269% and 347% over the plain concrete specimen (F0). Hence it was observed that, increasing the volume fraction of steel fiber increases the impact energy of concrete significantly, in both the first crack stage as well as failure stage (Mahmoud and Afrougsabet, 2010; Taner *et al.*, 2010; Alavi *et al.*, 2012). This proves that the steel fibers act as an effective crack arrestor in case of FRC, when an impact load is encountered. Thus the plain concrete exhibits an early brittle failure when compared to FRC which shows better ductile properties (Swamy and Jojagha, 1982).

The overall coefficient of variation for plain and FRC indicates that the concrete sample has a sufficient

quality control as shown in Table-3. However, the coefficient of variation in the range of 5% and 10% is adopted as an acceptable quality control which was suggested by Day (1999).

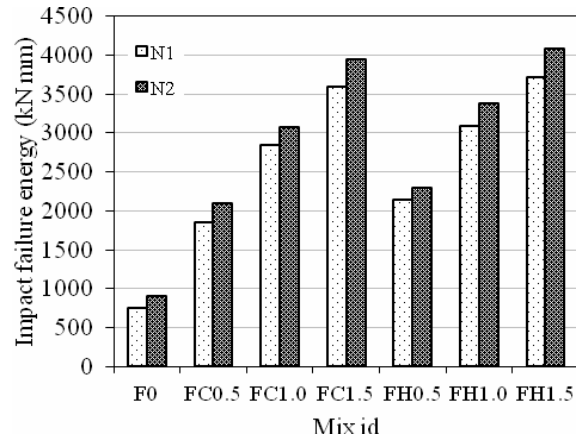


Figure-2. Impact failure energy of FRC.

Table-2. Results from drop weight test (blows).

Spec No	N1/N2						
	F0	FC0.5	FC1.0	FC1.5	FH0.5	FH1.0	FH1.5
1	32/40	87/100	137/147	175/92	99/108	147/158	177/192
2	36/41	89/102	139/149	177/194	105/111	149/163	181/198
3	40/48	93/104	141/154	177/194	107/114	154/167	185/203
4	42/50	95/106	143/156	179/194	111/119	157/177	190/210

Table-3. Statistical analyses of test results.

Item	N1/N2						
	F0	FC 0.5	FC 1.0	FC 1.5	FH0.5	FH1.0	FH1.5
Mean	38/45	91/103	140/152	177/194	106/113	152/166	183/201
Standard deviation	4/4	3/2	2/4	1/1	4/4	4/7	5/7
Coefficient of variance	10/10	3/2	2/2	1/1	4/4	3/4	3/3

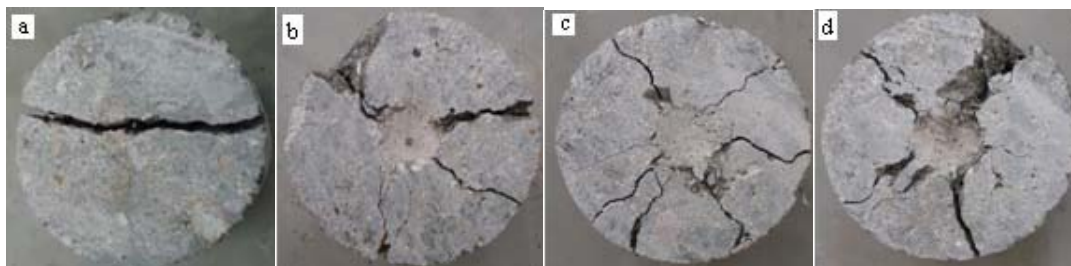


Figure-3. Comparison of failure pattern of specimens with different dosage of fiber (a) plain concrete, (b) 0.5% fiber, (c) 1% fiber, and (d) 1.5% fiber.



The mode of failure of concrete depends upon the matrix strength, aggregate strength and bond strength of fiber with aggregate matrix. Brittle behavior was observed in plain concrete specimens and it was broken into two pieces as shown in Figure-3(a). The FRC specimens displayed different mode of failure as shown in Figure-3 (b), (c) and (d). The mode of failure was changed from brittle to ductile behavior and this is consistent with previous studies (Swamy and Jojagha, 1982; Mohammadi *et al.*, 2008; Mahmoud and Afroughsabet, 2010; Taner *et al.*, 2010; Chen *et al.* 2011; Alavi *et al.*, 2012). It is clear from the Figure-3(a) to (d) by incorporating fiber to concrete mixtures the cracking pattern was changed from single crack to large number of multiple cracks, which displays beneficial effects of FRC used in structural engineering applications.

Weibull distribution

In the past few decades, several mathematical probability functions have been used for describing the fatigue and impact test data of concrete statistically (Nataraja *et al.*, 1999; Song *et al.*, 2004; Song *et al.*, 2005; Atef *et al.*, 2006). It has been proved in several investigations that, the two-parameter weibull distribution is most commonly used for describing the fatigue life of concrete (Singh and Kaushik, 2003; Raif Sakin *et al.*, 2008; Raman Bedi and Rakesh Chandra, 2009).

The cumulative distribution function

$$F_N(t) = 1 - \exp\left[-\left(\frac{t}{u}\right)^\alpha\right] \quad (5)$$

Where, n represents the specific value of the random variable N ; u represents the scale parameter; α represents shape parameter.

Taking natural logarithm twice on both sides of the Equation (5) gives

$$\ln\left[\ln\left(\frac{1}{F_N(t)}\right)\right] = \alpha \ln(t) - \alpha \ln(u) \quad (6)$$

The equation (6) can be rearranged as linear equation, where $Y = \ln\left[\ln\left(\frac{1}{F_N(t)}\right)\right]$, $X = \ln(t)$, $m = \alpha$ and $c = -\alpha \ln(u)$.

Several predefined empirical survivorship function has been used in different literatures for evaluating the value of L_N (Jayatilaka and De, 1979; Saghafi *et al.*, 2009).

$$L_N = 1 - \frac{t - 0.3}{k + 0.4} \quad (7)$$

$$R_N = 1 - F_N(t) \quad (8)$$

$$U_{RN} = u \cdot ((-\ln(R_N))^{\frac{1}{\alpha}}) \quad (9)$$

In the above equations (8 and 9); R_N denotes reliability or probability of survival and U_{RN} denotes the Impact energy based on the reliability.

In order to compute α and u , linear regression analysis was applied to the $\ln\left[\ln\left(\frac{1}{L_N}\right)\right]$ and $\ln(U)$ values and regression line obtained is shown in Figure-4. It can be seen from Figure-4 that, the plain concrete points does not appear to fit the line and this is the expected situation in linear regression method. The slope of the line for F0, FC0.5, FC1.0, FC1.5, FH0.5, FH1.0 and FH1.5 were 8.412, 39.94, 35.15, 114.5, 23.82, 20.16 and 26.29 which corresponds to the value of shape parameter α . When shape parameter $\alpha < 1.0$, $\alpha = 0$ and $\alpha > 1.0$ it indicates that, the material has a decreasing failure rate, constant failure rate and increasing failure rate, respectively. The shape factor (u) value was computed using the points at which the line intersects Y axis (-57.76, -305.9, -282.9, -949.3, -184.8, -164.3 and -219.1).

Table-4. Weibull probability of survival distribution.

Reliability level	F0 (kN mm)	FC0.5 (kN mm)	FC1.0 (kN mm)	FC1.5 (kN mm)	FH0.5 (kN mm)	FH1.0 (kN mm)	FH1.5 (kN mm)
0.99	555.31	1889.02	2744.88	3830.13	1929.59	2756.23	3494.66
0.9	734.26	2003.49	2934.65	3909.54	2129.65	3096.96	3821.39
0.8	802.77	2041.49	2997.97	3935.25	2197.81	3214.41	3932.04
0.7	848.80	2065.60	3038.24	3951.40	2241.51	3290.07	4002.82
0.6	885.83	2084.26	3069.45	3963.82	2275.57	3349.22	4057.88
0.5	918.56	2100.25	3096.22	3974.40	2304.92	3400.31	4105.27
0.4	949.55	2114.98	3120.90	3984.10	2332.08	3447.71	4149.08
0.3	980.88	2129.49	3145.24	3993.61	2358.97	3494.72	4192.40
0.2	1015.31	2145.02	3171.32	4003.75	2387.89	3545.40	4238.94
0.1	1059.48	2164.34	3203.80	4016.29	2424.06	3608.95	4297.08
0.01	1150.47	2202.23	3267.60	4040.68	2495.64	3735.19	4411.88

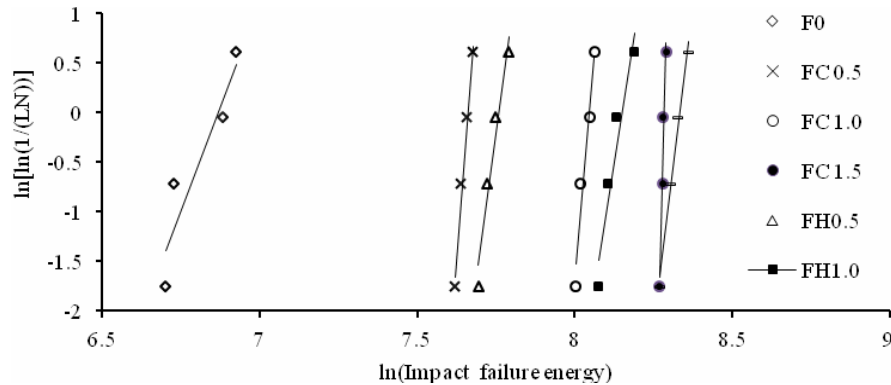


Figure-4. Weibull lines for plain and FRC.

Therefore α indicates, increasing failure rate of the material for every unit of increase in impact energy. Based on theoretical property, reliability value of 0.368 was obtained from Equation (8). Therefore 36.8% of the tested, plain and FRC specimens have impact energy of at least (952.01, 2119.6, 3128.6, 3987.1, 2340.65, 3462.69 and 4162.90 kN mm).

Table-4 shows the impact energy values of F0, FC0.5, FC1.0, FC1.5, FH0.5, FH1.0 and FH1.5 at failure stage that was approximately less than or equal to 555.31, 1889.02, 2744.88, 3830.13, 1929.59, 2756.23 and 3494.66 which offers a high reliability. When 0.9 reliability level was considered and value 0.9 was substituted in Equation (9), the corresponding impact energy value was shown in Table-4. As the reliability curve of plain and FRC predicts the impact failure strength values, additional cost involved to conduct experiments can be avoided.

CONCLUSIONS

The following conclusions were made from the experimental study.

- By adding 0.5%, 1.0% and 1.5% volume fraction of crimped steel fiber, the energy required to cause the visibility of first crack and failure was increased by 139%, 268%, 366% and 129%, 238%, 321%, respectively over plain concrete.
- When 0.5%, 1.0% and 1.5% volume fraction of hooked end steel fiber was added to concrete, the energy required to cause the visibility of first crack and failure was increased by 180%, 300%, 381% and 151%, 269%, 347% respectively compared to plain concrete.
- The results show that 1.5% volume fraction of steel fiber considerably increases the impact energy in case of FRC when compared to plain concrete.
- The incorporation of steel fiber to concrete, changes the failure pattern from brittle mode to ductile, which displays the beneficial effects of FRC, used in structural engineering applications.
- The impact failure strength variation in the PC and FRC has been modeled using weibull distribution. In this respect, the weibull distribution allows

researchers to describe the impact failure strength of FRC in terms of a reliability function.

- Based on theoretical property, reliability value is 0.368; therefore 36.8% of the tested, plain and FRC specimens have impact energy of at least (952.01, 2119.6, 3128.6, 3987.1, 2340.65, 3462.69 and 4162.90 kN mm). Hence it enables them to present the necessary impact failure strength that minimizes the number of experiments to be conducted to find probability of failure.

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