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AN ORDER LEVEL INVENTORY MODEL FOR A DETERIORATING ITEM WITH QUADRATIC TIME-VARYING DEMAND, SHORTAGE AND PARTIAL BACKLOGGING

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ABSTRACT

An Economic Order Quantity (EOQ) model is developed for a deteriorating item having time-dependent demand and shortage. The deterioration rate is assumed to rise linearly with time and time-varying demand rate is taken to be a quadratic function of time. Shortages are also assumed to be partially backlogged. The model is first developed and solved analytically and then the results are illustrated with numerical examples. Justifications for considering a time-quadratic demand are discussed elaborately.

Keywords: inventory, deterioration, quadratic demand, shortage, partial backlogging.

1. INTRODUCTION

The square-root formula for the EOQ was used in the inventory literature for a pretty long time. Wagner and Whitin [1] were the first to discuss the discrete case of the dynamic version of the EOQ model. Simple modification of the classical square-root formula in the case of time-varying demand was first discussed by Silver and Meal [2]. Later, Silver and Meal [3] developed an approximate solution procedure, known as the Silver-Meal-Heuristic, for the general case of a time-dependent demand pattern. The first analytical model for the classical no-shortage inventory policy for the case of a linear time-dependent demand was developed by Donaldson [4]. Significant contributions in this direction came from researchers like Silver [5], McDonald [6], Dave and Patel [7], Ritchie [8], Sachan [9], Mitra *et al*. [10], etc. Deb and Chaudhuri [11] were the first to incorporate shortages into the inventory lot-sizing problem with a linearly increasing time-varying demand. Subsequent contributions in this field came from the researchers like Goyal [12], Murdeshwar [13], Dave [14] and Hariga [15], among others. Whitin [16] considered fashion goods deteriorating at the end of a prescribed storage period. Ghare and Schrader [17] developed a model for an exponentially decaying inventory. Various types of order-level inventory developed a model for an exponentially decaying inventory. Various types of order-level inventory model for items deteriorating at a constant rate were discussed by Shah and Jaishwal [18], Aggarwal [19], Roychowdhury and Chaudhuri [20], Bahari-Kashani [21], etc. EOQ models for time-dependent rate of deterioration were studied by Goswami and Chaudhuri [22, 23], Hariga and Benkherouf [24], Jalan *et al*. [25], Lin *et al*. [26], etc. Wee [27] considered the inventory problem that deteriorates at a constant rate in an exponentially declining market over a fixed time horizon in which partial back ordering, a fixed-interval replenishment policy and a constant service level in each replenishment cycle were assumed. The justifications for considering a quadratic demand are discussed. Hariga and Al-Alyan [28], Benkherouf [29], Jalan and Chaudhuri [30], etc., also discussed inventory models with exponentially time-dependent demand patterns. In the present paper, we assume that the time-dependence of demand is quadratic and deterioration rate is linearly increasing in time. The other main assumptions of our model include a constant service level in each replenishment cycle and shortages with partial back ordering. Analytical solution of the model is discussed. The procedure of solving the model is illustrated with some numerical examples for increasing quadratic demand only.

2. ASSUMPTIONS AND NOTATIONS

- H is the total time horizon of the inventory system of a single item.
- The demand rate $D(t)$ at any time t is given by $D(t) = a + b\,t + c\,t^2$ where $a > 0$, $b, c \geq 0$.
- $\theta(t)$ is the time dependent deterioration rate and there is no repair or replacement of the units during the period H where $\theta(t) = \theta_1 + \theta_2\, t$, $0 \leq \theta(t) < 1$, $0 < \theta_1$; $\theta_2 << 1$.
- Shortages are allowed except for the last cycle. A fraction B ($0 < B < 1$) of the shortage is back ordered.
- Replenishment rate is infinite. The constant replenishment interval length is T.
- n is the number of replenishment during the entire period H and $T = \frac{H}{n}$.
- Lead time is zero.
- c_1 is the ordering cost per replenishment.
- c_2 is the inventory carrying cost per unit per unit of time.
- c_3 is the complete back ordering cost per unit per unit of time.
- c_4 is the cost of lost sales per unit.
- c_k is the deteriorated unit cost per unit. The $(i+1)$-th replenishment is made at time $T_i = \frac{H}{n} i$, $i = 0, 1, 2, \ldots, n-1$ with $T_0 = 0$ (A)

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- The inventory in the i-th cycle drops to zero at time t_i, i = 1, 2,...., n with $t_n = H$. The stock period $(t_i - T_{i-1})$ is assumed to be equal to a fraction r $(0 \le r \le 1)$ of the cycle time $(T_i - T_{i-1})$. From the above assumptions, we can express t_i as the convex combination of T_i and T_{i-1} as $t_i = rT_i + (1 - r)\, T_{i-1}$ for i = 1, 2,......,n-1 where $T_{i-1} \le t_i \le T_i$ and $0 \le r \le 1$: r is called a service level constant. (B)

3. FORMULATION AND SOLUTION

Let $I(t)$ be the instantaneous inventory (or shortage) level at any time t. Inventory level is depleted by the combined effect of demand and deterioration in the period of positive inventory. In the period of shortages, there is no deterioration effect. Therefore, instantaneous states of $I(t)$ are described by the following differential equations:

$$\frac{dI(t)}{dt} + (\theta_1 + \theta_2\, t) I(t) = -D(t),$$

$$T_{i-1} \le t \le t_i, \quad i = 1, 2, \ldots \ldots n \qquad (1)$$

with the boundary condition $I(t_i) = 0$;

$$\frac{dI(t)}{dt} = D(t), \quad t_i \le t \le T_i,$$

$$t_i \le t \le T_i, \quad i = 1, 2, \ldots \ldots n-1 \qquad (2)$$

with the boundary condition $I(t_i) = 0$. The solution of Equation (1) is:

$$I(t) = e^{-\left(\theta_1 t + \frac{1}{2}\theta_2 t^2\right)} \int_t^{t_i} e^{-\left(\theta_1 t + \frac{1}{2}\theta_2\, t^2\right)} dt,$$

$$T_{i-1} \le t \le t_i, \quad i = 1, 2, \ldots \ldots n \qquad (3)$$

The solution of Equation (2) is:

$$I(t) = \int_t^{t_i} D(t) dt,$$

$$t_i \le t \le T_i, \quad i = 1, 2, \ldots \ldots n-1 \qquad (4)$$

Therefore, the total inventory carried during the interval [T_{i-1} t_i] is:

$$I_i = \int_{T_{i-1}}^{t_i} [I(t)] dt, \quad i = 1, 2, \ldots\ldots, n \qquad (5)$$

The total shortage quantity during the interval [t_i T_i] is:

$$S_i = \int_{t_i}^{T_i} [\int_{t_i}^{t} D(u) du]\, dt, \quad i = 1, 2, \ldots\ldots, n-1 \qquad (6)$$

The total number of units deteriorated during the i -th replenishment cycle is:

$$D_i = I(T_{i-1}) - \int_{T_{i-1}}^{t_i} D(u)] du, \quad i = 1, 2, \ldots\ldots, n \qquad (7)$$

Total cost of the inventory system over the time horizon H is given by:

$$TC(n, r) = n\, c_1 + c_2 \sum_{i=1}^{n} I_i + c_k \sum_{i=1}^{n} D_i + c_3\, B \sum_{i=1}^{n-1} S_i$$

$$+ c_k \sum_{i=1}^{n} D_i + c_3\, B \sum_{i=1}^{n-1} S_i + c_3\, B \sum_{i=1}^{n-1} S_i$$

$$+ c_4 (1 - B) \sum_{i=1}^{n-1} \int_{t_i}^{T_i} D(u) du \qquad (8)$$

The total cost function $TC(n, r)$ given above is a function of two variables n and r where r $(0 \le r \le 1)$ is a continuous variable and n is a discrete variable. For any given value of n, the necessary and sufficient condition for TC *to* be minimum is:

$$\frac{dTC}{dr} = 0 \qquad (9)$$

The solution of Equation (9) gives the optimal solution r^* which minimize TC (n, r) provided it satisfies the sufficient condition $\frac{d^2TC}{dr^2} > 0$. Equation (9) is simply a polynomial equation in r. This equation can be easily solved by iterative method when the values of the parameters are prescribed. Hence, to solve the proposed model, n = 2, 3 ... are substituted in Equation (9) and the corresponding values of r $(0 \le r \le 1)$ for which TC is minimum is found. A list of the corresponding costs TC can be obtained from Equation (8) and minimum value of TC in the list would be the optimal TC. The values of n and r for the minimum value of TC are the optimal values of n and r, respectively.

4. COMPUTATIONAL RESULTS

(a) To illustrate the complete backlogging case, consider the base example: a = 200, b = 20, c = 2, c_1 = 150, c_2 = 60, c_3 = 20, c_4 = 90, B = 1, c_k = 120, θ_1 = 0.01, θ_2 = 0.001, H = 10 in appropriate units. Equation (9) is solved for r $(0 \le r \le 1)$ for different value of n and then substituting these values of n and r in (8), the corresponding values of the cost TC are obtained. Results obtained are shown in Table-2. In Table-2, the numerical values of the cost function show that cost function TC is

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convex and TC is minimum for n = 47. Hence the optimal values of n and r are respectively, n^* = 47 and r^* = 0.2441799 and minimum cost TC becomes $TC^*(n, r)$ = 13299.720. Putting r = 1 in Equation. (8), our model reduces to the case of no-shortage. In this case also, we obtain the optimal values of n and TC from (8), by finding the total cost from Equation (8), for different values of n and then find the minimum cost. This is shown in Table-1. From this table, we see that the optimal values of n and TC are respectively, n^* = 87, TC^* (n, r) = 25841.190.

(b) To illustrate the partial backlogging case, consider the base example: a = 200, b = 20, c = 2, c_1 = 150, c_2 = 60, c_3 = 20, c_4= 90, B = 0.7, c_k = 120, θ_1 = 0.01, θ_2 = 0.001, H = 10 in appropriate units. For these parameters, optimal values of n and r are respectively, n^* = 30 and r^* = 0.2298681 and minimum cost TC^* becomes $TC^*(n, r)$ = 105849.200 (shown in Table-4). For no-shortage case (r = 1), optimal values of n and TC are respectively, n^* = 75, $TC^*(n, r)$ = 120993.800 (shown in Table-3).

Table-1. Complete back-logging case (r = 1).

N	Cost	n	Cost
2	637077.100	61	27544.260
3	409769.100	62	27391.510
4	301805.600	63	27248.410
5	238892.500	64	27114.500
6	197757.600	65	26989.370
7	168791.900	66	26872.600
8	147310.400	67	26763.820
9	130758.100	68	26662.680
10	117623.600	69	26568.850
11	106955.800	70	26482.020
12	98126.310	71	26401.870
13	90703.480	72	26328.140
14	84380.880	73	26260.560
15	78935.080	74	26198.880
16	74199.260	75	26142.880
17	70046.390	76	26092.290
18	66378.060	77	26046.950
19	63116.790	78	26006.630
20	60200.790	79	25971.140
21	57580.200	80	25940.300
22	55214.280	81	25913.950
23	53069.450	82	25891.900
24	51117.820	83	25874.020
25	49335.990	84	25860.140
26	47704.150	85	25850.140
27	46205.510	86	25843. 860
28	44825.670	*87	*25841.190
29	43552.230	88	25841.990
30	42374.460	89	25846.160
31	41283.050	90	25853.580
32	40269.840	91	25864.140
33	39327.640	92	25877.740
34	38450.170	93	25894.290
35	37631.830	94	25913.670
36	36867.640	95	25935.820
37	36153.210	96	25960.640
38	35484.570	97	25988.040
39	34858.190	98	26017.960
40	34270.860	99	26050.300
41	33719.730	100	26085.000
42	33202.170	101	26122.000
43	32715.850	102	26161.210
44	32258.620	103	26202.580
45	31828.520	104	26246.050
46	31423.780	105	26291.550
47	31042.780	106	26339.020
48	30684.010	107	26388.420
49	30346.120	108	26439.680
50	30027.840	109	26492.760
51	29728.010	110	26547.610
52	29445.580	111	26604.180
53	29179.530	112	26662.410
54	28928.960	113	26722.280
55	28693.040	114	26783.730
56	28470.950	115	26846.720
57	28261.970	116	26911.210
58	28065.430	117	26977.180
59	27880.690	118	27044.560
60	27707.150	119	27113.350

Table-2. Complete back-logging case.

N	r	Cost
2	0.2448856	456014.100
3	0.2445216	239115.500
4	0.2443860	153652.200
5	0.2443200	110320.200
6	0.2442826	84901.540
7	0.2442589	68506.130
8	0.2442430	57203.310
9	0.2442316	49019.660
10	0.2442231	42868.040
11	0.2442167	38105.640
12	0.2442115	34330.450
13	0.2442074	31279.410
14	0.2442041	28773.790
15	0.2442012	26688.340
16	0.2441989	24932.810
17	0.2441969	23440.710
18	0.2441951	22162.050
19	0.2441936	21058.540
20	0.2441923	20100.410
21	0.2441911	19264.210
22	0.2441900	18531.210
23	0.2441891	17886.250
24	0.2441883	17317.010
25	0.2441875	16813.320
26	0.2441868	16366.730
27	0.2441861	15970.170
28	0.2441856	15617.690
29	0.2441851	15304.220
30	0.2441846	15025.460
31	0.2441841	14777.670
32	0.2441837	14557.670
33	0.2441833	14362.670
34	0.2441829	14190.250
35	0.2441826	14038.280
36	0.2441823	13904.880
37	0.2441820	13788.430
38	0.2441818	13687.450
39	0.2441815	13600.650
40	0.2441813	13526.860
41	0.2441810	13465.060
42	0.2441808	13414.310
43	0.2441806	13373.770
44	0.2441804	13342.700
45	0.2441802	13320.400
46	0.2441800	13306.260
*47	*0.2441799	*13299.720
48	0.2441797	13300.270
49	0.2441795	13307.430
50	0.2441794	13320.790
51	0.2441793	13339.940
52	0.2441791	13364.530
53	0.2441790	13394.230
54	0.2441789	13428.730
55	0.2441788	13467.760
56	0.2441786	13511.060
57	0.2441785	13558.380
58	0.2441784	13609.500
59	0.2441783	13664.210
60	0.2441782	13722.340

Table-3. Partial back-logging case (r = 1).

n	Cost	n	Cost
2	637077.100	61	121457.300
3	431769.100	62	121385.400
4	338368.200	63	121320.600
5	285548.500	64	121262.500
6	251757.700	65	121211.000
7	228354.500	66	121165.600
8	211224.400	67	121126.100
9	198165.500	68	121092.300
10	187895.600	69	121063.800
11	179618.500	70	121040.500
12	172813.800	71	121022.100
13	167127.700	72	121008.400
14	162310.900	73	120999.300
15	158183.100	74	120994.500
16	154610.400	*75	*120993.800
17	151491.600	76	120997.200
18	148748.500	77	121004.400
19	146319.900	78	121015.300
20	144157.300	79	121029.800

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21	142221.600	80	121047.600
22	140481.000	81	121068.800
23	138909.400	82	121093.100
24	137485.000	83	121120.500
25	136189.900	84	121150.800
26	135008.700	85	121183.900
27	133928.400	86	121219.800
28	132938.100	87	121258.400
29	132028.200	88	121299.500
30	131190.500	89	121343.100
31	130417.800	90	121389.000
32	129704.000	91	121437.300
33	129043.700	92	121487.700
34	128431.900	93	121540.400
35	127864.400	94	121595.100
36	127337.600	95	121651.800
37	126847.900	96	121710.500
38	126392.600	97	121771.100

Table-4. Partial back-logging case.

n	r	Cost
2	0.2562945	473664.500
3	0.2427338	275355.800
4	0.2381905	201987.100
5	0.2359006	166957.100
6	0.2345176	147544.500
7	0.2335910	135681.500
8	0.2329265	127915.000
9	0.2324264	122565.000
10	0.2320366	118733.700
11	0.2317241	115905.600
12	0.2314679	113767.200
13	0.2312541	112119.000
14	0.2310730	110829.000
15	0.2309175	109806.900
16	0.2307827	108989.400
17	0.2306647	108330.900
18	0.2305604	107797.800
19	0.2304676	107365.200
20	0.2303846	107013.800
21	0.2303098	106729.100
22	0.2302421	106499.600
23	0.2301805	106315.900
24	0.2301243	106171.000
25	0.2300728	106058.900
26	0.2300253	105974.900
27	0.2299814	105914.800
28	0.2299409	105875.700
29	0.2299032	105854.500
*30	*0.2298681	*105849.200
31	0.2298353	105857.700
32	0.2298046	105878.400
33	0.2297758	105909.900
34	0.2297488	105951.100
35	0.2297233	106000.800
36	0.2296994	106058.100
37	0.2296767	106122.300
38	0.2296553	106192.700

5. CONCLUDING REMARKS

In modeling of inventories with time-varying demand patterns, the researchers usually consider linearly or exponentially time-dependent demands. The demand rate function $D(t) = a + b\,t,\ a \geq 0,\ b \neq 0$ represents linearly trended demand implying steady increase (or decrease) in demand for the product. For exponential time dependence, $D(t) = a\,e^{b\,t}, a > 0, b \neq 0$ implying exponential increase (or decrease) in demand. An exponential rate being very high, it is doubtful whether the real market demand of any product can really rise (or decline) exponentially. There may be accelerated rise or decline in demand for a product depending on its nature. We are of the view that quadratic time-dependence in the form $D(t) = a + b\,t + c\,t^2$ is more realistic. Here $a\ (\geq 0)$ stands for the initial demand rate. We have $\frac{dD(t)}{dt} = b + 2ct$ and $\frac{d^2D(t)}{dt^2} = 2c$, now $\frac{dD(t)}{dt} = 0$ give $t = -\frac{b}{2c}$ which is positive only if b and c are of opposite signs. For $b > 0$ and $c < 0$, $D(t)$ possesses a maximum $(a - \frac{b^2}{4c})$ at $t = -\frac{b}{2c}$. The demand rate rises smoothly to a maximum and then falls down gradually. Seasonal products like winter cosmetics have this type of demand. With the beginning of the season, the demand starts growing, reaches a pick at the high season and then dies out as the season comes to an end. There is accelerated growth in demand for $b > 0$, $c > 0$. This type of demand is found to occur in the case of the state-of-the- art aircrafts, computers, machines and their spare parts. For $b < 0$, $c < 0$, demand undergoes accelerated decline which happens in the case of obsolete aircrafts, computers, machines and their spare parts. Thus quadratic time-dependence of demand is more general than other forms as it accommodates various natures of market demands.

In this paper, an EOQ model with increasing quadratic demand and increasing linear deterioration is examined. In the complete back ordering case, it is found that the optimal cost for with shortage case and without shortage case are 13299.72 and 25841.19 in 47 and 87 cycles respectively. But in partial back ordering case respective costs are 105849.20 and 120993.80 in 30 and 75 cycles. Hence in both the cases, shortages are considered to be better economically. For further study, many problems are worth considering in this model, for example when the lengths between replenishment cycles are different, when the deterioration rate may have a more complex distribution and when the set up cost, holding cost and back order cost are in unstable states.

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