ARPN Journal of Engineering and Applied Sciences

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RATE-TRANSIENT ANALYSIS FOR HYDRAULICALLY FRACTURED VERTICAL OIL AND GAS WELLS

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ABSTRACT

Several common reservoir production conditions result in flow at a constant pressure; then, a constantly changing well-flowing pressure is recorded. Nowadays, most well-test analysis methods assume constant-rate production especially since gas shale wells are normally tested by recording the flow rate values under constant pressure conditions. In such cases, well testing could be eliminated in many cases as being of little value or economically unjustifiable because of the resulting production loss when compared with what can be obtained from constant wellbore pressure production data. Then, this paper presents a transient-rate analysis for artificially fractured vertical wells flowing under constant pressure in homogenous deposits with circular/square shape. Expressions for reservoir characterization using both *TDS* and conventional techniques are introduced and successfully tested with field and synthetic examples.

Keywords: fractured wells, RTA, TDS, constant-pressure conditions, fracture conductivity.

INTRODUCTION

Multi-rate testing can be set as the main testing tool for reservoir characterization since it comprises the remaining tests. A pressure drawdown test can be defined as the simplest multi-rate test with a single non-zero flow rate. Besides, a pressure buildup test can be defined as a multi-rate test having two flow rates: one different than zero and another one with a value of zero. Multi-rate tests can have several flow rate variations either with regular or irregular changes in flow rate. If the flow rate changes continuously, then, the case of transient-rate analysis is obtained.

The behavior of a well operating at constant sand-face pressure is analogous to that of a well operating at constant flow rate. In a constant pressure flow testing, the well produces at a constant bottom-hole pressure and flow rate is recorded with time. Since rate solutions are found on basic flow principles, flow rate data can be used for reservoir characterization and different property estimations. Hence, this technique can be considered as an alternative to conventional well testing techniques: constant flow rate cases. However, they are customary used in decline-curve analysis.

Arps (1945) developed the standard exponential, hyperbolic, and harmonic decline equations. Fetkovich (1980) generated the dimensionless rate-time type curves for decline curve analysis of wells producing at a constant bottom-hole pressure. He demonstrated that decline curve analysis not only has a solid fundamental basis but also provides a tool with more diagnostic power than had been previously known. These type curves combined analytical solutions to the flow equations in the transient region and empirical rate relationships, proposed by Arps (1945), in the pseudosteady state region. A method for determining the skin effect from rate-time data was given by Earlougher (1977). With regard to heterogeneous reservoirs, numerous analytical and numerical solutions for constant pressure production conditions have been published using the Van Everdingen and Hurst (1949) famous solution.

Escobar, Rojas and Cantillo (2012) extended the conventional technique for rate-transient analysis in long and narrow homogeneous and naturally fractured reservoirs. Also, Escobar, Rojas and Bonilla (2012) and Escobar, Sanchez and Cantillo (2008) provided methodologies for transient-rate interpretation for elongated homogeneous and heterogeneous reservoir systems and gas reservoirs, respectively, following the philosophy of the *TDS* technique, Tiab (1993).

For hydraulically fractured wells, Cinco-Ley and Samaniego (1978) and Cinco-Ley, Samaniego and Dominguez (1978) presented one of the most important findings so-called “finite conductivity”. This model represents the general case comparing with the previous published models. In addition, Tiab (1994) applied the *TDS* technique, Tiab (1993), to fractured wells. Under a constant pressure production, the well intercepted by a vertical fracture has been also discussed in the literature. Extension of the *TDS* technique to hydraulically fractured oil wells was performed by Arab (2003).

In this work, both conventional and *TDS* techniques are extended for interpretation of rate-transient tests run in hydraulically vertical fractured gas and oil wells. The proposed methodologies were successfully tested with actual and synthetic data.

MATHEMATICAL FORMULATION

The following dimensionless parameters are used for the mathematical development:

Dimensionless time based on area, A:

$$t_{DA} = \frac{0.0002637kt}{\phi \mu c_t A} \tag{1}$$

Dimensionless time based on half-fracture length, x_f,

$$t_{Dxf} = \frac{0.0002637kt}{\phi \mu c_t x_f^2} \tag{2}$$

Dimensionless pseudotime, Agarwal (1979), based on half-fracture length, x_f,

$$t_{aD}(P_D) = \frac{0.0002637kt_a(P)}{\phi x_f^2} \tag{3}$$

Dimensionless oil flow reciprocal rate, $1/q_D$,

$$\frac{1}{q_D} = \frac{kh\left(P_i - P_{wf}\right)}{141.2q\mu B} \tag{4}$$

Dimensionless gas flow reciprocal rate, $1/q_D$,

$$\frac{1}{q_D} = \frac{kh\left[\Delta m\left(P_i\right) - \Delta m\left(P_{wf}\right)\right]}{1424qT} \tag{5}$$

Dimensionless fracture conductivity, C_{fD},

$$C_{fD} = \frac{k_f w_f}{kx_f} \tag{6}$$

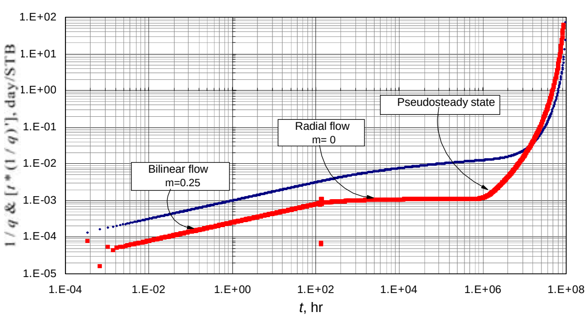

Figure-1. Dimensionless reciprocal rate and reciprocal rate derivative for a vertical well with a finite-conductivity fracture.

Transient-rate analysis for oil wells having a finite-conductivity fracture by the *TDS* technique

In such cases, the most remarkable flow regime seen at early times, if wellbore storage allows, is the bilinear. This is recognized by a slope of one fourth on the reciprocal rate derivative as indicated in Figure-1. The dimensionless reciprocal rate and reciprocal rate derivative governing equations for such flow are:

$$\frac{1}{q_D} = \left[\frac{2.722}{\sqrt{C_{fD}}}\right] t_{Dxf}^{0.25} \tag{7}$$

$$t_{Dxf} * (1/q_D)' = \frac{0.6805}{\sqrt{C_{fD}}} t_{Dxf}^{0.25} \tag{8}$$

An expression in oil-field units will results after plugging the dimensionless parameters given by Equations (2), (4) and (6) into Equation 6, thus,

$$\frac{1}{q} = \frac{48.968\mu B}{h\Delta P\sqrt{k_f w_f}\left(k\phi\mu c_t\right)^{0.25}} t^{0.25} \tag{9}$$

Which reciprocal rate derivative is given by

$$t*(1/q)' = \frac{12.242\mu B}{h\Delta P\sqrt{k_f w_f}\left(\phi\mu c_t k\right)^{0.25}} t^{0.25} \tag{10}$$

From which an expression to estimate the fracture conductivity is obtained using the reciprocal rate derivative read at a time of 1 hour.

$$k_f w_f = \frac{149.866}{\sqrt{\phi\mu c_t k}} \left\{\frac{\mu B}{h\Delta P[t*(1/q)']_{BL1}}\right\}^2 \tag{11}$$

Transient-rate analysis for oil wells having finite-conductivity fractures by the conventional method

Equation (7) suggests that the slope m_{BL} from a Cartesian plot of the one-fourth root of time versus the reciprocal rate can be used to estimate fracture conductivity:

$$k_f w_f = \left(\frac{48.968\mu B}{m_{BL} h\Delta P\left(\phi\mu c_t k\right)^{0.25}}\right)^2 \tag{12}$$

Transient-rate analysis for gas wells having a finite-conductivity fracture by the *TDS* technique

Once the dimensionless quantities given by Equations (2), (5) and (6) are replaced into Equation (7), the resulting expression for bilinear gas flow and its reciprocal derivative are:

$$\frac{1}{q} = \frac{493.94T}{h[\Delta m(P)]\sqrt{k_f w_f}\sqrt[4]{\phi(\mu c_t)_i k}} t^{0.25} \tag{13}$$

$$t*(1/q)' = \frac{123.49T}{h\Delta P\sqrt{k_f w_f}\sqrt[4]{\phi(\mu c_t)_i k}} t^{0.25} \tag{14}$$

Solving for the fracture conductivity from the above expression,

$$(k_f w_f)_{app} = \frac{15242.372}{\left(\phi\mu c_t k\right)^{0.5}} \left\{\frac{T}{h[\Delta m(P)][t*(1/q)']_{BL1}}\right\}^2 \tag{15}$$

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If using pseudotime, as defined by Equation (3), the above expression becomes:

$$(k_f w_f)_{app} = \frac{15242.372}{(\phi k)^{0.5}} \left\{ \frac{T}{h[\Delta m(P)][t*(1/q)']_{BL1}} \right\}^2 \quad (16)$$

Equations (15) or (16) are used to estimate the apparent fracture conductivity from the reciprocal rate derivative read at either the time of 1 hour or the pseudotime of 1 hr-psi/cp.

Transient-rate analysis for gas wells having a finite-conductivity fracture by the conventional technique

Equation (13) indicates that the slope m_{BL} from a Cartesian plot of the one-fourth root of either time or pseudotime versus the reciprocal rate can be used to estimate fracture conductivity from the following expressions using time and pseudotime, respectively,

$$(k_f w_f)_{app} = \left(\frac{493.94T}{m_{BL} h[\Delta m(P)] \sqrt[4]{\phi(\mu c_t)_i k}} \right)^2 \quad (17)$$

$$(k_f w_f)_{app} = \left(\frac{493.94T}{m_{BL} h[\Delta m(P)] \sqrt[4]{\phi k}} \right)^2 \quad (18)$$

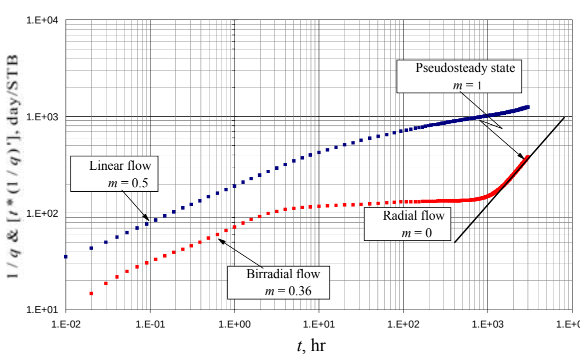

Figure-2. Dimensionless reciprocal rate and reciprocal rate derivative for a vertical well with an infinite-conductivity fracture.

Transient-rate analysis for oil wells having an infinite-conductivity fracture - *TDS* technique

Although, linear flow regime can also appear in low to medium finite-conductivity fractures, it is more common that either linear and/or birradial, Tiab (1994), develop as depicted in Figure-2.

The dimensionless reciprocal rate governing equation for linear flow, Arab (2003), is given as follows:

$$\frac{1}{q_D} = 2.7842\, t_{Dxf}^{0.5} \quad (19)$$

Which derivative results;

$$t_{Dxf} * (1/q_D)' = 2.3921\, t_{Dxf}^{0.5} \quad (20)$$

After plugging the dimensionless quantities defined by Equations (2) and (4) into the above expression will yield:

$$\frac{1}{q} = \frac{6.3836B}{k_f h \Delta P} \sqrt{\frac{\mu t}{\phi c_t k}} \quad (21)$$

Which reciprocal rate derivative results:

$$t*(1/q)' = \frac{3.1918B}{x_f h \Delta P} \sqrt{\frac{\mu t}{\phi c_t k}} \quad (22)$$

Therefore, solving for the half-fracture length results in an expression that uses the reciprocal rate derivative, extrapolated if necessary, at the time of 1 hour:

$$x_f = \frac{3.1918B}{h \Delta P [t*(1/q)_{L1}']} \sqrt{\frac{\mu}{\phi c_t k}} \quad (23)$$

Tiab (1994) introduced the concept and definition of birradial (or elliptical) flow regime. The definition of pressure derivative behavior is adapted here for transient-rate analysis, such as:

$$t_{DA} * (1/q_D)' = 0.769 \left(\frac{x_e}{x_f} \right)^{0.72} t_{DA}^{0.36} \quad (24)$$

From integration of Equation (24), it yields,

$$\frac{1}{q_D} = 2.1361 \left(\frac{x_e}{x_f} \right)^{0.72} t_{DA}^{0.36} \quad (25)$$

Replacing in Equation (25) -if the system is square- then $A = 4x_e^2$, the dimensionless parameters given by Equations (1) and (4) will result:

$$\frac{1}{q} = \frac{9.426B}{h \Delta P \left(\phi c_t x_f^2 \right)^{0.36}} \left(\frac{\mu}{k} \right)^{0.64} t^{0.36} \quad (26)$$

Which reciprocal rate derivative is given by

$$[t*(1/q)'] = \frac{3.93313\mu^{0.64}}{h \Delta P \left(\phi c_t x_f^2 \right)^{0.36}} \left(\frac{\mu}{k} \right)^{0.64} t^{0.36} \quad (27)$$

Solving for x_f, when the reciprocal derivative is read at a time of 1 hr,

$$x_f = \left[\frac{3.39313B}{[t*(1/q)']_{BR1} h \Delta P (\phi c_t)^{0.36}} \left(\frac{\mu}{k} \right)^{16/25} \right]^{50/36} \quad (28)$$

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Equation (28) is useful to estimate the half-fracture length by reading the reciprocal rate derivative during birradial flow regime at a time of 1 hour, extrapolated if needed.

Transient-rate analysis for oil wells having an infinite-conductivity fracture by conventional analysis

Equation (19) suggests that the slope m_L from a Cartesian plot of the square root of either time or pseudotime versus the reciprocal rate can be used to estimate half-fracture length by means of the following expression;

$$x_f = \frac{6.3836B}{m_L h\Delta P}\sqrt{\frac{\mu}{k\phi c_t}} \quad (29)$$

For infinite-conductivity cases, when birradial flow is present, the half-fracture length can be estimated from the slope m_{BR} of a Cartesian plot of the time to the power 9/25 or 0.36 against the reciprocal rate, using the following expression:

$$x_f = \left(\frac{9.426B}{m_{BR}h\Delta P\left(\phi c_t\right)^{0.36}}\left(\frac{\mu}{k}\right)^{16/25}\right)^{50/36} \quad (30)$$

Transient-rate analysis for gas wells having an infinite-conductivity fracture by the *TDS* technique

The below expression is obtained once the dimensionless parameters given by Equations (2) and (5) are replaced into Equation (19):

$$\frac{1}{q} = \frac{64.379T}{x_f h\left[\Delta m(P)\right]\sqrt{k\phi(\mu c_t)_i}}t^{0.5} \quad (31)$$

Which derivative is:

$$t*(1/q)' = \frac{32.1895T}{x_f h\left[\Delta m(P)\right]\sqrt{\phi(\mu c_t)_i k}}t^{0.5} \quad (32)$$

Solving for the half-fracture length;

$$x_f = \frac{32.1895T}{h\left[\Delta m(P)\right]\left[t*(1/q)'\right]_L\sqrt{\phi(\mu c_t)_i k}}t_L^{0.5} \quad (33)$$

As dealt before, if the value of the reciprocal rate derivative is read at the time of one hour, extrapolated if necessary, Equation (33) becomes:

$$x_f = \frac{32.1895T}{h\left[\Delta m(P)\right]\left[t*(1/q)'\right]_{L1}\left(\phi\mu_i c_{ti} k\right)^{0.5}} \quad (34)$$

If birradial flow is presented and considering a square reservoir, then $A = 4x_e^2$, then, the governing equation resulting from substituting Equations (1) and (5) into Equation (25), yields;

$$1/q = \frac{95.052T}{hk^{0.64}\left[\Delta m(P)\right]\left[\phi(\mu c_t)_i\right]^{0.36}x_f^{0.72}}t^{0.36} \quad (35)$$

The reciprocal rate derivative with respect to time of Equation (35) is given as follows:

$$[t*(1/q)']_{BR} = \frac{34.219T}{hk^{0.64}\left[\Delta m(P)\right]\left[\phi(\mu c_t)_i\right]^{0.36}x_f^{0.72}}t_{BR}^{0.36} \quad (36)$$

Solving for the half-fracture length, we obtain for time and pseudotime, respectively:

$$x_f = \left(\frac{34.219T}{hk^{0.64}\left[\Delta m(P)\right]\left[\phi(\mu c_t)_i\right]^{0.36}[t*(1/q)']_{BR1}}\right)^{50/36} \quad (37)$$

$$x_f = \left(\frac{34.219T}{hk^{0.64}\left[\Delta m(P)\right]\phi^{0.36}[t*(1/q)']_{BR1}}\right)^{50/36} \quad (38)$$

Transient-rate analysis for gas wells having an infinite-conductivity fracture by conventional analysis

The slope from a Cartesian plot of the square-root of time (or pseudotime) against the reciprocal rate allows obtaining the half-fracture length from observation of Equation (31), thus:

$$x_f = \frac{64.379T}{m_L h\left[\Delta m(P)\right]\sqrt{k\phi\mu c_t}} \quad (39)$$

$$x_f = \frac{64.379T}{m_L h\left[\Delta m(P)\right]\sqrt{kc_t}} \quad (40)$$

Notice that Equation (40) is given for pseudotime and Equation (36) is for regular time. Also, it follows for the case of birradial flow regime that Equation (35) suggests that a Cartesian plot of either time or pseudotime to the power 9/25 (or 0.36) will provide a slope, m_{BR}, which leads to find the half-fracture length:

$$x_f = \left(\frac{95.052T}{m_{BR}hk^{0.64}\left[\Delta m(P)\right]\left[\phi(\mu c_t)_i\right]^{0.36}}\right)^{50/36} \quad (41)$$

$$x_f = \left(\frac{95.052T}{m_{BR}hk^{0.64}\left[\Delta m(P)\right]\phi^{0.36}}\right)^{50/36} \quad (42)$$

Again, Equation (42) is given for pseudotime and Equation 41 is for regular time.

Pseudorradial flow regime

Arab (2003) demonstrated that the dimensionless reciprocal rate derivative during radial flow regime takes the value of 0.5,

$$[t_D*(1/q_D)']_R = 0.5 \tag{43}$$

From which, the permeability is estimated by reading the value of the reciprocal rate derivative during radial flow regime, $[t*(1/q)']_r$, and using the following expression:

$$k = \frac{70.6\mu B}{h\Delta P[t*(1/q)']_R} \tag{44}$$

And the skin factor is estimated by reading the reciprocal rate, $(1/q)_r$, read at any arbitrary time, t_r, during the radial flow regime, thus:

$$s = 0.5\left\{\frac{(1/q)_R}{[t*(1/q)']_R} - \ln\left(\frac{kt_R}{\phi(\mu c_t)_i r_w^2}\right) + 7.43\right\} \tag{45}$$

The permeability Equation for gas well was presented by Escobar *et al*. (2008),

$$k = \frac{711.5817T}{h\left[\Delta m(P)\right]\left[t\times(1/q)'\right]_R} \tag{46}$$

Notice that Equation (45) can be applied for gas wells, but in this case, it is referred as apparent skin since inertial flow conditions have to be taken into account. However, for gas flow using pseudotime the resulting apparent skin equation is:

$$s' = 0.5\left\{\frac{(1/q)_R}{[t*(1/q)']_R} - \ln\left(\frac{kt_a(P)_R}{\phi r_w^2}\right) + 7.43\right\} \tag{47}$$

Intersection points

It was found in this study that the governing equation for the reciprocal rate during pseudosteady-state regime is given by:

$$\left[t*(1/q_D)'\right]_{PSS} = 3\pi(t_{DA})_{PSS} \tag{48}$$

Taking the derivative of Equation (4) (oil wells) and plugging this result into Equation (48) along with the dimensionless time quantity given by Equation (1) and solving for A allows to obtain an expression to find the drainage area by reading the reciprocal rate derivative $[t*(1/q)']_{pss}$ at any arbitrary point during the unit-slope late pseudosteady:

$$A = \frac{Bt_{PSS}}{2.85h\Delta P\phi c_t[t*(1/q)']_{PSS}} \tag{49}$$

For the case of gas wells, the resulting equation is given for actual time by:

$$A = \frac{3.54kTt_{PSS}}{h\Delta m(P)\phi(\mu c_t)_i[t*(1/q)']_{PSS}} \tag{50}$$

And for pseudotime;

$$A = \frac{3.54kTt_a(P)_{PSS}}{h\Delta m(P)\phi(\mu c_t)_i[t*(1/q)']_{PSS}} \tag{51}$$

Equations (49) through (51) are also used by reading the reciprocal rate derivative at a value of 1 hr or 1 hr-psi/cp.

The point of intersection formed by the pseudosteady-state reciprocal derivative straight line (Equation 48) with the bilinear flow regime derivative, Equation (8), allows finding expressions for estimating the drainage area:

$$A = \frac{\sqrt{k_f w_f}}{34.892}\left(\frac{t_{BLPSSi}}{\phi\mu c_t}\right)^{0.75} k^{0.25} \tag{52}$$

Equation (52) applies also to gas wells using actual time considering that the gas viscosity and total compressibility are evaluated at initial conditions. For pseudotime, the resulting equation is:

$$A = \frac{\sqrt{k_f w_f}}{34.89}\left(\frac{t_a(P)_{BLPSSi}}{\phi}\right)^{0.75} k^{0.25} \tag{53}$$

The point of intercept between the birradial (Equation 24) and pseudosteady state (Equation 48) derivative lines allows finding another useful equation to estimate the drainage area in oil and gas reservoirs:

$$A = \left(\frac{kt_{BRPSSi}x_f^{1.125}}{34.649\phi\mu c_t}\right)^{16/25} \tag{54}$$

For gas wells with pseudotime,

$$A = \left(\frac{kt_a(P)_{BRPSSi}x_f^{1.125}}{34.649\phi}\right)^{16/25} \tag{55}$$

The point of intersection formed by the pseudosteady-state reciprocal derivative straight line (Equation 48) with the linear flow regime reciprocal rate derivative, Equation (20), leads to the estimation of the drainage area:

$$A = \frac{x_f}{5.79}\sqrt{\frac{kt_{LPSSi}}{\phi\mu c_t}} \tag{56}$$

Which is also good for gas wells if the viscosity and total compressibility are given at initial conditions. If pseudotime is used, the resulting equation would be:

$$A = \frac{x_f}{5.79}\sqrt{\frac{kt_a(P)_{LPSSi}}{\phi}} \tag{57}$$

Intersection of both bilinear (Equation 8) and linear reciprocal rate derivative (Equation 20) lines will provide:

$$k = \left(\frac{k_f w_f}{x_f^2}\right)^2 \frac{t'_{BLLi}}{869.375\phi\,\mu c_t} \tag{58}$$

Intersection of both bilinear (Equation 7) and linear reciprocal rate (Equation 19) lines will provide:

$$k = \left(\frac{k_f w_f}{x_f^2}\right)^2 \frac{t_{BLLi}}{13910\phi\,\mu c_t} \tag{59}$$

Intersection of both bilinear (Equation 8) and linear reciprocal derivative (Equation 20) lines with birradial (Equation 24) will provide:

$$k_f w_f = 12.759 k^{0.7714} x_f^{1.429}\left(\frac{\phi\mu c_t}{t_{BLBRi}}\right)^{0.2143} \tag{60}$$

The above equation assumes that $A = 4x_e^2$, then, both area and reservoir length cancelled out. Intersection of linear (Equation 20) and birradial reciprocal derivative (Equation 24) lines leads to:

$$\frac{x_f^2}{k} = \frac{t_{LBRi}}{39\phi\mu c_t} \tag{61}$$

The intersection point between the radial flow dimensionless reciprocal derivative line (Equation 43), with the bilinear flow (Equation 8), linear flow (Equation 20) and birradial flow (Equation 24) reciprocal derivative lines allow to obtain:

$$t_{RBLi} = 1677\frac{\phi\mu c_t}{k^3}(k_f w_f)^2 \tag{62}$$

$$\frac{x_f^2}{k} = \frac{t_{LRi}}{1207\phi\mu c_t} \tag{63}$$

$$\frac{x_f^2}{k} = \frac{t_{RBRi}}{4587\phi\mu c_t} \tag{64}$$

Equations (58) through (64) apply to both oil and gas using real time. For pseudotime, these equations become, respectively:

$$k = \left(\frac{(k_f w_f)_{app}}{x_f^2}\right)^2 \frac{t'_{BLLi}}{869.375\phi} \tag{65}$$

$$k = \left(\frac{(k_f w_f)_{app}}{x_f^2}\right)^2 \frac{t_{BLLi}}{13910\phi} \tag{66}$$

$$(k_f w_f)_{app} = 12.759 k^{0.7714} x_f^{1.429}\left(\frac{\phi}{t_{BLBRi}}\right)^{0.2143} \tag{67}$$

$$\frac{x_f^2}{k} = \frac{t_{LBRi}}{39\phi} \tag{67}$$

$$t_{RBLi} = 1677\frac{\phi}{k^3}(k_f w_f)_{app}^2 \tag{68}$$

$$\frac{x_f^2}{k} = \frac{t_{LRi}}{1207\phi} \tag{69}$$

$$\frac{x_f^2}{k} = \frac{t_{RBRi}}{4587\phi} \tag{70}$$

The interception of the reciprocal rate derivatives formed by the radial flow regime, Equation (43), and the unit-slope line forced to draw by the late pseudosteady-state line, Equation (48), leads to:

$$A = \frac{kt_{RPPSi}}{201.2\phi\mu c_t} \tag{71}$$

Which works for oil and gas with real time. For gas with the pseudotime function, Equation (71) becomes,

$$A = \frac{kt_a(P)_{RPPSi}}{201.2\phi} \tag{72}$$

Finally, if bilinear flow exists, then, fracture conductivity can be estimated. If linear flow regime exists then fracture half-length can be estimated. For cases such cases where exists only one of these two flow regimes, the fracture parameters can be found from another by:

$$k_f w_f = \frac{3.31739k}{\dfrac{e^s}{r_w} - \dfrac{1.92173}{x_f}} \tag{73}$$

$$x_f = \frac{1.92173}{\dfrac{e^s}{r_w} - \dfrac{3.31739k}{k_f w_f}} \tag{74}$$

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EXAMPLES

Example-1

Arab (2003) reports a transient-rate test run in a fractured well which reciprocal rate and reciprocal rate derivative are provided in Figure-3. Relevant information is given in Table-1. It is required to characterize such test.

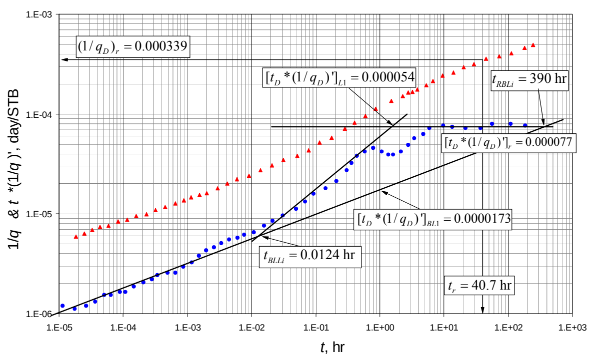

Figure-3. Reciprocal rate and reciprocal rate derivative versus time for example-1.

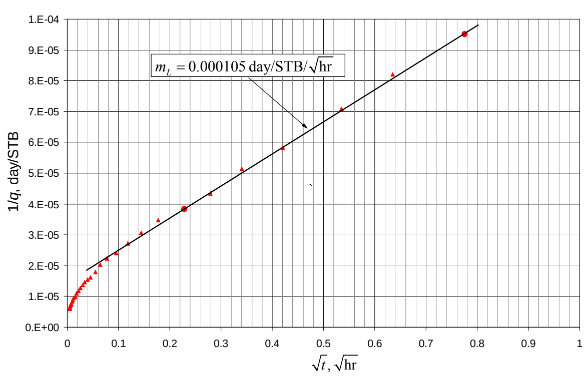

Figure-4. Reciprocal rate and reciprocal rate derivative versus the square root of time for example 1.

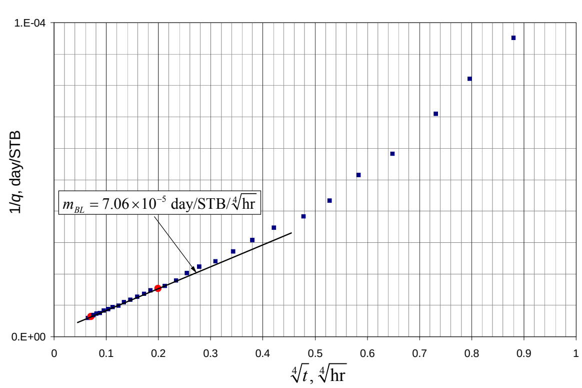

Figure-5. Reciprocal rate and reciprocal rate derivative versus the fourth root of time for example-1.

Table-1. Relevant information for example-1.

Parameter	Value	Parameter	Value
C (bbl/psi)	0	μ_o (cp)	0.85
r_w (ft)	0.3	x_f (ft)	110
h (ft)	30	C_{fD}	10
ϕ (%)	20	P_i (psi)	5200
T (°F)	212	P_{wf} (psi)	3500
B, rb/STB	1.05	c_t (psi^{-1})	0.000031
k (md)	15		

Solution by *TDS* technique

The flowing information was read from Figure-3.

$t_R = 40.7$ hr
$t_{BLLi} = 0.0124$ hr
$(1/q)_R = 0.000339$ day/STB
$[t*(1/q)']_R = 0.000077$ day/STB
$[t*(1/q)']_{BL1} = 0.0000173$ day/STB
$[t*(1/q)']_{L1} = 0.000054$ day/STB

Permeability and skin factor are found from Equations (44) and (45), respectively;

$$k = \frac{70.6(0.85)(1.05)}{(30)(1700)(\ 0.000077\ )} = 16 \text{ md}$$

$$s = 0.5\left\{\frac{3.39\times10^{-4}}{0.000077} - \ln\left(\frac{(16)(40.7)}{(0.2)(0.85)(3.1\times10^{-5})(0.29^2)}\right) + 7.43\right\}$$
$$s = -4.64$$

The half-fracture length is found with Equation (23),

$$x_f = \left[\frac{3.192(1.05)}{(30)(1700)(0.000054)}\right]\sqrt{\frac{(0.85)}{(0.2)(3.1\times10^{-5})(16)}} = 112 \text{ ft}$$

Equation (11) is used to find the fracture conductivity;

$$k_f w_f = \frac{149.866}{\left[(0.2)(0.85)(3.1\times10^{-5})(16)\right]^{1/2}}\left\{\frac{(0.85)(1.05)}{(30)(1700)0.0000173}\right\}^2$$
$$k_f w_f = 18711 \text{ md-ft}$$

Finally, fracture conductivity is estimated with Equation (6),

$$C_{fD} = \frac{18711}{(111)(16.5)} = 9.1$$

Solution by conventional analysis

The slope, m_L, of the Cartesian plot given in Figure-4 is 0.000105 day/STB/hr$^{0.5}$. Equation (29) is used to find the half-fracture length:

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$$x_f = \frac{6.3836(1.05)}{(0.000105)(30)(1700)}\sqrt{\frac{0.85}{(15)(0.20)(0.000031)}}$$

$$x_f = 119.9 \text{ ft}$$

Also, the slope m_{BL} of 7.06x10^{-5} day/STB/hr$^{0.25}$ read from the Cartesian plot of Figure-5 allows determining the fracture conductivity using Equation (12),

$$k_f w_f = \left(\frac{48.968(0.85)(1.05)}{7.06\times10^{-5}(30)(1700)(0.2*0.85*0.00031*15)^{0.25}}\right)^2$$

$$k_f w_f = 16589 \text{ md-ft}$$

Finally, Equation (6) is used to estimate de dimensionless fracture conductivity;

$$C_{fD} = \frac{16589}{(119.9)(15)} = 9.23$$

Example 2

Figure-6 presents synthetically generated reciprocal rate and reciprocal rate derivative for a vertical gas well having a finite-conductivity fracture. Other important data concerning this test is presented in Table-2.

Table-2. Fluid, reservoir and well information for example 2.

Parameter	Value	Parameter	Value
r_e, ft	4000	μ_g (cp)	0.017033
r_w (ft)	0.3	x_f (ft)	200
h (ft)	20	$k_f w_f$, (md-ft)	1200
ϕ (%)	5	P_i (psi)	2000
T (°F)	212	P_{wf} (psi)	1800
B, rb/STB	1.05	c_g (psi^{-1})	0.00051
k (md)	1.5	γ_g	0.9
$\Delta m(P)$, (psi^2/cp)			51235000

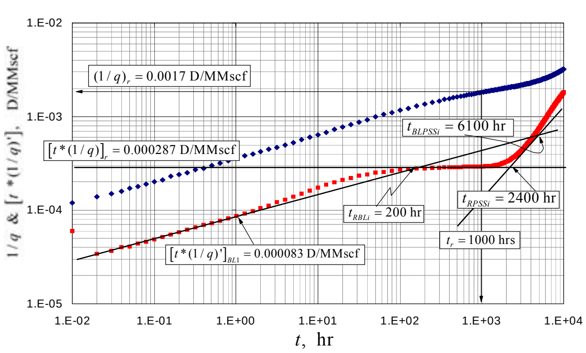

Figure-6. Reciprocal rate and reciprocal rate derivative versus time for example-2.

Solution by *TDS* technique

The flowing information was read from Figure-6.

t_r = 1000 hr
t_{RBLi} = 200 hr
t_{RPSSi} = 2400 hr
t_{BLPSSi} = 6100 hr
$(1/q)_r$ = 0.0017 day/STB
$[t*(1/q)']_r$ =0.000287 day/STB
$[t*(1/q)']_{BL1}$ = 0. 0.000083 day/STB

Equations (46) and (45) lead to determine a permeability value of 1.62 md and a pseudo skin factor of -5.5, respectively. Then, an apparent fracture conductivity value of 1180 md-ft is found with Equation (17). Equation 74 leads to estimate a half-fracture length of 211.6 ft. The time of intersection between the reciprocal rate derivatives of radial and bilinear flow regimes results to be 197.62 hr which is very close to the value of 200 hr read from Figure-6. A drainage area of 992.5 acres is estimated with Equation (53) which corresponds to an external radius of 3709.6 ft. Again, Equation (71) is used to obtain a drainage area of 982.42 Acres with translates into 3690.8 ft.

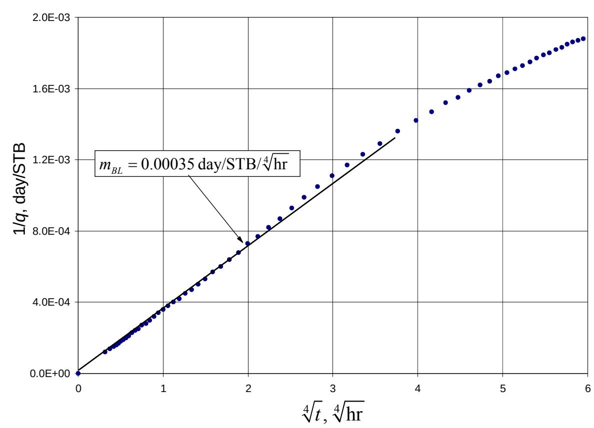

Figure-7. Reciprocal rate and reciprocal rate derivative versus the fourth root of time for example-2

Solution by conventional analysis

The slope, m_{BL}, of the Cartesian plot given in Figure-4 is 0.00035 day/STB/hr$^{0.25}$. Equation (17) is used to find the apparent fracture conductivity which results to be 1061.2 md-ft.

Synthetic example-3

A Transient rate test was simulated with the information given in Table-4. Figure-8 presents reciprocal rate and reciprocal rate derivative against time for this simulation. It is required to characterize such test.

VOL. 9, NO. 5, MAY 2014 ISSN 1819-6608
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Table-3. Well, fluid and reservoir parameters for synthetic example-3.

Parameter	Value	Parameter	Value
C (bbl/psi)	0	k (md)	0.28
r_w (ft)	0.6	μ_o (cp)	1.414
h (ft)	16.4	x_f (ft)	200
ϕ (%)	12	P_i (psi)	5000
r_e (ft)	5000	P_{wf} (psi)	2500
B, rb/STB	1.2	c_t (psi^{-1})	1.31x10^{-5}

Solution by *TDS* technique

The flowing information was read from Figure-8.

t_R = 795.04 hr
$(1/q)_R$ = 0.0271 day/STB
$[t*(1/q)']_R$ = 0.0099 day/STB
$[t*(1/q)']_{BR1}$ = 0.00085 day/STB
$[t*(1/q)']_{L1}$ = 0.000832 day/STB

Permeability and skin factor are estimated with Equations (44) and (45). These are 0.293 md and -2.12, respectively. A fracture-half length of 199.6 md was found with Equation (23). Also, Equation (28) allowed to find another estimation of the half-fracture length of 170.6 ft. Fracture conductivity is found with Equation (73) to be 5. Md-ft.

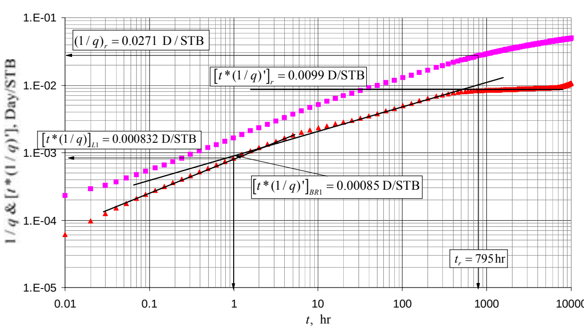

Figure-8. Reciprocal rate and reciprocal rate derivative versus time for example-3.

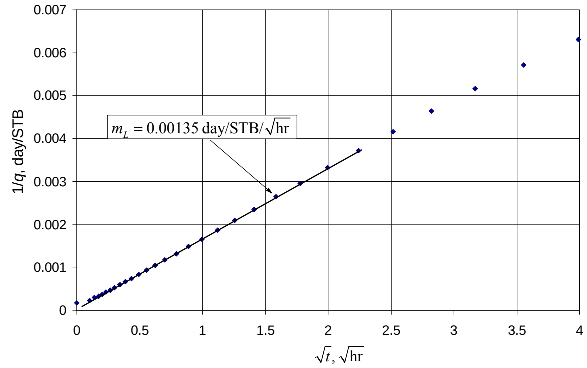

Figure-9. Reciprocal rate and reciprocal rate derivative versus the square root of time for example-3.

Solution by conventional analysis

The slope, m_L, of the Cartesian plot given in Figure-9 is 0.00135 day/STB/hr$^{0.5}$ which allows finding a half-fracture length of 206.9 ft by mans of Equation (29).

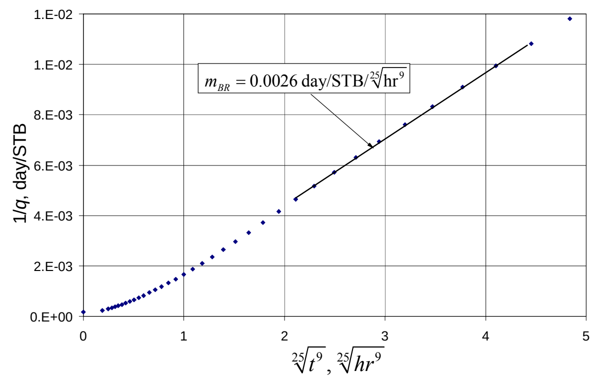

Figure-10. Reciprocal rate and reciprocal rate derivative versus the time to the power 9/25 for example-3.

Moreover, the slope, m_{BR}, of the Cartesian plot given in Figure-10 is 0.0026 day/STB/hr$^{0.36}$. This is used to find a half-fracture length of 148.7 ft by mans of Equation (30).

COMMENTS OF RESULTS

Table-4 present a summary of the main results obtained from the worked examples and compared to the reference values. It is observed a good match between the results and the reference values. A higher deviation is seen in example-2 (gas well) since the obtained values corresponds to apparent fracture conductivity which has to be corrected due to inertial effects.

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Tabla-4. Comparison of main results from examples.

Parameter	Reference value	This study *TDS*	This study Conventional
Example-1			
x_f, ft	110	112	119.9
$k_f w_f$, md-ft	16500	18711	16589
Example-2			
x_f, ft	200	211.6	-
$(k_f w_f)_{app}$, md-ft	1200	1180	1061.2
r_e, ft	4000	3700	-
Example-3			
x_f, ft	200	199.6	206.9-148.7

CONCLUSIONS

Both *TDS* and conventional techniques were complemented to characterize transient rate tests in hydraulically-fractured vertical hydrocarbon Wells. The new expression was successfully tested with field and simulated data.

ACKNOWLEDGEMENTS

The authors gratefully thank the Most Holy Trinity and the Virgin Mary mother of God for all the blessing received during their lives.

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Nomenclature

A	Drainage area, ft^2
B	Oil volumen factor, rb/STB
C	Wellbore storage coefficient, bbl/psi
C_{fD}	Dimensionless fracture conductivity
c_t	System compressibility 1/psi
h	Formation thickness, ft
k	Permeability, md
$k_f w_f$	Fracture conductivity, md-ft
$(k_f w_f)_{app}$	Apparent fracture conductivity, md-ft
$1/q$	Reciprocal rate, D/STB
$1/q_D$	Dimensionless reciprocal rate
P_i	Initial reservoir pressure, psi
P_{wf}	Well-flowing pressure, psi
P	Pressure, psi
r_e	Drainage radius, ft
r_w	Wellbore radius, ft
s	Skin factor
s'	Apparent skin factor
t	Time, hr
t'	Time read on derivative curves for bilinear
t_D	Tiempo adimensional
t_D*1/q_D'	Dimensionless reciprocal rate derivative
$t*(1/q)'$	Reciprocal rate derivative, Day/STB

Greek

Δ	Change
γ_g	Ga specific gravity
ϕ	Porosity
ρ	Density, lbm/ft^3
μ	Viscosity, cp

Sufijos

g	Gas
i	Intercept, initial
D	Dimensionless
$BLPSSi$	Bilnear-pseudosteady-state intercept
BL	Bilinear
$BL1$	Bilinear at 1 hr or 1 psi-hr/cp
$BLBRi$	Bilinear-birradial intercept
$BLLi$	Bilinear-linear intercept
$BLPSSi$	Bilinear-pseudosteady state intercept
BR	Birradial
$BR1$	Birradial at 1 hr or 1 psi-hr/cp
L	Linear
$L1$	Linear at 1 hr or 1 psi-hr/cp
$LBRi$	Linear-birradial intercept
LRi	Linear-radial intercept
$LPSSi$	Linear-pseudosteady-state intercept
o	Oil
PSS	Pseudosteady-state
R,r	Pseudorradial
$RBLi$	Radial-Bilinear intercept
$RBRi$	Radial-Birradial intercept
$RPSSi$	Radial-pseudosteady-state intercept
w	Well