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PROBABILISTIC APPROACH TO RELIABILITY EVALUATION OF LIFTING WIRE ROPES

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ABSTRACT

Wire ropes are used for different applications in many industrial domains, for instance, lifting system. Depending on the conditions of use, wire ropes are being degraded with direct consequences are significant changes of geometric and mechanical characteristics of its components. This results in a reduction in the resistance capacity of the wire rope with time, which could bring failure. Two parameters are susceptible to depict this degradation: a continuous variable which is damage and a statistical variable which is reliability. Our work consists of studying the impact of the breaking of the wires which constitute the wire ropes on its duration. For that we will establish a model that will allow us to connect the two parameters (Damage and reliability) and we will thus broaden this link to the case of compound systems. We are equally proposing to develop a new model which permits providing the reliability of a wire rope in multiple levels of damage of its components. The method adopted is a multi-scale approach with a total decoupling between the scale of the wire and the wire rope.

Keywords: steel wire rope, reliability, damage, fraction of life, probabilistic model.

1. INTRODUCTION

A wire rope (Figure-1) is generally constituted of many strands helically arranged around the central core in a layer or multiple overlying. The strand itself is composed of a lot of steel wires regularly disposed around a central core in a layer or multiple overlying. A wire rope could be composed of one strand, and then we are talking about a mono-strand wire rope, or a helical wire rope.

Figure-1. Schema illustrating the different component of the wire rope.

The wire ropes are very available in the industrial domain. They constitute the essential element of the lifting systems. Their mechanical characteristics change during its use. In this respect, the securities of people who use them directly depend on their states [1, 2].

The sudden breaking of the wire ropes results in major disorders related to many aspects like the loss of humans or materials .Consequently, it is very important to predict their mechanical performance before using them. In this study, our purpose is essentially reliable with the use of a mathematical model based on a probabilistic approach. Therefore; our objective is being able to describe the fatigue behavior of every wire to deduce that of the wire rope.

2. MULTI SCALE APPROACH

A wire rope is a group of interconnected or interdependent elements, so that the state of the wire rope depends on the states of its components [3]. In this way it is related to the most complex composed systems. This signifies that all modeling of the wire rope will be a multi-scale approach.

According to the study realized by Al achachi [4, 5], a suspension wire rope can be considered as a system made of a group of strands disposed in parallel. A strand is itself made of a group of stub disposed in series. Each stub of strand is composed of wires disposed in parallel. The study of the behavior of a wire rope is consequently a multi-scale study in which we can distinguish the scale of the wire, the scale of the strand and that of the wire rope. The systemic schema of a suspended wire rope is therefore a system: parallel (n strands) - series (p stub of strands) - parallel (n' wires). The choice can be justified as follow:

- The behavior of the wire governs the behavior of the wire rope;
- The wires are twisted together, a broken wire has the capacity to re-anchor on a given length, called re-anchor length, and which defines the stub's dimension;
- The behavior of the strand is profoundly linked to the behavior of the weakest stub (the series system);
- Since the strands are disposed in parallel, the resistance of a wire rope depends on their individual resistance and the distribution of the mechanical load.

On the other hand, the realized study by Kolowrocki [6] consists of developing a modeling allowing the estimation of a wire rope's duration of life, where we can distinguish the scale of the strand, the scale of the layer of strands and that of the wire rope. He

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considers that the wire rope is a mixed system (series-parallel). The choice of the series-parallel system is justified by:

- The exterior layer of a wire rope is made of strands having diameters superior than those of the interior layer;
- The failure of one of these strands leads to the failure of the wire rope (series system). These are connected in parallel with the interior layer (parallel system). Thus, we can say that the wire rope make a series parallel system.

3. THE RELIABILITY OF COMPOSED SYSTEMS

The reliability of a material is a statistic parameter. It represents the probability of the survival of this material. That is to say the probability of not facing any kind of failure (the accomplishment of the required function) in relation to the conditions of use given, during an interval of time given. We represent it R (t).

Considering the components of a system E_i $i = 1, 2,..., n$, $n \in N$ having a functions of reliability: $R_i(t) = P(T_i > t)$, $t \in (-\infty, \infty)$, where T_i, $i = 1, 2,..., n$, are independent random variables representing the lives of the components E_i with the functions of the distribution :
$Fi(t) = P(T_i \le t)$, $t \in (-\infty, \infty)$.

The functions of reliability of simplest systems are defined as follow [6]:

- For a series system:

$$R_n(t) = \prod_{i=1}^{n} R_i(t), t \in (-\infty, +\infty). \quad (1)$$

- For a parallel system:

$$R_n(t) = 1 - \prod_{i=1}^{n} F_i(t), t \in (-\infty, +\infty). \quad (2)$$

- For a series-parallel system:

$$R_{l_1, l_2, \dots, l_{k_n}}(t) = 1 - \prod_{i=1}^{k_n}[1 - \prod_{j=1}^{l_i} R_{ij}(t)], t \in (-\infty, +\infty). \quad (3)$$

k_n is the number of parallel subsets and l_i the number of series components.

- For a majority logic system m/n (m material among n functions), the reliability function is:

$$R(t) = \sum_m [\prod_{i \in A_m} R_i] * [\prod_{i \in A_m}(1 - R_i)]. \quad (4)$$

With A_m all arrangement (1, 2….m) include at least k materials in service.

When it comes to regular homogeneous systems, the reliability function will be:

- For a series system:

$$R_n(t) = [R(t)]^n, t \in (-\infty, +\infty). \quad (5)$$

- For a parallel system:

$$R_n(t) = 1 - [F(t)]^n, t \in (-\infty, +\infty) \quad (6)$$

- For a series-parallel system:

$$R_{k_n, l_n} = 1 - \{1 - [R(t)]^{l_n}\}^{k_n} \quad , t \in (-\infty, +\infty) \quad (7)$$

- For a parallel-series system:

$$R_{k_n, l_n} = \{1 - [1 - R(t)]^{k_n}\}^{l_n} \quad , t \in (-\infty, +\infty) \quad (8)$$

- For a majority logic system:

$$R(t) = \sum_{k=m}^{n} C_n^k R_i^{\,k}.(1 - R_i)^{n-k}. \quad (9)$$

Where: $C_n^k = \frac{n!}{k!(n-k)!}$

4. LAW OF PROBABILITY USED IN RELIABILITY

For predicting the life cycle of wire ropes, it is necessary to choose the appropriate statistic model to describe the duration of life of test samples. Three models are generally used for the description of wire ropes's life duration. These models are respectively the Gauss law, the exponential law and the Weibull law [7]. The first law where the distribution of the failure is centered around an average value in the third phase of their life. The exponential law is used only if we have a constant failure rate that is to say in the second phase of the component's life. The Weibull law which is the most used one in representing the wire ropes's duration of life, because it is a flexible law that can adjust with all sorts of experimental results. It covers the case where the failure rate is variable and thus allows adjusting with periods of "youthfulness" and to different forms of aging.

- Reliability according to the Weibull law:

$$R(t) = exp\left[-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right]. \quad (10)$$

Where β = Parameter of form
η = Parameter of scale
γ = Parameter of position

- Density of probability: For $t > \gamma$

$$f(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1}.e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}}. \quad (11)$$

- Function of dividing:

$$F(t) = 1 - e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}}. \quad (12)$$

- Instant rate failure:

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$$Z(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1-F(t)} = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1}. \quad (13)$$

5. RELIABILITY IN FUNCTION OF THE FRACTION OF LIFE

We can express the reliability R (t) in function of the fraction of life (β) which has as an expression:
$\beta = n/Nf$.

Therefore we will consider the time like an increment succession of period (τ) thus $T = n.\tau$, $\eta = Nf.\tau$.

Where n = instant cycle's amount
τ = time between two successive cycles of loading.
η = spreading of the distribution.
Nf = number of cycles cumulated in the breaking.

Exploiting this discretization of time $T = n.\tau$;
$\eta = Nf.\tau$ with $\gamma = 0$ and replacing it in the model of Weibull [7, 9], which has appeared as the most capable one for adjusting with the failure's emergence phenomenon, we take as an expression of reliability:

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}}. \quad (14)$$

We signal to the factor of form β with λ so as not to mix it with the fraction of life
($\beta = n/Nf$).
We obtain:

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\lambda}}. \quad (15)$$

Thus:

$$R(t) = exp(-\beta)^{\lambda}. \quad (16)$$

Figure-2 represents the graphic representation of the reliability in function of the fraction of life (β).

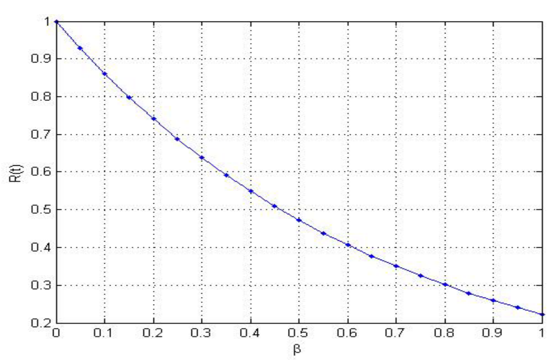

Figure-2. Graphic representation of the reliability in function of the fraction of life (β).

This curve describes well the decreasing of the reliability during functioning of an element. We also remark that for a fraction of life β=1 the reliability is equal to a non-zero value. This value can be attributed to a residual reliability just before the breaking of the material.

6. BEHAVIOR LAW

6.1. Miner law

The simplest and the best known approach to describe the evolution of damage at fatigue under constant amplitude, is the linear rule of the damage said: Miner Act [10, 11]. It is a law in which the damage varies linearly in function of the fraction of life. Miner supposed that the damage noted D is a linear function of the fraction of life β according to the following relationship:

$$D = \beta = \frac{n}{Nf}. \quad (17)$$

Where
n = instant cycle's amount.
F = number of cycles cumulated in the breaking.

The concept is described to be linear because of the linear relationship between the damage D and the fraction of life β. This law says that the interruption of the device occurs when the history of solicitations it underwent, caused partial damage such that their sum reaches the value 1.

$$D = \sum_{i=1}^{p} \frac{ni}{Ni} = 1. \quad (18)$$

6.2. Reliability in function of damage

The theory of damage in function of fraction of life β is the mechanic model chosen to translate the damage of the wire rope through fatigue. This based on a synthesis of different theories which describes the damage under loads of fatigue [8, 9] and which has as an expression:

$$D = \frac{\beta}{\text{[illegible]}}. \quad (19)$$

Where $\gamma = \Delta\sigma/\sigma 0$; $\gamma u = \sigma e/\sigma 0$
$\Delta\sigma$ = amplitude of the solicitation
$\sigma 0$ = endurance limit of the virgin material
σe = limit instant endurance

We put: $\propto = \frac{\gamma - \left(\frac{\gamma}{\gamma u}\right)^{8}}{\gamma - 1}$.

This gives the expression of the reliability in function of damage:

$$R(t) = exp\left(-\left(\frac{\propto .D}{1-D(1-\propto)}\right)^{\lambda}\right). \quad (20)$$

Figure-3 illustrates the graphic representation of the reliability in function of the damage for: $\gamma = \frac{\Delta\sigma}{-} = 1.5$; $\gamma_u = \frac{\sigma_u}{-} = 1.8$; $\lambda = 2$.

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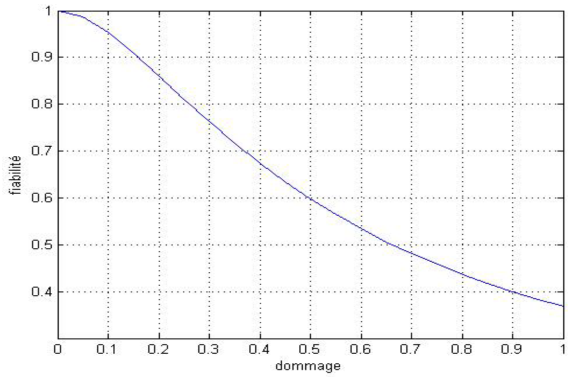

Figure-3. Graphic representation of the reliability in function of damage.

After graphic reading of this curve, we remark that for a damage that equals 1, the reliability is not zero value. Thus, the damage theory considers that the damage reaches its maximal value "1" when it has appearance of a macroscopic crack, but the material keeps a certain resistance translated by a non zero reliability.

7. NUMERICAL APPLICATION

In what follows, we will consider a steel wire rope of 6 mm diameter of a 6*7 type (6strands 7 wires) with a core in textile.

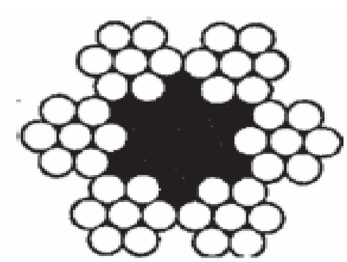

Figure-4. Steel wire rope of a 6*7 type with a textile core.

7.1. Series parallel system (Kolowrocki model)

Considering a series system which has some components or some sub systems in parallel, the system becomes series-parallel in which the reliability equation in function of the fraction of life is the following:

$$R_{SP} = \left[1 - \left(1 - exp(-\beta)^{\lambda}\right)^{p}\right]^{s}. \qquad (21)$$

Where s = the number of blocs in series.
p = the number of blocs in parallel.

In the first case we consider that the wire rope (6*7) is a series-parallel system. We take s=6, the number of series branches which correspond to the number of the strand, and considering p=7 the number of parallel branches which is the number of wires. Figure-5 represents the reliability in function of fraction of life of this wire rope according to the series-parallel model.

Substituting β by $\frac{\alpha . D}{1-D(1-\alpha)}$ in the equation (16), we obtain the reliability's equation of the wire rope according to the damage. The Figure-6 represents the reliability in function of damage of this wire rope according to the series-parallel model.

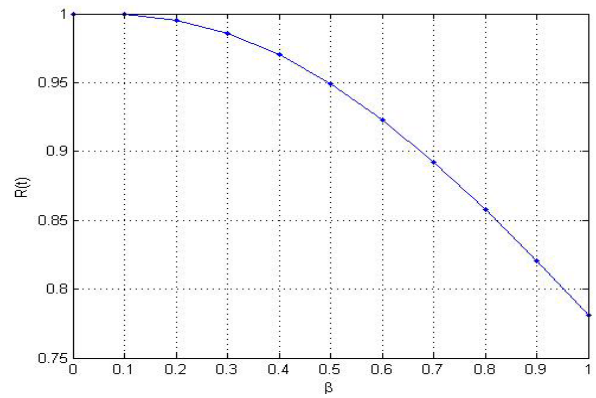

Figure-5. Reliability in function of the fraction of life of the wire rope of (6*7) type according to the series-parallel model.

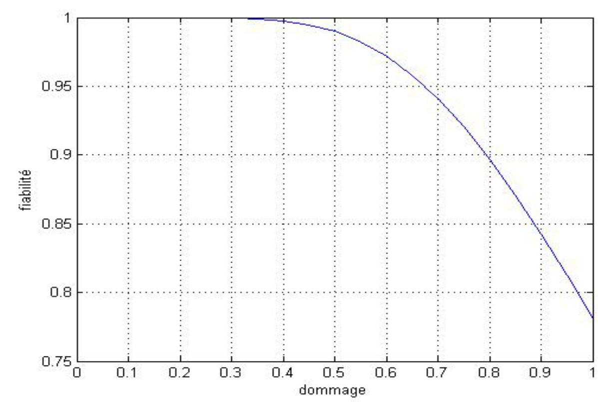

Figure-6. Reliability in function of damage of the wire rope (6*7) type according to the series- parallel model.

7.2. Series-parallel-series system (Al achachi model)

The expression of the reliability in function of the fraction of life of a parallel-series-parallel system is written as follow:

$$R_{psp} = 1 - \left\{1 - \left[1 - \left(1 - exp\ (-\beta)^{\lambda}\right)^{p}\right]^{s}\right\}^{p} \qquad (22)$$

Where s = the number of blocs in series.
p = the number of blocs in parallel.

Considering that the wire rope (6 * 7) is a parallel-series-parallel system. Figures 7 and 8 respectively represent the reliability depending on the fraction of life and reliability based on the damage of the previous wire rope according to the parallel-series-parallel model.

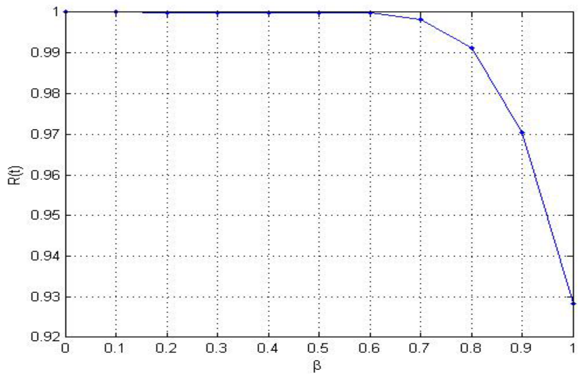

Figure-7. Reliability in function of the fraction of life of the wire rope of (6*7) type according to the parallel-series-parallel model.

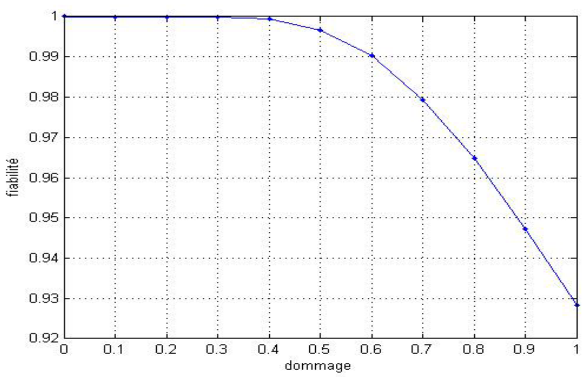

Figure-8. Reliability in function of damage of the wire rope (6*7) type according to the parallel-series-parallel model.

After studying these curves, we remark that the reliability of the system is influenced by the parallel branches. The opposite is observed: the system is more reliable if we increase the number of parallel blocs, and less reliable if we increase the number of series blocs.

8. A PROPOSAL OF A NEW MODELING OF A WIRE ROPE

A wire rope can be considered as a system constituted of a group of strands disposed of a majority logic system. Each strand is itself constituted of a group of wires disposed in parallel. The method adopted is a multi-scale approach where we distinguish the scale of wire, the scale of strand and that of the wire rope.

The schema proposed of a suspended wire rope is then a system: majority logic / parallel. The choice of this system is justified by:

- AA broken strand does not lead to the failure of the wire rope. However, starting from a certain number of broken strands, the wire rope can be declared as being failed.
- The wires are twisted together, a broken wire has the capacity to re-anchor on a given length, called re-anchor length, and which defines the stub's dimension.

$$R(t) = 1 - \left(1 - \sum_{k=m}^{n} C_n^k R_i^{\,k} . (1 - R_i)^{n-k}\right)^p . \qquad (23)$$

With p: number of wire*number of stubs; n: total number of strands and m: minimal threshold of the number of functional strands.

9. A COMPARATIVE STUDY OF THE PROPOSED MODELS

In order to show the failure criteria based on the number of unacceptable broken wires. We proceeded to a comparison of our model to those proposed by Al Achahchi and Kolowrocki. The first considers that the wire rope is like a parallel-series-parallel system. The second considers it as a series-parallel system, on a steel wire rope of (6*7) type (6 strands 7 wires). The Figures 9, 10 and 11 illustrate this comparison taking the number of unacceptable broken wires as criteria of failure.

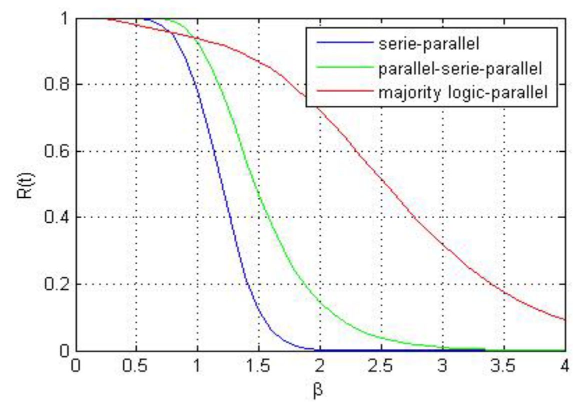

Figure-9. Case where the criteria of failure is six broken wires.

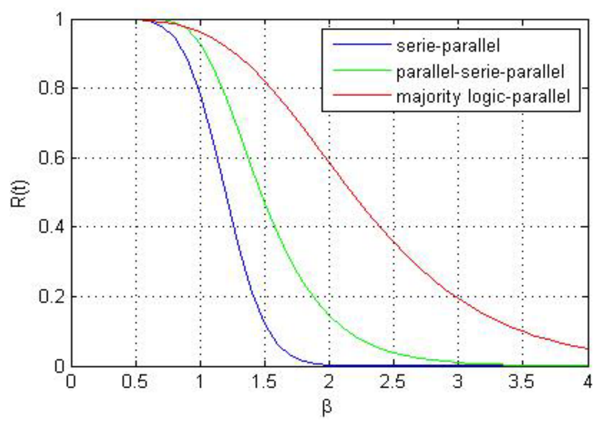

Figure-10. Case where the criteria of failure is three broken wires.

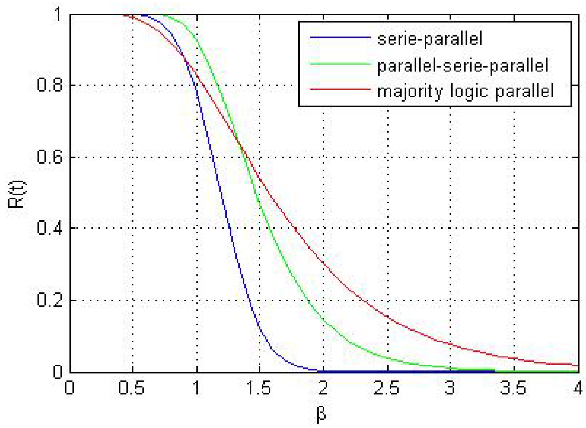

Figure-11. Case where the criteria of failure is one broken wire.

These curves show that the two first models (those proposed by AL Achachi and Kolowrocki) do not change and consequently do not take into consideration the failure criteria. Yet the proposed model (majority logic/parallel) appears as being well adapted to the real situation of wire rope's use and takes into consideration the degradation of the wire rope in function.

CONCLUSIONS

This work is elaborated to make a link between reliability and damage through fatigue. This link allows associating to each stage of damage the corresponding reliability. For each particular type of wire rope application, the occurrence of unacceptable number of broken wires is the action adopted for the damage of fatigue evaluation. Consequently, our study's purpose was developing a modeling which allows predicting the resistance capacity of a wire rope in different levels of damage of its components. Our contribution is essentially reliable by the use of a new model which determines the reliability of the wire rope taking into account the number of tolerated broken wire in the wire rope. In this respect, our objective is to be able to describe the mechanical behavior of each wire so as to deduce that of the wire rope.

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