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APPLICATION OF IMPROVED MULTISTAGE VEHICLE ROUTING PROBLEM WITH TIME WINDOW

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ABSTRACT

This paper presented an application of improved multistage vehicle routing problem with time window. By using this improved method, we can solve a multistage vehicle routing problem issue. We applied this model for two layers multistage. First layer consist of only one depot which distribute items among the distributors, second layer consist of several distributors which distribute items among several retailers. Adaptation to Larsen model was for second layer, which consist of several distributors which distribute to several retailers. Meanwhile Larsen model only worked on one depot which distribute among several distributors. In this model, we worked on two steps. First step is to solve second layer problems. We must determine the delivery area of retailers among the distributors by combine all possible path to minimize distance within capacity vehicle constraint and time window constraint. Next step was to solve first layer problem. We worked with Larsen model for solving the first layer. Using this improved multistage vehicle routing problem with time window helped to solve multistage vehicle routing problem as well as minimize distance.

Keywords: multistage, mathematical modelling, time window, vehicle routing problems.

INTRODUCTION

Logistic activities are much needed these days. Many manufacturing activities involved distribution of finished goods to distributors, distributors to retailers and retailers to customers. Third party logistics as logistic partner run this distribution business to overcome the complexity of this distribution issues. Many kinds of daily activities involved on vehicle routing problems application such as: news paper distribution, water gallon distribution, and mail delivery.

Many distribution and transport logistic problems could be express as a vehicle routing problem which objective function is minimize total distance of the network or minimize cost of transportation under known demand. Each node will be visited exactly once under the vehicle capacity constraint [1].

Vehicle Routing Problem (VRP) can be portray as the problem of scheming optimal route from one or several node to a amount of node to minimize total distance under several constraints [2]. Vehicle Routing Problem with Time Window (VRPTW) is a form of VRP problems with time window restriction. Some of the practical examples of VRPTW are postal deliveries, school bus way, and vendor distribution in just in time manufacturing [3, 4].

As a result of value and complexities in a real life, VRPTW attract much thought from researchers. Much VRPTW works has been done both for optimization and heuristic methods. Most of heuristic methods developed to defeat the VRP as NP-hard problems. Such of works can be founded in [1, 3, 5]. Meanwhile, very few works on VRP mathematical modeling, Larsen model was one of it.
Multistage VRPTW is a network consists of two or more layers of distribution network. First layer consist of one depot which distribute items to several distributors. Second layer usually consist of several distributors which distribute items to several retailers. Our main contribution is that we develop model for multistage vehicle routing problem with time window.

PROPOSED MULTISTAGE VRPTW MATHEMATICAL MODEL

Larsen, 1999 developed a single depot VRPTW model. Here, we focus on developing Larsen, 1999 model for multistage VRPTW. Figure-1 is network for multistage vehicle routing problem. O is a Depot. A and B are distributors, C, D, E, F are retailers.

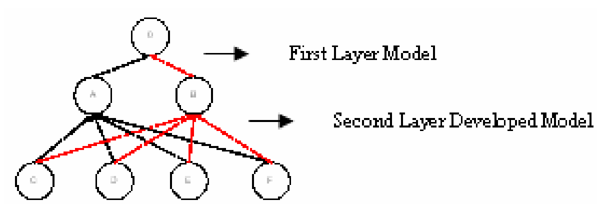

Figure-1. Multistage vehicle routing problem Network.

The Improved Multistage vehicle routing problem with time window is solved by two steps, as follows:

Step1. Solving the solution for second layer. The second layer consists of distributors and retailers. Each distributor must distribute item to retailers which minimize total distance. There were several possible solutions among the distributors to distribute item to retailers. All possible solutions is modelled in Model VRPTW for second layer.

Step 2. Solving the solution for first layer using Larsen Model. First layer consist of one depot and several distributors. Depot must distribute items to several distributors. Solution for this step is using Larsen model.

Mathematical Model VRPTW for second layer as follows:

Objective Function

$$\min Z = \sum_{k\in V_h}\sum_{i\in C}\sum_{j\in C} Y_{ij} X_{ijk} + \sum_{k\in V_h}\sum_{i\in D}\sum_{j\in C} Y_{hi} X_{hik} + \sum_{k\in V_h}\sum_{i\in C} Y_{iN+h} X_{iN+hk}$$

$\forall h \in Y \quad \forall h \in Y \quad i \neq j$ (1)

Subject to constrains:

$$\sum_{k\in V_h}\sum_{j\in C} X_{ijk} = 1 \quad (2)$$

$\forall i \in C \ \forall h \in D \ i \neq j$

$$\sum_{i\in C} X_{hik} = 1 \quad (3)$$

$\forall h \in D \ \forall k \in V_h$

$$\sum_{i\in C} X_{iN+hk} = 1 \quad (4)$$

$\forall h \in D \ \forall k \in V_h$

$$\sum_{h\in D} X_{hik} - \sum_{j\in C} X_{ijk} = 0 \quad (5)$$

$\forall i \in C \ \forall k \in V_h \quad i \neq j$

$$\sum_{i\in C} W_i \sum_{j\in C} X_{ijk} \le Q_k \quad (6)$$

$\forall k \in V_h \quad \forall h \in D \quad i \neq j$

$$\sum_{k\in V_h}\sum_{i\in C} W_i X_{hik} + \sum_{k\in V_h}\sum_{j\in C}\sum_{i\in C} W_j X_{ijk} \le Q_h \quad (7)$$

$\forall h \in D$

$$S_{ik} + t_{ij} - K\left(1 - X_{ijk}\right) \le S_{jk} \quad (8)$$

$\forall i,j \in N \quad \forall k \in v$

$$a_i \le S_{ik} \le b_i \quad (9)$$

$\forall i \in N \quad \forall k \in v$

$$X_{hik},\ X_{ijk} \in \{0,1\} \quad (10)$$

The equation (1) is the objective function, to minimize the total distance between node i to node j and the others node by vehicle k and ensure to minimize distance from distributors to several retailers. Constraint (2) represents that each retailers is visited just only once, Constraint (3) means that each vehicle k leaves node, Constraint (4) represents that each vehicle k arrive on node, Constraint (5) ensure that vehicle k depart and leave to the node N+1, Constraint (6, 7) is the capasity restriction for every vehicle k, Constraint (8) means that vehicle k has to depart node i during t_{ij} to node j if take a trip from node i to node j, K is a large number. Constraint (9) states a time window restriction, Constraint (10) is a binary integer value for the decision variable.

A group of vehicles are represented by V, V=1, 2,…, v and a set of retailers are represented by C, C=a,b,….z. Depot represented by node h (driving-out depot) and N+h (returning depot). The set of arcs (A) correspond to connections among depot and retailers and vice versa, the set of vertice correspond to 0, 1,…n+1 is denoted N. Each arc (i,j), where i ≠ j. We related a distance Y_{ij} and each customer demand (W_i), each vehicle has a capacity Q_k. Each depot and retailers has a time window $[a_i, b_i]$, which is assumed identical. All vehicle can not depart before a_0 and must arrive before b_i. Model had two decision variables x and s. For each arc (i, j), which i ≠ j, i ≠ n+ 1; j ≠ 0, and each vehicle (k) we identify x_{ijk} as:

$$X_{ijk} = \begin{cases} 0, \text{if vehicle k does not drive from node i to node j} \\ 1, \text{if vehicle k drives from node i to node j} \end{cases}$$

The decision variable s_{ik} is described as start time to service customer i with vehicle k. It is assumed $a_0 = 0$ and hence $s_{ok} = 0$ for all k. The purpose here is that to minimize total distance to each route, with subject to constraint as follows:

a) Each node is used exactly once.
b) Time windows are restricted to constraint.
c) Vehicles capasity are not exceeded.

Mathematical Model VRPTW for first layer as follows (Larsen model):

Objective Function:

$$\text{Min} = \sum_{k\in v}\sum_{i\in N}\sum_{j\in N} Y_{ij} X_{ijk} \quad (11)$$

Subject to constraints:

$$\sum_{j\in N} X_{yjk} = 1 \quad (12)$$

$\forall i \in C$

$$\sum_{i\in N} X_{izk} = 1 \quad (13)$$

$\forall k \in v$

$$\sum_{k\in v}\sum_{j\in N} X_{ijk} = 1 \quad (14)$$

$\forall i \in C \quad i \neq j$

$$\sum_{i\in N} X_{ihk} - \sum_{j\in N} X_{hjk} = 0 \quad (15)$$

$\forall h \in C \quad \forall k \in v$

$$\sum_{i\in C} W_i \sum_{j\in N} X_{ijk} \le Q_k \quad (16)$$

$\forall k \in v$

$$S_{ik} + t_{ij} - K\left(1 - X_{ijk}\right) \le S_{jk} \quad (17)$$

$\forall i,j \in N \quad \forall k \in v$

$$a_i \le S_{ik} \le b_i \quad (18)$$

$\forall i \in N \quad \forall k \in v$

$$X_{ijk} \in \{0,1\} \quad (19)$$

$\forall i,j \in N \quad \forall k \in v$

The equation (11) is a objective function, to minimize the total distance between node i to node j. Constraint (12) ensure that each vehicle k departs the depot, Constraint (13) ensure that each vehicle k will be

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back to depot , Constraint (14) represents that each distributors is visited only once, Constraint (15) ensure that vehicle k depart and leave to the node N+1, Constraint (16) is the capasity restriction for every vehicle k, Constraint (17) means that vehicle k has to depart node i during t_{ij} to node j if take a trip from node i to node j, K is a large number. Constraint (18) ensure a time window restriction, Constraint (19) is a binary integer value.

APPLICATION OF THE MULTISTAGE VRPTW MATHEMATICAL MODEL

PT. Tirta Bahagia is a aqua gallon distributor company which distribute aqua gallon in Surabaya district area. Distribution system consists of depot/manufacturing, distributors, and retailers. Depot supervised 3 distributors, and distributors supervised 14 retailers. In this case, distribution each area for distributors have not decided yet. This models works to solve distribution area for each distributor. Second layers model works first as solving the area for distributor to minimize total distance. Furthermore, first layers model will be solved using Larsen model. The time window is from 08.00 WIB and close at 14.00 WIB. It is assumed the velocity of the vehicle 30 km/hours.There is 1 vehicles for servicing from depot to distributors with 1000 box capasity and there are 3 vehicles for servicing from ditributors to retailers with capasity 200 box each.

Table-1. Distance between Node (km).

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**
A		18	28	12	4.7	10.5	5.13	8.32	21.2	22.5	34.56	44.7	37.4	25.7	27.27	15.7	27.2
B	18	0	16	19	13	18.4	16.2	12.01	10.8	11.5	16.51	18.9	16.4	8.14	9.67	5.4	5.22
C	26	16	0	18	26	17.0	23.4	26.50	9.72	8.82	7.74	14.9	7.78	18.7	24.07	12.8	24.0
D	10	19	18	0	15	3.87	13.9	19.17	10.3	14.9	19.44	21.8	22.5	29.2	27.81	18.5	21.8
E	4.8	13	26	14	0	12.37	2.25	3.375	14.7	18.9	26.64	29.2	20.6	27.7	21.87	11.7	15.9
F	10	18	17	3.6	12	0	10.1	15.3	6.43	11.1	22.77	17.9	18.6	25.3	23.94	14.7	17.9
G	5.2	16	23	13	2.5	9.82	0	5.625	17.0	21.1	28.89	31.5	22.9	30.0	24.12	13.9	18.2
H	8.4	11	26	18	3.6	15	5.63	0	12.51	16.6	24.39	27.0	18.4	25.5	19.62	12.4	15.7
I	21	10	9.9	10	15	6.13	17.0	12.5	0	5.22	11.7	19.1	14.3	17.2	17.86	7.15	12.6
J	22	11	9.0	14	19	10.8	21.1	16.64	10.1	0	9.9	15.9	15.3	13.1	18.135	6.25	12.3
K	34	16	7.9	19	26	22.4	28.9	24.38	16.1	9.69	0	10.4	3.46	13.3	18.72	15.7	17.1
L	44	18	15	21	29	17.6	31.5	27.03	15.5	15.7	10.13	0	7.11	12.0	16.42	21.9	21.8
M	37	16	7.9	22	20	18.3	22.9	18.39	13.3	15.0	3.155	7.21	0	4.90	9.31	14.8	14.7
N	25	8.0	18	29	28	25.0	30.0	25.50	18.3	12.9	13.01	12.1	5.00	0	6.12	11.5	10.21
O	27	9.5	24	27	22	23.6	24.13	19.61	6.46	17.9	18.41	16.2	9.41	6.32	0	13.14	9.81
P	15	5.3	13	18	12	14.4	13.9	12.45	12.5	6.04	15.395	22.06	14.95	11.7	12.94	0	7.69
Q	27	5.2	24	21	16	17.6	18.2	15.695	0.21	12.16	16.88	21.9	14.8	10.41	9.61	7.68	0

Table-2. Demand for retailers.

Node	Demand (Box)
A	573
B	322
C	662
D	36
E	35
F	25
G	28
H	7
I	28
J	21
K	17
L	17
M	18
N	35
O	17
P	16
Q	15

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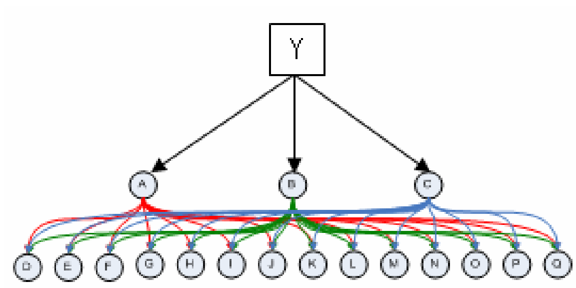

Figure-2. Network Distribution of PT. Tirta Bahagia.

Y as depot, A, B, C as distributors, D - Q as retailers

Step 1. Solving for second layer.

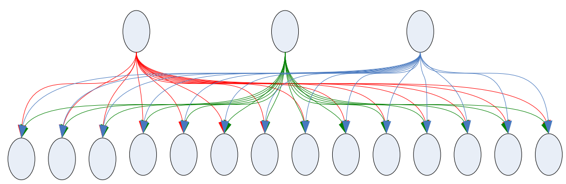

Figure-3. Network Distribution of second layer.

Mathematical Model VRPTW for second layer result:

a. Route 1(vehicle 1):
A - G - E - H - P - J - I - F - D - A
Total Distance = 54.8 km
Total Load = 196 box
Total time = 173.89 minutes
b. Route 2(vehicle 2): B - Q - O - N - B
Total Distance = 29.1km
Total Load = 67 box
Total time = 88.99 minutes
c. Route 3(vehicle 3): C - L - M - K - C
Total Distance = 33.0 km
Total Load= 52 box
Total time = 82.52 minutes

Step 2. Solving for first layer

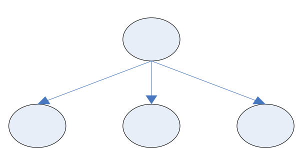

Figure-4. Network Distribution of first layer.

Y as depot, A, B, C as distributors

Mathematical Model VRPTW for first layer result:

a. Route 1 : Y - A - Y
Total Distance = 152.6 km
Total Load = 573 box
Total time = 305.3 minutes
b. Route 2: Y - C - B - Y
Total Distance = 184.1 km
Total Load = 984 box
Total time = 346.2 minutes

Below is the result of The Multistage VRPTW Mathematical Model:

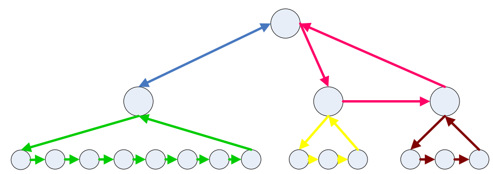

Figure-5. Distribution as result of improved multistage VRPTW.

CONCLUSIONS

The result of improved VRPTW had been verified as follows:

a) Every node visited only once with only one vehichle at time.
b) All demand had already fulfilled and capasity of the vehicle for each path not exceed.
c) Time Windows restriction was filled.

The result shown that all constraint had fullfilled to minimize total distance for multistage vehicle routing problems. Validation of the result showed that solving of the second layer was consistent with distributors and retailers nearnest location. Therefore, the model could work well.

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