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CHOICE OF PRODUCTION MEASURING INSTRUMENTS BASED ON TECHNO-ECONOMIC ANALYSIS, TAKING INTO ACCOUNT THE TYPE I ERROR AND TYPE II ERROR

Michael A. Bolotov, Elena A. Kapenkina, Nikolay V. Rusanov and Vadim A. Pechenin
Samara State Aerospace University, Aircraft Engine Faculty, Samara, Russia

ABSTRACT

Measuring instruments are integral parts of production process. Their accuracy and cost parameters influence the quality and prime cost of the products made. One of the most important tasks is a reasonable choice of the measuring instruments' usage for re-equipment of the existing or equipment of newly engineered production facilities. Control of details with compound shape is performed with the usage of specialized equipment, analog instruments or modern measuring mens that are supported by computer means. As a rule, the cost and the accuracy of modern measuring instruments that are supported by computer means are higher than that of the general instruments. That's why the question emerges, whether big investments to the measuing instruments would recover. In this work the model of a resonable choice of production measuring instruments is suggested, which considers their accuracy and cost parameters. Measuring instruments' accuracy is considered by means of modeling error I and error II occurrences. As a selection criterion we use general expenditures for control, which account for control performance and expenditures, connected with rejection of accepted parts or mistaken acceptance of unacceptable details as accepted ones. Production process' modeling is performed with the usage of the following models: "White noise", "Linear trend", "Fan process" and "Wiener process". In the course of modeling it was concluded that the usage of modern measuring instruments, characterized by high accuracy and high cost, is preferable.

Keywords: production measurment, production error, accuracy of measuring instruments.

1. INTRODUCTION

Production process can not be an ideal process without flaws. Liao (2007) studied production processes that don't exclude the presence of defected items. He divided them into two groups: controllable processes and uncontrollable ones. In this work we will analyze both types of processes and their combination in the form of accounting sporadic and systematic contributing errors, the presence of which influence control errors of the I and the II type.

The accuracy of measuring instruments can be defined while using the procedures of uncertainties' estimation. Evdokimov (2014) was defining technological process' accuracy, taking into account influencing technological factors and specifications of products, which were selected by Zhdanov (2013) and Ulanov (2009) as structural limitations.

The quality and prime cost of the production manufactured depend among others on accuracy parameters of measuring instruments. Effective selection of assents allows substantially decrease prime cost of the products.

Savio (2012) compared two variants of control procedure's implementation, based on initial investments, payback periods and number of parameters measured. Khan (2011), while describing EOQ model that considers presence of defective goods in the batch, for defining measuring process' cost used the amount of expenses for control performance, as well as the expenditures related to control errors, the probability of which may be calculated in case the batch size is known (Porteus, 1986; Urban, 1998). However, this work will calculate such probabilities via realization of simulation model of quality control process, with consideration of technological process' accuracy.

2. MATERIALS AND METHODS

2.1. Description of the simulation model, which takes into account the accuracy of the production process

In this work we will use statistical modeling method named "Monte Carlo", and input data for model's creation will be random values, specified by corresponding distributional laws, since collection of mass technological information is hindered and decrease of information volume is undesirable since it will lead to less reliable results.

"Monte Carlo" method is a common name for the group of numerical methods, based on acquisition of great number of realizations of equilibrium process, which is formed in such way that its probabilistic characteristics coincide with equivalent values of solvable problem. This method allows solving problems, the conditions of which contain uncertainty element.

Principle of the method lies in the fact that with the help of random number generator ECM simulates situations or processes that are possible under conditions of the problem and which lead to this or that outcomes. Furthermore, sought quantity takes on some values (for instance, "0" or "1"). All events or nearly so will manifest themselves in case occasional development of one and the same state is scrutinized multiple times. Let's study a mathematical apparatus, which "Monte Carlo" method is based on, which consists of two limiting theorems of probability theory: Bernoulli and Chebyshev's apparatuses, which depict the following key point: with the help of an adequate probability model accurate solution for the problem is interpreted either as P-probability of some event, or an expected mean M of some random value, which are formed in keeping with the number n, which stands for a large number of independent experiments performed.

Formularization of Bernoulli theorem:

$$\lim_{n\to\infty} P\left(\left|\frac{m}{n} - P\right| < \varepsilon\right) = 1, (1)$$

where n is the number of experiments, P is probability of event occurrence, $\varepsilon > 0, \frac{m}{n}$ is a relative frequency of the event.

Formularization of Chebyshev's theorem:

$$\lim_{n\to\infty} P\left(\left|\frac{1}{n}\sum_{i=1}^{n} x_i - M\right| < \varepsilon\right) = 1, (2)$$

where M is a expected value.

Then, based on the law of large numbers, relative event frequency (according to formula 1) or mean value (according to formula 2) are taken for approximate solution of this problem. The only weakness of the Monte Carlo method lies in its slow convergence, at this; statistical error is defined by the n value. Often this method requires a large number of experiments, and, thus, powerful computing resources for obtaining accurate estimations, although against explosive development of ECM technologies, on a number of cases it wouldn't be a grave disadvantage.

The process of simulative modeling requires reproduction of random values with their distributive laws.

The possibility of conduction of large number of process' realizations (which is formed in such way that its probabilistic characteristics coincide with equivalent values of the problem being solved) make "Monte Carlo" an effective method for solving the set task of accuracy definition. The model's essence lies in the fact that at some initial conditions (batch size, tolerance zone for a certain geometric parameter, accuracy of technological process and accuracy of measuring equipment) a simulation of errors' occurrence is performed in case there are processes that are different from the point of view of their accuracy.

It was taken that production errors can be described with the help of normal law or law with equal probability, and measuring errors are subject to beta-distribution and normal distribution, respectively.

Modeling of production process' accuracy with further control of geometric parameters consists of the following stages:

a) Input parameters are set, like: batch size, allowance for a certain parameter, production process' accuracy, measuring instrument's accuracy;
b) In case technological process is modeled with specified random and/or systematic errors, also the values characterizing the speed/limit of error's component are set for random and systematic components, respectively.
c) The array of production errors is created. The following variants of formation may be distinguished:

(i) Without consideration of occasional and systematic components of the error (the model of production process called "White noise"). In this case a range of production errors is received via generation of array of random values, which can be described via normal law or law with equal probability (depending on requirements of the process that is simulated).

The following specified input parameters are required for array's generalization, which is being a statistical expectation of distribution of random values' generated sequences:

- batch size that defines the array's length;
- technological process' accuracy parameter, which stands as a statistical expectation of distribution of random values' generated sequences.

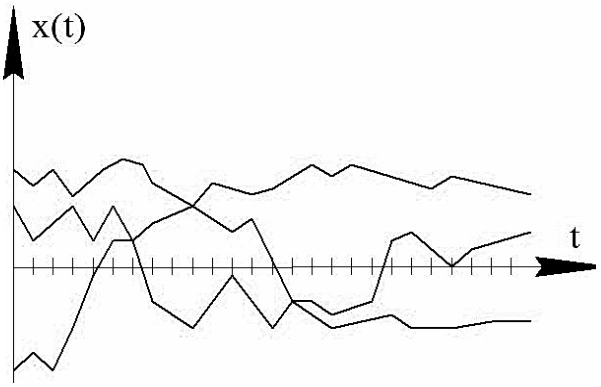

Figure-1. Increase of production error over time within the framework of "White noise" model.

Figure-1 displays correlation of production error's value with the time within the framework of "White noise" model.

(ii) Considering error's systematic compound (production process' model called "Linear trend"), each element in the array, generation of which is described in point a), is added with the parameter characterizing the growth rate of systematic compound of production error.

With each further element the value increases and at the moment when it reaches threshold value (the limit of growth of error's systematic compound), the process of increasing stops and starting from the next element it commences again, which is a simulation, for example, of change of blunted lathe tool.

Figure-2 displays correlation of production error's value with the time within the framework of "Linear trend" model.

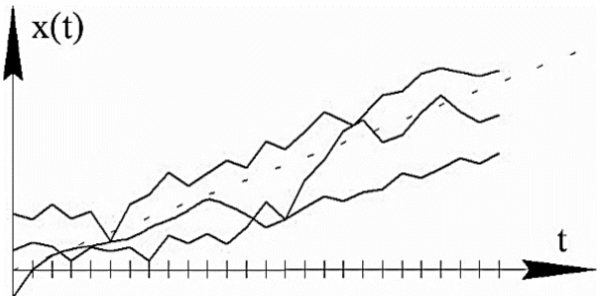

Figure-2. The growth of production error with the time within the framework of "Linear trend" model.

(iii) Considering error's systematic compound (production process' models called "Fan process" and "Wiener process"), generation of array of production errors is performed in the way similar to the one, described in the a) point, however, statistical expectation of distribution of random values' generated sequences is specified not only by input parameter of technological process' accuracy, but also by a parameter of bias of error's random compound, which also has its limit of growth. This is typical for "Fan process" model. Figure-3 shows how the value of production error depends on the time, typical for this model.

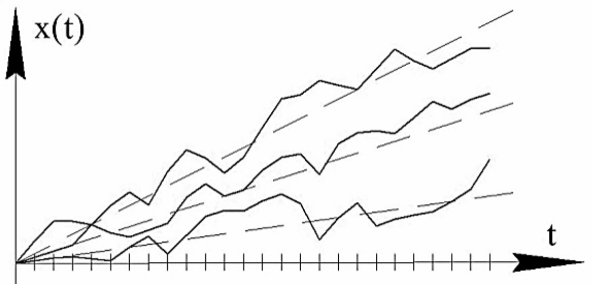

Figure-3. The growth of production error with the time within the framework of "Fan process" model.

In "Fan process" model, considering characteristics of this process, accounting of random error's random compound is considered by multiplying technological process' accuracy parameter by root from the order number of this element minus 1. Growth value is also limited by input parameter.

Figure-4 demonstrates how the value of production error depends on the time, typical for this model.

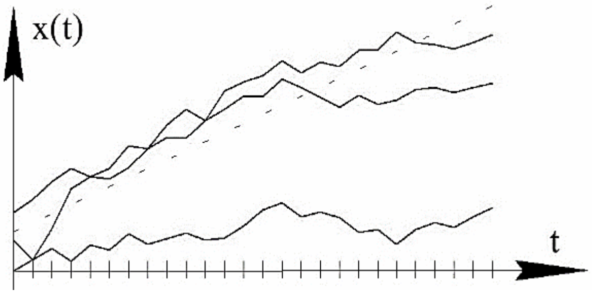

Figure-4. The growth of production error with the time within the framework of "Wiener process" model.

It should be noted that the most realistic model of production process is "Wiener process" one, since it tales into account random and systematic compounds of the error. "Wiener process" model, as it was mentioned above, is applicable to grinding method.

d) Formation of measuring instruments' error array is conducted similarly to formation of production error for "White noise" model. The difference lies in the initial data - statistical expectation for formation of values' random sequences, which follow the normal law, is an accuracy parameter of measuring instrument. In case of formation of values' random sequences array, which is subject to description of beta-law, initial data are represented by values of "A" and "B" coefficients.
e) Simulation of checking detail's suitability is performed via successive comparison of each element from the array of error's random compounds with specified input parameter - tolerance limit's bound: if a certain value of the error is less or equal to tolerance limit's bound, the detail deems to be acceptable, otherwise the detail is considered to be defected. Data about acceptance and rejection of details are recorded in logical variables in the following way: accepted detail is recorded as «false» (0), and rejected detail is recorded as «true» (1).
f) Simulation of making measurements and, thus, creation of overall error's array, which is equal to the sum of array values of production errors and measuring errors, corresponding to each other.
g) The task of identification of type I and II errors is modeled with consideration of occurrence of the following four events:

- Acceptable detail was deemed to be acceptable (no quality error);
- Acceptable detail was deemed to be defective (type I error);
- Defective detail was deemed to be defective (no quality error);
- Defective detail was deemed to be acceptable (type II error).

For this purpose details are divided into initially accepted and initially defective details. In the cycle overall error's value of each element is compared with tolerance zone's limit. The event related to acceptance of a detail has two possible outcomes.

The first outcome. Acceptable detail was deemed to be acceptable. So, the variable is in the array that is responsible for storing data regarding presence/absence of errors, takes on "0" value.

The second outcome. Acceptable detail was deemed to be defective. Thus, the variable is in the array that is responsible for storing data regarding presence/absence of errors, since it's type I error. In this case the error counter is added with 1.

The opposite event, connected with defective details, has two outcomes as well. The variable is in the array that is responsible for storing data regarding presence/absence of errors, takes on "0" value.

The first outcome. Defective detail was deemed to be defective. The variable is in the array that is responsible for storing data regarding presence/absence of errors, since it's type II error. In this case the error counter is added with 1.

The second outcome. Defective detail was deemed to be acceptable.

h) Probabilities of occurrence of type I and II errors are calculated as correlation of corresponding counter functions' values with overall number of details.
i) Probability of occurrence of defected detail is calculated. For this all array's elements are summed, in which defected details are defined as "1", and acceptable details are defined as "0", then we get the value that is equal to overall number of defected items in the batch. Probability of occurrence of defected detail is calculated as relation of overall number of defected details to the batch size.

2.2. Mathematical model

For estimation of type I and II errors mathematical model was developed. It is presented by expressions (3) and (4).

$$PI = 2\cdot\int_{0}^{E}\int_{E}^{\infty} fTP(\overline{P})\cdot fm\left(x_m - x, \sigma_m, \mu_m\right)dxdx_m, \qquad (3)$$

where $fTP(\overline{P})$ is a frequency function of a probability, which corresponds to reviewed type of technological process with parameter vector,
fm is a frequency function of a probability, which corresponds to measuring instrument,
x is a current value of reviewed geometric parameter in technological process,
xm is a current value of measured amount,
σm is a mean-square deviation, characterizing random error component of measuring instrument,
μm is a statistical expectation, characterizing random error component of measuring instrument.

$$PII = 2\cdot\int_{E}^{\infty}\int_{0}^{E} fTP(\overline{P})\cdot fm\left(x_m - x, \sigma_m, \mu_m\right)dxdx_m, \qquad (4)$$

Frequency distribution functions for the "White noise" type of technological process were chosen in accordance with Gaussian law and uniform law. For other technological processes compositional functions of frequency distribution were defined in the following way. For "Linear trend" type of technological process it was represented by formula (5), Figure-5 demonstrates the type of distribution.

$$fs\left(x,\sigma,\mu,vs,ms\right) = \frac{\int_{0}^{\frac{ms}{vs}} fN\left(x,\sigma,\mu+vs\cdot t\right)dt}{\int_{-\infty}^{\infty}\int_{0}^{\frac{ms}{vs}} fN\left(x,\sigma,\mu+vs\cdot t\right)dtdx}, \qquad (5)$$

where vs is a growth rate of error's systematic component in technological process,
ms is growth limit of error's systematic component in technological process,
t is reviewed moment of time, in which technological process is being performed.

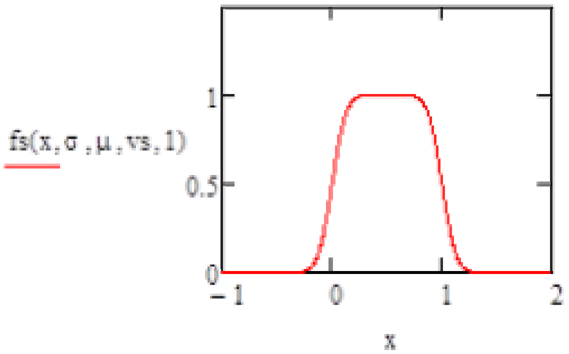

Figure-5. Distribution view for technological process "Linear trend" type.

Technological process of "Fan process" type is represented by formula (6), Figure-6 demonstrates the type of distribution.

$$T = \min\left(\frac{ms}{vs}, \frac{mr}{vr}\right), \qquad (6)$$

$$fr\left(x,\sigma,\mu,vs,ms,vr,mr\right) = \frac{\int_{0}^{T} fN\left(x,\sigma+vr\cdot t,\mu+vs\cdot t\right)dt}{\int_{-\infty}^{\infty}\int_{0}^{T} fN\left(x,\sigma+vr\cdot t,\mu+vs\cdot t\right)dtdx},$$

where vr a growth rate of error's random component in technological process,
mr is growth limit of error's random component in technological process.

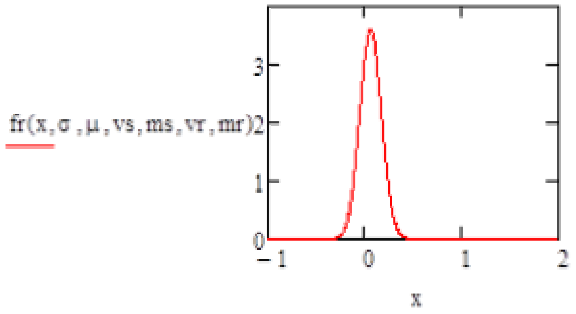

Figure-6. Distribution view for technological process "Fan process" type.

Technological process of "Wiener process" type is represented by formula (7); Figure-7 demonstrates the type of distribution.

$$fvin(x,\sigma,\mu,vs,ms,vr,mr)=\frac{\int_{0}^{T} fN\left(x,\sigma+vr\cdot\sqrt{t},\mu+vs\cdot\sqrt{t}\right)dt}{\int_{-\infty}^{\infty}\int_{0}^{T} fN\left(x,\sigma+vr\cdot\sqrt{t},\mu+vs\cdot\sqrt{t}\right)dtdx}. \qquad (7)$$

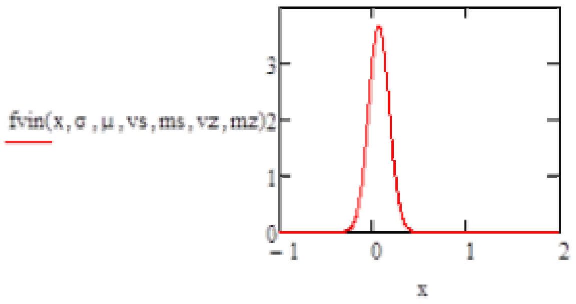

Figure-7. Distribution view for technological process "Wiener process" type

2.3. Description of the model of choice of production measuring instruments

Economical comparison of two different measuring methods is based on correlation between measurement performance and damages, caused by faulty control, including damages, connected with errors of I and II type. Comparison is performed as a calculation of expenses for control, considering damages, related to faulty control, for both measuring methods;

Expenses for control of one unit, with consideration of control errors, per each of the variants, are presented in formulas 3 and 4.
IC1=EC1+EEI1+EEII1, (3)
IC2= EC 2+EEI2+EEIII2, (4)
Where EC1/ EC2 are expenses for control, EEI1/ EEI2 are expenses, connected with type I errors;
EEII1/EEIII2 are expenses, connected with type II errors.
Expenses for control of one unit of products in the sample are calculated according to formula of technological operation's prime cost:
Ccontr= (3contr+Od+Or+E)•tmeas (5),
Where 3contr stands for controller's salary; Od are expenses for deprecation of equipment; Or stands for the expenses for repair of the equipment, E stands for expenses on electricity; tmeas is the time of one measurement.
Expenses, connected with type I errors, are calculated according to the following formula:
EOIi= cr•(1 - p)•y•m1, (6)
where cr are damages, incurred because of faulty rejection of one unit; p is probability of defected detail's occurrence; y is a batch size; m1 is probability of occurrence of type I error.
Expenses, connected with type II errors, are calculated according to the following formula:

EOIIi= ca•p•y•m2, (7)

Where cr are damages, connected with absence of one defected detail, p is probability of defected detail's occurrence; y is a batch size; m2 is probability of occurrence of type II error.

Probability values of occurrence of type I and II errors, as well as probability of occurrence of defected item are received by realization of simulation model of serial products' quality control, which considers accuracy of technological process and measuring instruments that were described above. Expenses, connected with damages incurred because of incorrect rejection of one item were assumed equal to mean value of prime cost of one GTE compressor blade.

Expenses, associated with omission of one defected item were assumed equal to 2 mean values of prime cost of one GTE compressor blade.

3. RESULTS

3.1. Results, obtained in the course of simulation model's realization

Measuring of six important geometric parameters with known tolerance zones from the batch of 1000 items was taken as an example of simulation model's realization. It should be noted that data on rates and limits of growth of production error's random and systematic compounds is statistical generalization of measurements of compressor blades' geometrical parameters, which had been conducted on Coordinate-measuring machine before.

Figure-8 demonstrates the results of experimentation within the framework of simulation model that concern manifestation of error I type.

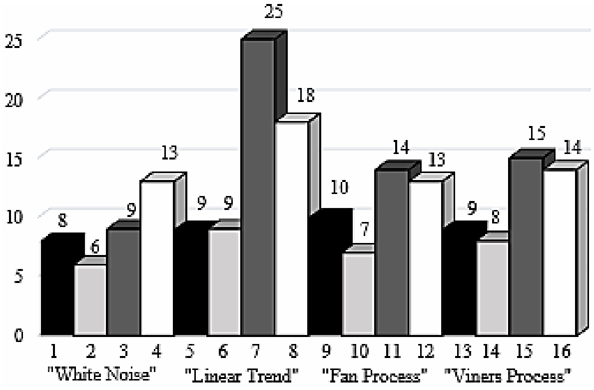

Figure-8. The number of erroneously rejected details that were in fact acceptable.

Figure-9 presents the number of erroneously rejected details that were in fact acceptable for various processes in case of various combinations of distribution laws.

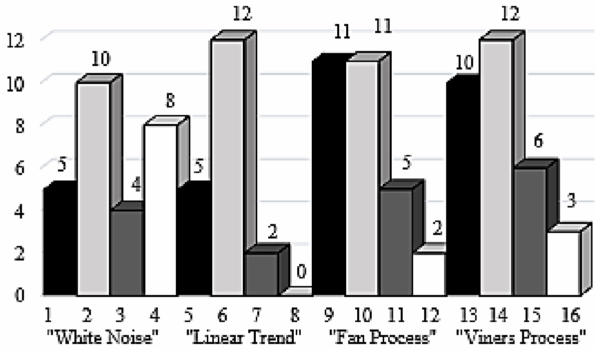

Figure-9. The number of defective details that were accepted.

Figure-10 demonstrates the number of truly defected details.

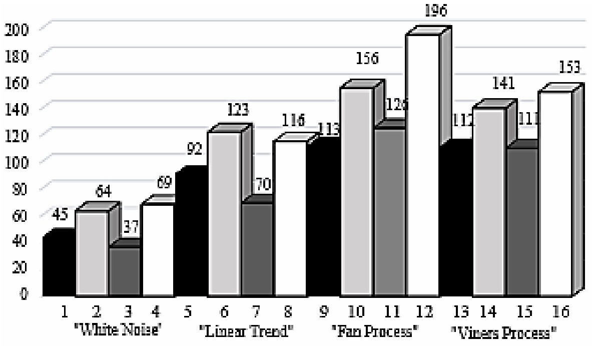

Figure-10. The number of truly defected details.

Order numbers 1, 5, 9, 13 display distribution of production errors and measuring errors under normal law. Order numbers 2, 6, 10, 14 display combination of distribution of measuring errors under normal law and equal distribution of production errors.

Order numbers 3, 7, 11, 15 display combination of beta-distribution of measuring errors and normal distribution of production errors.

Order numbers 4, 8, 12, 16 display combination of beta-distribution of measuring errors and equal distribution of production errors.

The results obtained may lead us to the following conclusion:

- Probability of type I errors is high in case of beta-distribution of measuring errors. Maximum value can be seen in case of “Linear trend” process at normal distribution of technological errors. Minimal value can bee seen in “White noise” process at normal distribution of measuring errors and law of equal distribution of technological errors;
- Probability of type II errors is high in case of normal distribution of measuring errors. Maximum value can be seen in case “Linear trend” and “Wiener process” are used at equal distribution of technological errors. Minimal value can bee seen in “Linear trend” process at beta-distribution of measuring errors and law of equal distribution of technological errors;
- Probability of defective details increases with increase of number of errors that are considered in models, and its maximum value can be seen under the law of equal distribution of technological errors, which is caused by distribution density of random values’ probabilities at this law of distribution.

3.2. An example of choosing optimum alternative of control

Table-1 presents calculated values of overall expenses, connected with control of complex units’ geometry, for 2 variants of its realization. The first variant suggests overall cost of outdated measuring instruments that require less qualification and, thus, fewer expenses for control procedure. The second variant suggests modern instruments, including control-measuring machine. The instruments themselves cost much, which means that costs, associated with deprecation and qualification requirements are higher than that in the first variant. However, considering high accuracy of modern measuring instruments, the benefit can be obtained on account of decrease of expenses, associated with control errors.

Table-1. Comparison of control-performing variants.

The name of the parameter	**Numerical value**	
	The first variant	**The second variant**
Expenses for control of one unit of product, rub.	120,00	150,00
The cost of type I error per one unit of product, rub.	2 000,00	2 000,00
The cost of type II error per one unit of product, rub.	4 000,00	4 000,00
Probability of rejection	0,0623	0,0623
Probability of type I error’s occurrence	0,0134	0,0084
Probability of type II error’s occurrence	0,0301	0,0113
Batch size, pcs.	17 000	17 000
Overall expenses, rub.	554 851,76	315 828,44

4. DISCUSSIONS

The type of distributional law and model of production process have an impact on type I and II errors, which in their turn have much impact on expenses, associated with control of geometric parameters. Thus, obtained results show that reduction of probability of type I error’s occurrence by 37, 31%, and reduction of probability of type II error’s occurrence by 62,46% balance the increase in value of control procedure by 25%. Furthermore, overall expenses, associated with both control and its errors decrease as well - by 43, 08% (by 239 023, 32 rubles). This data was received in the course of realization of simulation model of the process that considers both systematic and random components of errors. It means that large-volume investments into modern measuring instruments are repaid through reduction of cost, connected with control errors.

5. CONCLUSIONS

This work described simulation model of product quality control that considers accuracy of technological process and measuring instruments. Described models allow defining probability of type I and II errors at various types of technological processes. Besides, existing types of technological processes were examined; the main error types were identified, as well as corresponding laws of random values’ distribution. Corresponding conclusions were made as for the results, obtained in the course of models’ realization.

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