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ON TRANSFORMATION OF HYSTERESIS IN DAMPER RINGS MADE OF “METAL RUBBER” PRESSURE-TESTED WIRE MATERIAL UNDER PRECESSIONAL LOADING CONDITIONS

Yury Konstantinovich Ponomarev
Samara State Aerospace University, Russia, Samara, Moskovskoe shosse, Russia

ABSTRACT

This work is aimed at demonstration of considerable transformation of hysteresis in structural damping systems with dry friction in case of vibrator movement pattern type change. A damper ring made of pressure-tested wire material “metal rubber” used for damping of pipelines, turbomachine rotors and engine components of air vehicles was chosen as a survey target for this work. There was elaborated a mathematical model of deformation of “metal rubber” material based on which a model of a damper ring being deformed by a vibrator (a rotor journal) along the ellipse trajectories with a possibility to change smoothly the correlation between the elliptical semi-axes from zero to one was developed. At that the trajectory form is being changed from a straight line to a circular curve and hysteresis in its projections onto the coordinate axes is being transformed from a pattern typical for any structural damping system with peaked vertices to an ellipse-like pattern peculiar to the viscous friction systems.

Keywords: hysteresis, structural damping, metallic rubber, energy dissipation, vibration protection.

1. INTRODUCTION

Lately the researchers pay increased attention to theoretical problems of hysteretic systems. And while there is quite a significant number of works dedicated to the hysteretic systems statics and dynamics yet remain many blind spots in this question.

(Voigt) [1] Spoke of a hypothesis that internal friction forces in materials have viscous nature, i.e. hysteresis within tension-deformation (σ - ε [sigma - epsilon]) coordinates is represented by an ellipse-like relationship. For a definite time this hypothesis became a regular practice in engineering analysis.

However some researchers put it into question. It was observed that Voight model is unable to represent relaxation which is a significant feature of real materials. New models appeared (Maxwell model, Poynting-Thompson model) [4]. The mentioned models were created based on viscoelastic schemes which resulted in dissipated energy dependence on frequency. Real materials do not naturally have such dependence.

Pian and Hallowell [2], as well as Goodman and Klumpp [3] for the first time presented original mathematical models of hysteresis for elastic band extension and double-layered beam bend.

Within the framework of these models it was demonstrated that diffusion of energy in multi-layered systems exceeds considerably inner friction in materials. There were found optimum modes of the layers compression ensuring maximum energy diffusion. The mentioned results gave a powerful impulse to research of elastic hysteretic systems and their use in vibration protection equipment [5, 6, 8, 11-18].

In the eighties of the past age one more feature of mechanical dissipative systems was revealed, i.e. dependence of hysteresis form on a movement trajectory type [6, 7]. It was proved that unidirectional deformation of elastic damping device results in hysteresis loop patterns with intermittent hardness change in the points of the movement velocity value sign change [9, 10]. When a variable point of an elastic damping device (vibrator) moves along a closed circular path hysteresis loops obtain an ellipse-like pattern [19].

The author of this article set himself a task to analyze a mechanism of smooth change of hysteresis patterns and parameters deriving from the same depending on variations of correlation between the journal movement elliptical trajectories semi-axes for the journal mounted in a support with a damper ring made of “metal rubber” material [8].

2. PROBLEM STATEMENT

Let’s consider a mechanism of hysteresis occurrence in a support with a damper made of MR material (Figure-1).

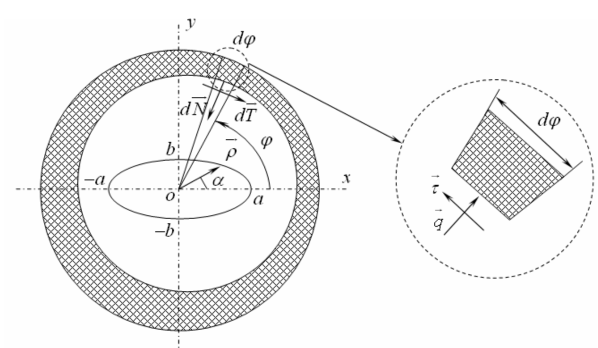

Figure-1. Design model of a damper ring.

The law of motion of the journal center will be expressed in a parametric form:

$$\begin{cases} \Delta x(t) = x_0 + a\cdot\cos(t), \\ \Delta y(t) = y_0 + b\cdot\sin(t). \end{cases} \quad (1)$$

In expression (1) x_0 and y_0 - an elliptical trajectory center (permanent displacement), a and b - values of the horizontal and vertical ellipse semi-axes, t - parameter assuming the values from 0 to 2π [2pi].

By projecting the displacement vector $\vec{\rho}$ [ro] onto the normal and tangent lines of a random infinitesimal element positioned at an angle of φ [fi] (Figure-1) we'll obtain the law of temporal variation of pressurization of the damper element $q(\varphi,t)$ [qu(fi, t)] and the journal tangential displacement against the element $\tau(\varphi,t)$ [tau(fi, t)].

$$\begin{cases} q(\varphi,t)=\Delta+\Delta x(t)\cdot\cos(\varphi)+\Delta y(t)\cdot\sin(\varphi), \\ \tau(\varphi,t)=-\Delta x(t)\cdot\sin(\varphi)+\Delta y(t)\cdot\cos(\varphi). \end{cases} \quad (2)$$

In expression (2) Δ [delta] - value of interference for the damper mounted in coaxial space between the support and the journal. The interference is determined according to the following expression:

$$\Delta=h_0-\left(R_{SUP}-R\right), \quad (3)$$

where h_0 - the damper thickness before mounting, R_{SUP} - the support radius, R - the journal radius.

By plugging in the law of temporal variation of the displacement vector (1) into expression (2) and by grouping the multipliers at cos (t) and sin (t) we'll obtain:

$$\begin{cases} q(\varphi,t)=\left[\Delta+x_0\cdot\cos(\varphi)+y_0\cdot\sin(\varphi)\right]+\left[a\cdot\cos(\varphi)\right]\cdot\cos(t)+\left[b\cdot\sin(\varphi)\right]\cdot\sin(t), \\ \tau(\varphi,t)=\left[-x_0\cdot\sin(\varphi)+y_0\cdot\cos(\varphi)\right]+\left[-a\cdot\sin(\varphi)\right]\cdot\cos(t)+\left[b\cdot\cos(\varphi)\right]\cdot\sin(t). \end{cases} \quad (4)$$

It is not difficult to determine on the basis of (4) the values of pressurization amplitude q_{ampl} and tangential journal displacement against the damper elements τ_{ampl} [tau ampl] as well as of the derivatives q and τ [tau] with respect to t parameter, that are necessary for computation of reactions:

$$\begin{cases} q_{ampl}(\varphi)=\sqrt{\left[a\cdot\cos(\varphi)\right]^2+\left[b\cdot\sin(\varphi)\right]^2}, \\ \tau_{ampl}(\varphi)=\sqrt{\left[-a\cdot\sin(\varphi)\right]^2+\left[b\cdot\cos(\varphi)\right]^2}. \end{cases} \quad (5)$$

$$\begin{cases} \dfrac{dq(\varphi,t)}{dt}=-\left[a\cdot\cos(\varphi)\right]\cdot\sin(t)+\left[b\cdot\sin(\varphi)\right]\cdot\cos(t), \\ \dfrac{d\tau(\varphi,t)}{dt}=-\left[-a\cdot\sin(\varphi)\right]\cdot\sin(t)+\left[b\cdot\cos(\varphi)\right]\cdot\cos(t). \end{cases} \quad (5)$$

$$\begin{cases} \nu q(\varphi,t)=\text{signum}\left(\dfrac{dq(\varphi,t)}{dt}\right), \\ \nu\tau(\varphi,t)=\text{signum}\left(\dfrac{d\tau(\varphi,t)}{dt}\right). \end{cases} \quad (6)$$

The positive value of dq/dt derivative corresponds to the loading path at the hysteresis loop occurring in the damper element due to inner friction in the MR material, and the negative value - to the falling unloading path (Figure-2). Similarly when the sign of $d\tau/dt$ [dtau/dt] derivative changes from positive to negative a transfer from loading to unloading occurs at the hysteresis loop occurring as a result of interfacial friction of the damper element against the body members (Figure-3).

In accordance with formula (6) the loading parameters νq [nu q] and $\nu\tau$ [nu tau] at time of loading are equal to 1 and during unloading are equal to (-1).

For discovering reactions relating to known displacements it is necessary to work out the law associating deformations and the MR material tension in the context of hysteresis. To this effect let's use the model of MR material offered in works [6, 8] and supplement it with a correction for small deformation amplitudes when the loading and unloading processes have no time to join the boundary process:

$$\sigma\left(\xi,\xi_0,\xi_{ampl},\nu\right)=\sigma_E(\xi)+\frac{1}{2}\cdot\nu\cdot\sigma_{NE}(\xi)- \\ -\nu\cdot\sigma_{NE}(\xi_0)\cdot\left[\exp\left(-5\frac{|\xi-\xi_0|}{a_0(\xi_0)}\right)-\frac{1}{2}\exp\left(-5\frac{2\xi_{ampl}}{a_0(\xi_0)}\right)\right], \quad (7)$$

where ξ - current deformation, ξ_0 - initial deformation, ξ_{ampl} - deformation amplitude, σ_E - the material elastic curve, σ_{NE} - hysteresis loop thickness (non-elastic tension unit), a_0 - residual deformation[6, 8, 13, 17].

As a part of calculation procedure it was assumed that the damper elements did nor experience shear deformations. The multiplier kf which makes allowance for predisplacement effect is used for approximate calculation of friction forces occurring at the boundary of the MR material and the journal.

$$kf\left(\tau,\tau_0,\tau_{ampl},\nu\right)=\nu\cdot\left[1-2\exp\left(-k_{PD}\cdot|\tau-\tau_0|\right)+\exp\left(-k_{PD}\cdot\tau_{ampl}\right)\right], \quad (8)$$

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where - k_{PD} = the coefficient setting an integral zone value for predisplacement.

Now when distribution of normal tension and friction coefficients is known it is possible to determine normal and tangential reactions of the damper elements by multiplying tensions and the elements areas.

$$\begin{cases} dN(\varphi,t) = \sigma\left[\dfrac{q(\varphi,t)}{h_0}, \dfrac{q_0(\varphi,t)}{h_0}, \dfrac{q_{ampl}(\varphi)}{h_0}, \nu q(\varphi,t)\right] \cdot B \cdot \left[R + q(\varphi,t)\right] \cdot d\varphi, \\ dT(\varphi,t) = f \cdot kf\left[\tau(\varphi,t), \tau_0(\varphi,t), q_{ampl}(\varphi), \nu\tau(\varphi,t)\right] \cdot dN(\varphi,t). \end{cases} \quad (9)$$

Computations according to formulas (4) - (9) result in getting hysteresis loops for the infinitesimal damper elements (Figures 2, 3).

By projecting the normal and tangential parts of the element reaction onto the coordinate axes,

$$\begin{cases} dR_x(\varphi,t) = dN(\varphi,t) \cdot \left[-\cos(\varphi)\right] + dT(\varphi,t) \cdot \left[\sin(\varphi)\right], \\ dR_y(\varphi,t) = dN(\varphi,t) \cdot \left[-\sin(\varphi)\right] + dT(\varphi,t) \cdot \left[-\cos(\varphi)\right], \end{cases} \quad (10)$$

and by integrating these projections along the whole damper length we'll be able to obtain a complete damper reaction.

$$\begin{cases} R_x(t) = \displaystyle\int_0^{2\pi} \frac{dR_x(\varphi,t)}{d\varphi} d\varphi, \\ R_y(t) = \displaystyle\int_0^{2\pi} \frac{dR_y(\varphi,t)}{d\varphi} d\varphi. \end{cases} \quad (11)$$

Angular position α [alpha] of the displacement vector $\vec{\rho}$ [ro], angular position β [beta] of the reaction vector $\vec{R}$ and displacement of phases between them γ [gamma] may be determined as follows:

$$\alpha(t) = \arccos\left[\frac{\Delta x(t)}{\sqrt{\Delta x(t)^2 + \Delta y(t)^2}}\right] \cdot \text{signum}\left[\Delta y(t)\right], \quad (12)$$

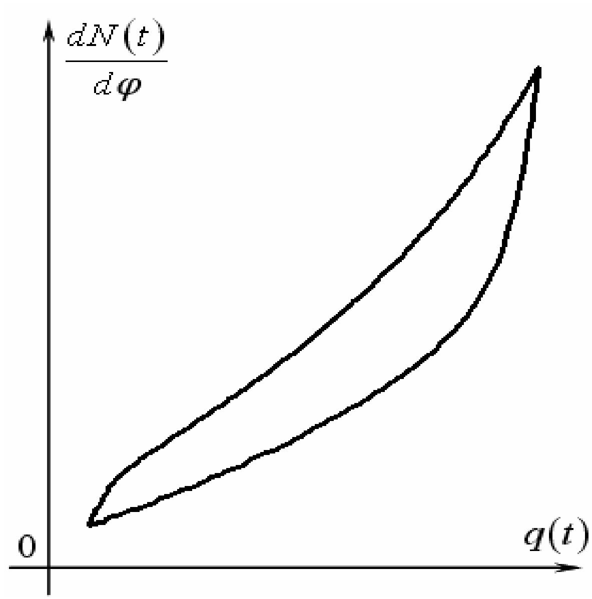

Figure-2. Typical hysteresis in a damper element due to inner friction in the MR material.

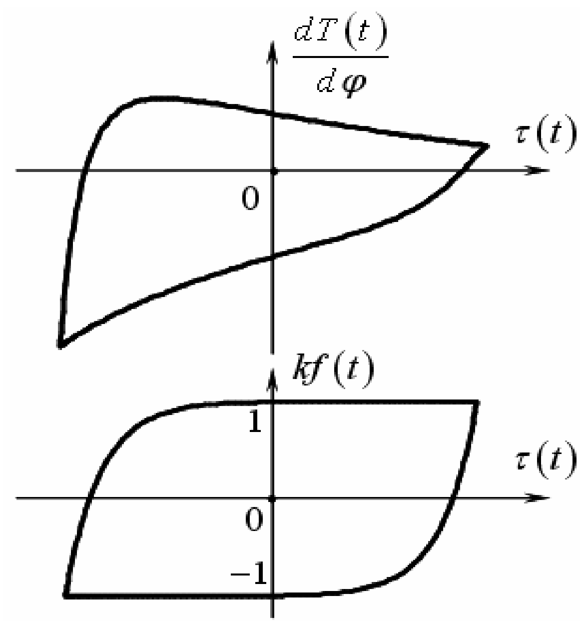

Figure-3. Typical hysteresis due to interfacial friction of a damper element against body members and the corresponding law of change of kf friction coefficient multiplier.

$$\beta(R_x, R_y) = \arccos\left[\frac{R_x}{\sqrt{R_x^2 + R_y^2}}\right] \cdot \text{signum}\left[P_y\right], \quad (13)$$

$$\gamma = \beta - \alpha. \quad (14)$$

Since the damper reaction $\vec{R}$ and displacement of the journal center $\vec{\rho}$ are vectors in regard of which there is no determined division operation, the damper hardness should be found out in a complex form. In this case a real part of hardness corresponds to the elastic damper reaction and an imaginary part to non-elastic related to the journal displacement vector module.

$$R_{elast.}(t) = R_x(t)\cdot\cos[\alpha(t)] + R_y(t)\cdot\sin[\alpha(t)], \qquad (15)$$
$$R_{non-elast.}(t) = -R_x(t)\cdot\sin[\alpha(t)] + R_y(t)\cdot\cos[\alpha(t)],$$

$$c^*(t) = \frac{R_{elast.}(t) + i\cdot R_{non-elast.}(t)}{\sqrt{\Delta x(t)^2 + \Delta y(t)^2}}. \qquad (16)$$

Dissipated energy is determined by means of integrating the products of reactions R_x and R_y by the corresponding increments $d\Delta x(t)/dt$ and $d\Delta y(t)/dt$ with respect to t parameter:

$$\begin{cases} W_x = \int_0^{2\pi} R_x(t)\dfrac{d\Delta x(t)}{dt}dt, \\ W_y = \int_0^{2\pi} R_y(t)\dfrac{d\Delta y(t)}{dt}dt. \end{cases} \qquad (17)$$

The moment of resistance to the journal movement is determined as a sum of the products of forces R_x and R_y by the corresponding displacement increments $d\Delta x(t)/dt$ and $d\Delta y(t)/dt$:

$$M_{resist.}(t) = R_x(t)\frac{d\Delta x(t)}{dt} + R_y(t)\frac{d\Delta y(t)}{dt}. \qquad (18)$$

Total dissipated energy for the loading cycle:

$$W = W_x + W_y = \int_0^{2\pi} M_{resist.}(t)dt. \qquad (20)$$

Potential energy of elastic deformation and absorption coefficients will be determined with use of conventional formulas:

$$\begin{cases} PE_x(t) = \dfrac{1}{2}\Delta x(t)\cdot R_x(t), \\ PE_y(t) = \dfrac{1}{2}\Delta y(t)\cdot R_y(t), \\ PE(t) = PE_x(t) + PE_y(t). \end{cases} \qquad (19)$$

$$\begin{cases} \Psi_x = W_x\cdot\max[PE_x(t)], \\ \Psi_y = W_y\cdot\max[PE_y(t)], \\ \Psi = W\cdot\max[PE(t)]. \end{cases} \qquad (20)$$

3. STUDY RESULTS

On the basis of the algorithm set forth herein and with the aid of the integrated mathematical software package Mathcad there was developed a program and a comprehensive study of elastic hysteresis characteristics and their derivatives (phase displacement, hardness, dissipated and potential energy, absorption coefficients) was carried out.

Figure-4 represents curves of normal and tangential pressures.

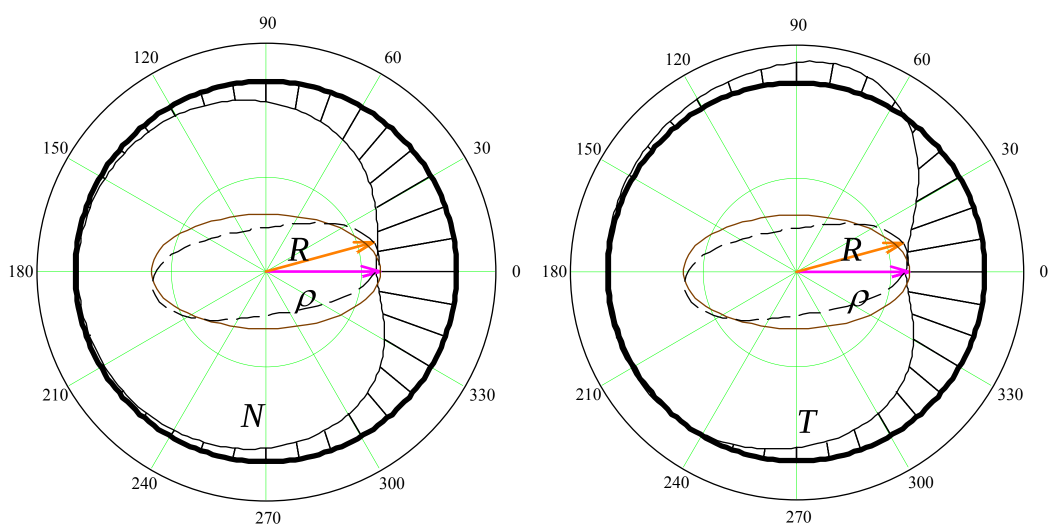

Figure-4. Curve of normal forces N ant tangential forces T in a damper.

At time of the force vector rotation around the journal center the movement vector drops behind the force vector. At that the whole curve of pressure onto the journal on the part of MR material is rotating synchronously with the force vector and undergoes qualitative change in the course of the displacement vector movement.

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Table-1 represents the hysteresis loop patterns for different correlations of the elliptical trajectory semi-axes and the corresponding hardness values c^* showed at the complex plane.

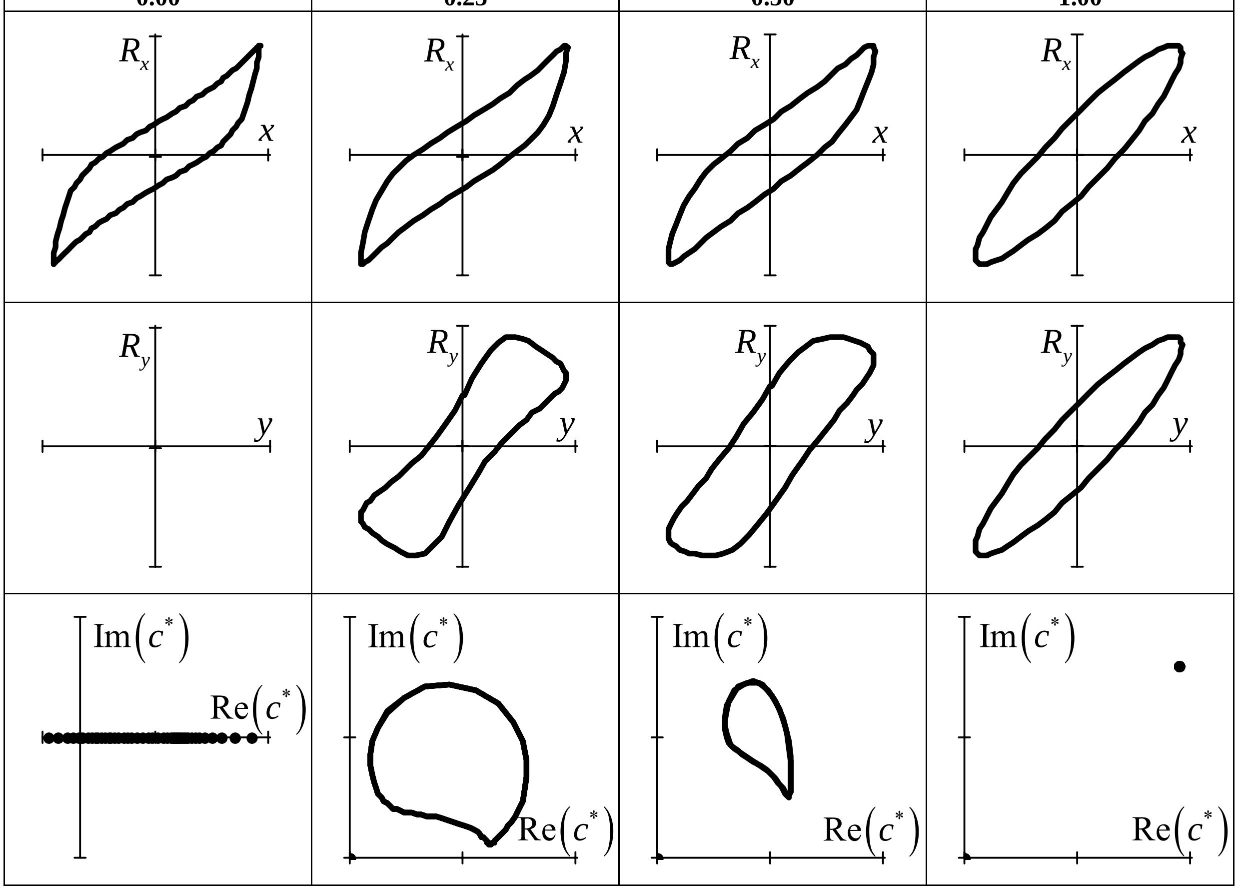
As it can be seen from Table-1 during uniaxial loading ($b/a=0$) the hysteresis loop in projection onto x axis is represented as a typical pattern with pointed edges at time of change of deformation velocity sign, and in projection onto y axis - as a point at the origin. With gradual increase of the coefficient b/a from 0 to 1 the form of trajectory changes from a straight line to a circular curve and hysteresis in the projections onto x and y axes is being transformed into an elliptic pattern.

Figure-5 shows diagrams with the relative deformation amplitudes $\overline{a}=a/\Delta$ put onto the abscissa axis and the absorption coefficients ψ put onto the ordinate axis for five different correlations of the elliptical trajectory semi-axes b/a.

It can be seen from Figure-5 that the absorption coefficient has the highest value when the journal moves along a circular trajectory ($b/a=1$). At that the maximum values of the absorbance coefficients for various correlations of the semi-axes are being reached at the same relative deformation amplitude $\overline{a}$.

Table-1. Transformation of hysteresis in a damper ring.

Correlation of the elliptical trajectory semi-axes b/a			
0.00	**0.25**	**0.50**	**1.00**



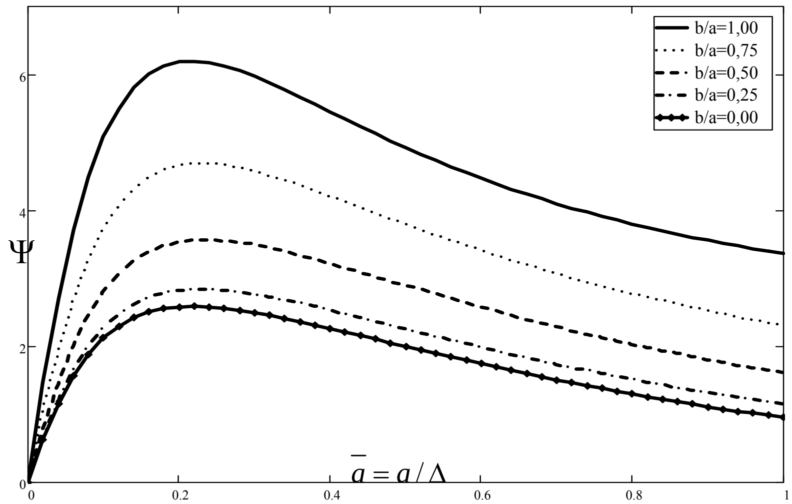


Figure-5. Influence of the elliptical trajectory semi-axes correlation on the damper absorption coefficient.

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5. RESTRICTIONS IN APPLICATION OF THEORY AND THE OFFERED MATHEMATICAL MODELS

The work outlines only the class of elliptic movement trajectories of the journals with dampers which has extreme trajectories built by cyclic relocation along a straight line and a circular curve. Hypocycloid- and epycycloid-like relocation is rather interesting class of movement trajectories of the journals of the elastic damping supports. But due to mathematical complexity and awkward computation this class was not analyzed in the present work. It is planned to describe this material in future articles of the author and his students.

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