VOL. 9, NO. 10, OCTOBER 2014 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

AN ANISOTROPIC COSMOLOGICAL MODEL FILLED WITH PERFECT FLUID IN A MODIFIED BRANS-DICKE THEORY OF GRAVITATION

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ABSTRACT

We present a new Cosmological solution for an anisotropic homogeneous Bianchi type-1 Cosmological model in modified Brans- Dicke theory with variable cosmological constant. We discussed the physical and geometrical properties of this model for radiation era in detail.

Keywords: anisotropic, bianchi type-1, perfect fluid, brans-dicke theory.

1. INTRODUCTION

There are many alternative theories and extensions of the Einstein's general relativity. Among them Brans and Dicke [1] theory of gravitation is well known modified version of Einstein's theory. It is a scalar tensor theory in which the gravitational interaction is mediated by a scalar field Φ as well as the tensor field g_{ij} of Einstein's theory. Many authors have studied the problems with cosmological solutions involving time dependent cosmological term and Brans-Dicke field. The work of Singh and Rai [2] gives a detailed discussion of Brans-Dicke cosmological models. In particular, spatially homogeneous Bianchi models in Brans-Dicke theory in the presence of perfect fluid with or without radiation are quite important to discuss the early stages of evolution of the universe. Nariai [3], Belinskii and Khalatnikov [4], Reddy and Rao [5], Banerjee and Santos [6], Shri Ram [7], Shri Ram and Singh [8], Berman *et al.* [9], Reddy [10], Reddy and Naidu [11], Adhav *et al.* [12], Rao *et al.* [13, 14], Endo and Fukui [15] and Rai, Rai and Singh [16] are some of the authors who have investigated several aspects of this Brans-Dicke theory and discussed in detail. Some authors like Bergmann [17] and Wagoner [18] proposed the variable cosmological term Q in an explicit function of a scalar field Φ .

The Brans-Dicke field equations with cosmological term Q are:

$$G_{ij} + g_{ij}Q = \frac{8\pi}{\varphi}T_{ij} + \frac{\omega}{\varphi^2}\left(\phi_i\phi_j - \frac{1}{2}g_{ij}\phi_k\phi^k\right) + \frac{1}{\phi}(\phi_{i;j} - \phi\,()\tag{1}$$

$$\phi = \frac{8\pi\mu T}{(2\omega+3)}\tag{2}$$

$$Q = \frac{(2\omega+3)}{4}\frac{(1-\mu)}{\mu}\frac{\phi}{\phi} = \frac{8\pi(1-\mu)}{4\phi}T\tag{3}$$

where the constant μ shows how much this theory including $Q(\phi)$ deviates from that of Brans and Dicke and as usual ω is coupling constant and T_{ij} is the energy-momentum tensor for a viscous fluid distribution [19]. Covariant derivative with respect to the metric g_{ij} is denoted by semicolons and partial differentiation with respect to the coordinate x^i is denoted by comas. Then under the conformal transformation:

$$g_{ij} \rightarrow \bar{g}_{ij} = \phi g_{ij}\tag{4}$$

the equations (1)-(3) go to the form

$$\bar{G}_{ij} + \bar{g}_{ij}\bar{Q} = 8\pi\bar{T}_{ij} + \frac{1}{2}(2\omega+3)\left(\Lambda_i\Lambda_j - \frac{1}{2}\bar{g}_{ij}\Lambda_k\Lambda^k\right)\tag{5}$$

$$\bar{\Box}\Lambda = \frac{8\pi\mu\bar{T}}{(2\omega+3)},\ \Lambda = log\phi\tag{6}$$

$$\bar{Q} = \frac{(2\omega+3)}{4}\cdot\frac{(1-\mu)}{\mu}\bar{\Box}\Lambda = \frac{8\pi(1-\mu)}{4\phi}\bar{T}\tag{7}$$

where all barred and unbarred quantities are defined in terms of metric $\bar{g}_{ij}$ and g_{ij} respectively.

In this paper we discuss Bianchi type-I perfect fluid cosmological models in a scalar-tensor theory proposed by Brans and Dicke. We obtain solution of the field equations for radiation era assuming that the deceleration parameter q is a constant.

2. FIELD EQUATIONS

The Bianchi type-I metric is considered as

$$ds^2 = -dt^2 + A^2(t)\,dx^2 + B^2(t)\,dy^2 + C^2(t)\,dz^2\tag{8}$$

where $A, B, C,$ are functions of $x^4 = t$ only. The energy-momentum tensor $(\bar{T}_{ij})$ for perfect fluid distribution is given by

$$\bar{T}_{ij} = (\bar{\rho} + \bar{p})\bar{v}_i\bar{v}_j + \bar{p}\bar{g}_{ij}\tag{9}$$

together with

VOL. 9, NO. 10, OCTOBER 2014 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

$$\bar{g}_{ij}\bar{v}^i\bar{v}^j = -1 \qquad (10)$$

where $\bar{p}$ and $\bar{\rho}$ are the proper pressure and energy density respectively and $\bar{v}^i$ are the components of the fluid four-velocity. We assume the coordinates to be commoving so that $\bar{v}^1 = \bar{v}^2 = \bar{v}^3 = 0$ and $\bar{v}^4 = 1$. Scalar field Λ is also taken to be a function of t only. The field equations (5) and (6) turn into

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} + \bar{Q} = 8\pi\bar{p} + \frac{(2\omega+3)}{4}\Lambda_4^2 \qquad (11)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A}\frac{C_4}{C} + \bar{Q} = 8\pi\bar{p} + \frac{(2\omega+3)}{4}\Lambda_4^2 \qquad (12)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} + \bar{Q} = 8\pi\bar{p} + \frac{(2\omega+3)}{4}\Lambda_4^2 \qquad (13)$$

$$\frac{A_4}{A}\frac{B_4}{B} + \frac{B_4}{B}\frac{C_4}{C} + \frac{A_4}{A}\frac{C_4}{C} + \bar{Q} = -8\pi\bar{\rho} - \frac{(2\omega+3)}{4}\Lambda_4^2 \qquad (14)$$

$$\Lambda_{44} + \Lambda_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{8\pi\mu(\bar{\rho} - 3\bar{p})}{(2\omega+3)} \qquad (15)$$

The suffix '4' stands for ordinary time-derivative of the concerned quantity.
From (11) and (12), we get

$$\frac{B_{44}}{B} - \frac{A_{44}}{A} + \frac{B_4}{B}\frac{C_4}{C} - \frac{A_4}{A}\frac{C_4}{C} = 0 \qquad (16)$$

From (12) and (13), we get

$$\frac{C_{44}}{C} - \frac{B_{44}}{B} + \frac{A_4}{A}\frac{C_4}{C} - \frac{A_4}{A}\frac{B_4}{B} = 0 \qquad (17)$$

First integral of (16) and (17) are

$$\frac{A_4}{A} - \frac{B_4}{B} = \frac{k_1}{ABC} \qquad (18)$$

and

$$\frac{B_4}{B} - \frac{C_4}{C} = \frac{k_2}{ABC} \qquad (19)$$

where k_1 and k_2 are the constants of integration.

Let R be the average scale factor of the Bianchi type-1 universe, i.e.

$$R^3 = ABC \qquad (20)$$

The Hubble parameter H, volume expansionθ, deceleration parameter q and shear σ for the metric (1) can be written as:

$$H = \frac{R_4}{R} \qquad (21)$$

$$\theta = u^i_{;i} = \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$$

$$\theta = 3H$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$$

$$= \frac{1}{2}\left[\left(\frac{A_4}{A}\right)^2 + \left(\frac{B_4}{B}\right)^2 + \left(\frac{C_4}{C}\right)^2\right] - \frac{\theta^2}{6}$$

where

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;\alpha}P^\alpha_j + u_{j;\alpha}P^\alpha_i\right) - \frac{1}{3}\theta P_{ij}$$

Here the projection tensor P_{ij} has the form

$$P_{ij} = g_{ij} - u_i u_j$$

$$\sigma^2 = \frac{k_1^2 + k_2^2 + k_1 k_2}{3R^6}$$

$$q = -\frac{RR_{44}}{R_4^2} \qquad (22)$$

3. SOLUTIONS OF THE FIELD EQUATIONS

The system of equations (2.4) - (2.8) supply only five equations in six unknowns (A, B, C,$\bar{\rho}$ $\bar{p}$ andΛ). One extra equation is needed to solve the system completely. We take deceleration parameter q is a constant.
Now integrating equation (22), we get

$$R = (at)^{\frac{1}{m}} \qquad (23)$$

where a and m are constant and m = 1+q
Using equations (18) and (19), we obtain

$$\frac{B}{A} = e^{\frac{k_1 m}{a(m-1)}(at)^{\frac{m-1}{m}}} \qquad (24)$$

$$\frac{C}{B} = e^{\frac{k_2 m}{a(m-1)}(at)^{\frac{m-1}{m}}} \qquad (25)$$

Dividing (24) by (25) gives

$$\frac{B^2}{AC} = e^{\frac{m(k_1-k_2)}{a(m-1)}(at)^{\frac{m-1}{m}}}$$

$$\frac{B^3}{ABC} = e^{\frac{m(k_1-k_2)}{a(m-1)}(at)^{\frac{m-1}{m}}}$$

$$\frac{B^3}{(at)^{\frac{3}{m}}} = e^{\frac{m(k_1-k_2)}{a(m-1)}(at)^{\frac{m-1}{m}}}$$

$$B = (at)^{\frac{1}{m}}\, e^{\frac{m(k_1-k_2)}{3a(m-1)}(at)^{\frac{m-1}{m}}} \qquad (26)$$

VOL. 9, NO. 10, OCTOBER 2014 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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And

$$A = (at)^{\frac{1}{m}}\, e^{\frac{-m(2k_1+k_2)}{3a(m-1)}(at)^{\frac{m-1}{m}}} \tag{27}$$

$$C = (at)^{\frac{1}{m}}\, e^{\frac{m(k_1+2k_2)}{3a(m-1)}(at)^{\frac{m-1}{m}}} \tag{28}$$

Metric (8) can be written as

$$ds^2 = -dt^2 + (at)^{\frac{2}{m}}\, e^{\frac{-2m(2k_1+k_2)}{3a(m-1)}(at)^{\frac{m-1}{m}}}\, dx^2 + (at)^{\frac{2}{m}}\, e^{\frac{2m(k_1-k_2)}{3a(m-1)}(at)^{\frac{m-1}{m}}}\, dy^2$$

$$+ (at)^{\frac{2}{m}}\, e^{\frac{2m(k_1+2k_2)}{3a(m-1)}(at)^{\frac{m-1}{m}}}\, dz^2 \tag{29}$$

For radiation era $\bar{\rho} = 3\bar{p}$ and $k_2 = -2k_1$, Metric (29) can be written as

$$ds^2 = -dt^2 + (at)^{\frac{2}{m}}\left[dx^2 + e^{\frac{2mk_1}{a(m-1)}(at)^{\frac{m-1}{m}}}\, dy^2 + e^{\frac{-mk_1}{a(m-1)}(at)^{\frac{m-1}{m}}}\, dz^2\right] \tag{30}$$

The Hubble parameter H, volume expansion θ and shear σ for the metric (30) can be written as

$$H = \frac{R_4}{R} = \frac{a}{m}(at)^{-1}$$

$$\theta = \frac{3a}{m}(at)^{-1}$$

$$\sigma = \left[\frac{18k_1^2}{54}(at)^{\frac{-2}{m}} + \frac{3a^2}{m^2}(at)^{-2}\right]^{\frac{1}{2}}$$

Using (26), (27), (28), we have

$$\Lambda = \frac{mt^{\frac{m-3}{m}}}{m-3}$$

where $m < 3$ (31)

From (14), we have

$$\bar{Q} = -\frac{3}{m}\frac{(2\omega+3)}{4}\frac{(1-\mu)}{\mu}t^{\frac{-(m+3)}{m}} \tag{32}$$

Using (6) and (4) finally from (14), we get

$$\bar{\rho} = -\frac{1}{8\pi}e^{\frac{2m}{m-3}t^{\frac{m-3}{3}}}\left[k_3(at)^{\frac{-2}{m}} + k_4(at)^{\frac{-(1+m)}{m}} + k_5(at)^{-2} + \frac{2(\omega+3)}{4}t^{\frac{-6}{m}} - \frac{3(1-\mu)(2\omega+3)}{4m\mu}t^{\frac{-(1+m)}{m}}\right] \tag{33}$$

4. TRANSFORMATIONS OF SOLUTIONS

Under the transformation given by [15]:

$$\phi \to \bar{\phi} = e^{\Delta},\ \bar{\rho} \to \rho = \phi^2\bar{\rho}$$

$$\bar{Q} \to Q = \phi\bar{Q},\ \bar{g}_{ij} \to g_{ij} = \frac{1}{\phi}\bar{g}_{ij}$$

$$\bar{v}^i \to v^i = \phi^{\frac{1}{2}}\bar{v}^i$$

We obtained

$$Q = e^{\frac{m}{m-3}t^{\frac{m-3}{3}}}\left(-\frac{3}{m}\frac{(2\omega+3)}{4}\frac{(1-\mu)}{\mu}t^{\frac{-(m+3)}{m}}\right)$$

$$\rho = e^{\frac{2m}{m-3}t^{\frac{m-3}{3}}}\left[-\frac{1}{8\pi}e^{\frac{2m}{m-3}t^{\frac{m-3}{3}}}\left\{k_3(at)^{\frac{-2}{m}} + k_4(at)^{\frac{-(1+m)}{m}} + k_5(at)^{-2} + \frac{2(\omega+3)}{4}t^{\frac{-6}{m}} - \frac{3(1-\mu)(2\omega+3)}{4m\mu}t^{\frac{-(1+m)}{m}}\right\}\right]$$

$$g_{11} = (at)^{\frac{2}{m}}e^{\frac{-m}{m-3}t^{\frac{m-3}{3}}}$$

$$g_{22} = (at)^{\frac{2}{m}}e^{\frac{2mk_1}{a(m-1)}(at)^{\frac{m-1}{m}} - \frac{m}{m-3}t^{\frac{m-3}{3}}}$$

$$g_{33} = (at)^{\frac{2}{m}}e^{-\left(\frac{m}{m-3}t^{\frac{m-3}{3}} + \frac{mk_1}{a(m-1)}(at)^{\frac{m-1}{m}}\right)}$$

$$g_{44} = -e^{\frac{-m}{m-3}t^{\frac{m-3}{3}}}$$

$$v^1 = v^2 = v^3 = 0,\ v^4 = e^{\frac{m}{2(m-3)}t^{\frac{m-3}{3}}}$$

5. DISCUSSIONS

The Hobble parameter, pressure, density, scalar field, cosmological constant and the cosmological term $(\bar{Q})$ are singular at $t = 0$ for $m < 3$. For $\mu = 1$ the cosmological term $(\bar{Q})$ vanish and the model (30) reduces into a Brans- Dick one of the model in general relativity. After transformation we get the same scenario.

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