



ANALYSIS OF SUSCEPTIBILITY TO SUFFUSION

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ABSTRACT

In the modern world of science and technology, rapid growth has escalated the need for specific and appropriate techniques so that construction activities ensure comprehensive solutions to each and every problem faced by the construction industry. It is in this respect a study of suffusion has been taken up. A study of collapse of dams, erosion of embankments and damages to buildings due to earthquakes lead us to conclude that suffusion has played a major role in all the above setbacks. Suffusion occurs due to voids and uneven particles in soils leading to seepage force causing damages. This requires analysis of geometric criteria of the soil and hydraulics that causes suffusion. This project attempts at assessment of susceptibility to suffusion through geometric criteria. To have deeper insight and finer evaluation, five different soil samples were analyzed using different methodologies proposed by nine authors. While six methodologies have confirmed that all the samples are suffusive, three methodologies have shown variations and out of these three, Burenkova's methodology is widely used for assessment. Unfortunately this methodology was found to give unsafe results. Wan and Fell (2008) refined Burenkova method and even this refined method was found to give unsafe results. Therefore attempts have been made to refine Burenkova method. Taking up 101 gradations from other studies and 5 from current study, a broader analysis has been done and improved limits have been suggested. Different models have been proposed for widely-graded and gap-graded soils. New models have been proposed based on two ideas. The first idea is that the d_{90}/d_{60} value should increase as d_{90}/d_{15} value increases. Second idea is that at lower values of d_{90}/d_{15} the stable zone will be small and as the d_{90}/d_{15} increases stable zone also increases. Among the five models proposed for widely-graded soils, model 2 is found to be most appropriate and among four models proposed for gap-graded soils, model 1 is found to be most appropriate. Along with refinement of Burenkova's method, the authors also propose a new better method for finding susceptibility of soils to suffusion. The authors use division between D and d_5 for the same. D represents higher diameter at gap location for gap graded soils or higher diameter corresponding to highest value of division between two successive diameters with a difference of 10% (first division alone be for a difference of 5%, between d_{10} and d_5). d_5 is the representative of fine grains while D is the representative of voids. D/d_5 value less than 4 corresponds to stable soils while greater than 6 corresponds to unstable soils and between 4 to 6 indicates transition zone.

Keywords: suffusion, internal stability, burenkova, suffosion.

INTRODUCTION

Suffusion refers to internal erosion of the soil. It is a multidisciplinary issue governed by the principles of soil mechanics and hydraulics. It occurs when the voids in a soil are large enough to allow the movement of the fine particles. Widely-graded soils with particles ranging from clay to gravel size, having concavely upward particle size distribution profile, and gap-graded soils are usually susceptible to suffusion. When the voids are sufficiently large and there is enough seepage force, the finer particles may get dislocated. This dislocation of finer particles widens the voids at certain points and there is a local change in gradation of the soil. Suffusion may ultimately result in soil piping and collapse of structures.

Suffusion may lead to formation of voids in foundation, increase in pore pressure and backward erosion (Fell *et al*, 2005). Formation of voids in the foundation results in settlement of crest and formation of sinkhole. Backward erosion causes piping. Increase in pore pressure leads to slope instability and blow out.

The major cause of dam failures in U.S. is stated to be internal erosion of embankments or their foundations (U.S. Department of the Interior, 2012). A case study of a railway embankment in Southern Italy says that suffusion

together with concentrated pipe erosion led to the failure of the embankment (Polemio and Lollino, 2011). Suffusion could trigger earthquake-induced damage of geotechnical structures. Failure of fills due to suffusion caused damages to houses, roads and railways during 2007 Noto Hanto Earthquake (Horikoshi *et al*, 2012). It is therefore necessary that appropriate attention is paid to suffusion to eliminate future problems.

The objective of this study is to evaluate the susceptibility of soils to suffusion by different methods. The scope of this work is restricted to qualitative analysis of susceptibility of soils to suffusion using geometric criteria.

EXPERIMENTAL STUDIES

In this study, five different soils designated as S1 to S5 were selected. These soils were characterized for their particle size distribution for checking susceptibility to suffusion. The particle-size distribution of the soils was determined using the wet sieve analysis and hydrometer analysis as per ASTM D 422-63. The following geometric criteria reported by earlier researchers are used to assess the susceptibility of soils to suffusion:



- a) Kenney and Lau proposed the following criteria (Wan and Fell, 2008)

$$(H/F)_{\min} \geq 1 \quad (1)$$

where H is increment of % passing that occurs over a designated grain size interval of D to 4D
F is the % passing at grain size D (F should be $\leq 30\%$)

- b) Burenkova suggested the following formula (Wan and Fell, 2008)

$$0.76\log(h'') + 1 < h' < 1.86\log(h'') + 1 \quad (2)$$

$$\text{where } h' = d_{90} / d_{60} \quad (3)$$

$$h'' = d_{90} / d_{15} \quad (4)$$

d_{90} is diameter (in mm) corresponding to 90% mass passing.

d_{60} is diameter (in mm) corresponding to 60% mass passing.

d_{15} is diameter (in mm) corresponding to 15% mass passing.

- c) Istomina put forward the following criterion (Li, 2008)

$$C_u \leq 10 - \text{Soil is internally stable} \quad (5)$$

$$C_u \geq 20 - \text{Soil is internally Unstable} \quad (6)$$

$$10 < C_u < 20 - \text{Soil is deemed a transition material} \quad (7)$$

where C_u is the coefficient of uniformity = d_{60} / d_{10}

d_{60} is diameter (in mm) corresponding to 60% mass passing.

d_{10} is diameter (in mm) corresponding to 10% mass passing.

- d) Kezdi postulated the condition given below (Li, 2008)

$$D'_{15} / d'_{85} \leq 4 \quad (8)$$

at gap location for gap-graded soils and $(H/F)_{\min}$ point for widely-graded soils.

where D_{15} is the diameter (in mm) of the 15% mass

passing in the coarser fraction

d'_{85} is the diameter (in mm) of the 85% mass passing in the finer fraction.

- e) Liu gave the following criterion (Li, 2008)

$$P < 25 - \text{internally unstable} \quad (9)$$

$$P = 25 \text{ to } 35 - \text{transition condition} \quad (10)$$

$$P > 35 - \text{internally stable} \quad (11)$$

where P is the mass passing (%) at the gap location for gap graded soils and mass passing (%) at the division diameter

$$d_f = \sqrt{d_{70}d_{10}} \text{ for continuously graded soils.}$$

- f) Mao suggested following condition (Li, 2008)

$$p_f < 100 \left(\frac{1}{4(1-n)} \right) - \text{internally unstable} \quad (12)$$

$$p_f \geq 100 \left(\frac{1}{4(1-n)} \right) - \text{internally stable} \quad (13)$$

where p_f is the mass passing at the gap location (%) for gap-graded soils and mass passing (%) at the division

diameter $d_f = 1.3\sqrt{d_{85}d_{15}}$ for continuously graded soils

n is porosity

- g) Busch and Luckner proposed the following equation (Hudak, 2009)

$$d_s = 0.27\sqrt[3]{U \cdot e \cdot d_{17}} \quad (14)$$

where d_s is largest grain that can undergo suffusion (in mm)

U is co-efficient of uniformity

e is void ratio

d_{17} is diameter (in mm) corresponding to 17% mass passing.

- h) Patrasev devised the following equation (Semar *et al.*, 2010)

$$d_s \geq 0.77d_{po} \quad (15)$$

$$d_{po} = 0.455(1 + 0.05C_u) \cdot \sqrt[3]{C_u \cdot e \cdot d_{17}} \text{ for } C_u \leq 25 \quad (16)$$

$$d_{po} = 0.16(3 + \sqrt[3]{C_u \cdot \log(C_u)}) \cdot \sqrt[3]{C_u \cdot e \cdot d_{17}} \text{ for } C_u > 25 \quad (17)$$

where d_s is the largest suffusive grain size diameter (in mm)

d_{po} is the effective opening size of the structure (in mm)

C_u is co-efficient of uniformity

e is void ratio

d_{17} is diameter (in mm) corresponding to 17% mass passing.

- i) Ziem proposed the following formula (Semar *et al.*, 2010)

$$d_{\min} = 0.409\sqrt[3]{C_u \cdot e \cdot d_{17}} \quad (18)$$



where C_u is co-efficient of uniformity

e is void ratio

d_{17} is diameter (in mm) corresponding to 17% mass passing.

RESULTS AND DISCUSSIONS

The particle size distribution of the soils selected in the study is given below in Figures 1, 2, 3, 4 and 5.

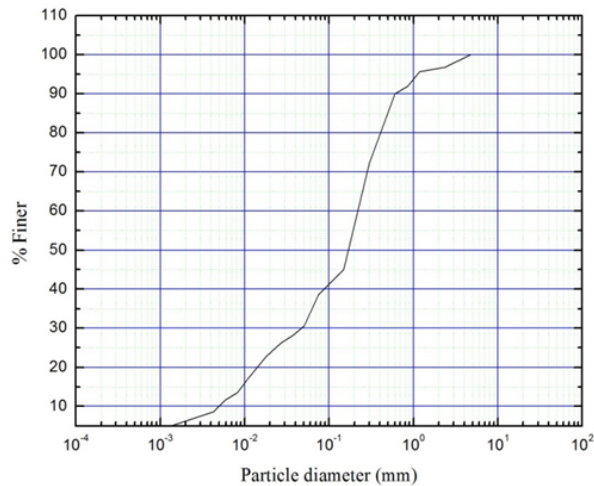


Figure-1. Particle grain size distribution curve for soil S1.

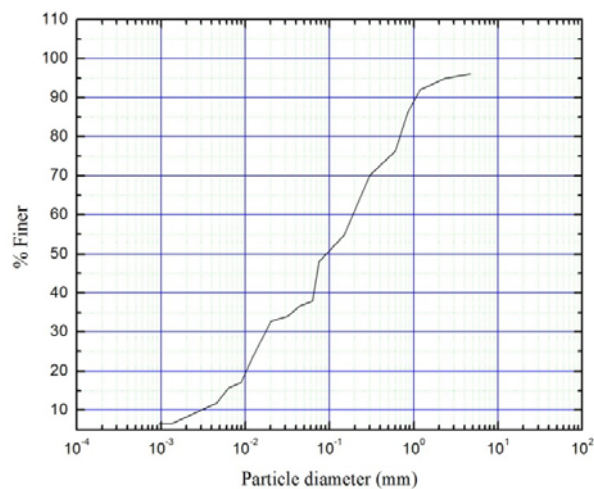


Figure-2. Particle grain size distribution curve for soil S2.

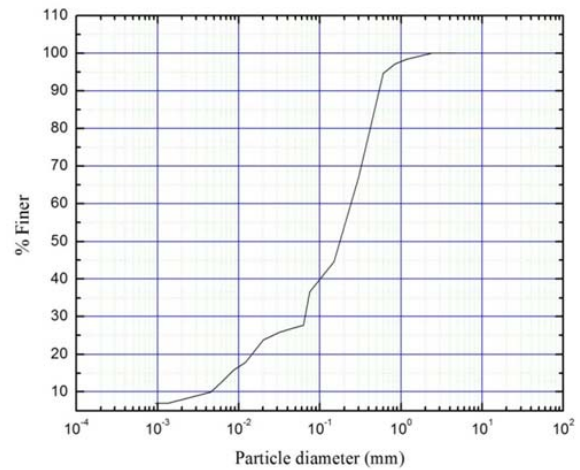


Figure-3. Particle grain size distribution curve for soil S3.

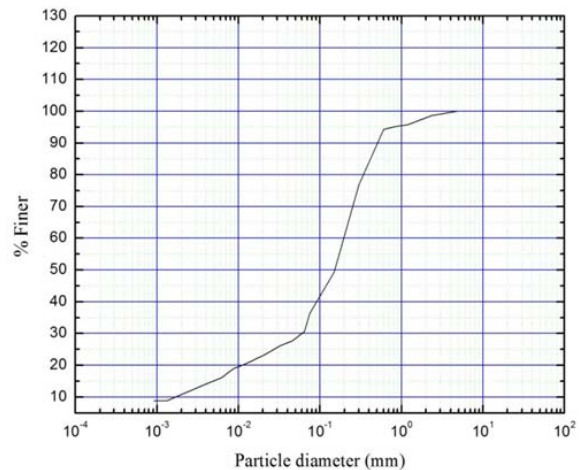


Figure-4. Particle grain size distribution curve for soil S4.

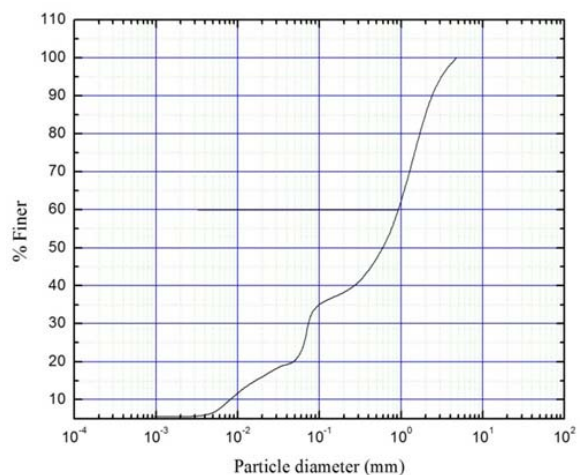


Figure-5. Particle grain size distribution curve for soil S5.

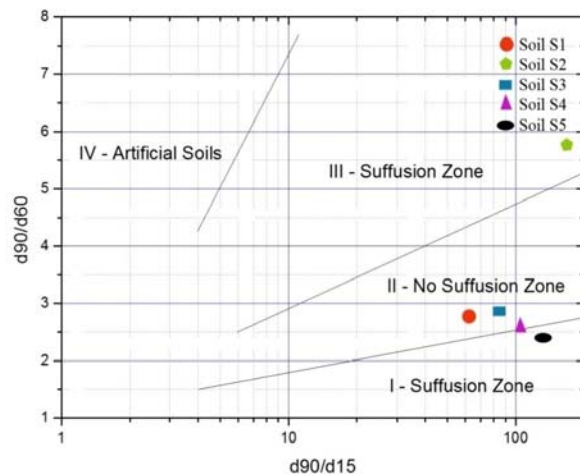
Susceptibility of the soils to suffusion based on different criteria is given below in Table-1.

**Table-1.** Susceptibility of the soils to suffusion.

Soil	S1	S2	S3	S4	S5
Kenney and Lau (1986)	S	S	S	S	S
Istomania (1957)	S	S	S	S	S
Kezdi (1979)	S	S	S	S	S
Burenkova (1993)	NS	S	NS	NS	S
Liu (2005)	T	T	T	S	NS
Mao (2005)	S	NS	S	S	S
Busch and Luckner (1972)	S	S	S	S	S
Patrasev (1981)	S	S	S	S	S
Ziem (1969)	S	S	S	S	S

S - Suffusive; NS - Non-suffusive; T - Transition Condition

Analysis of the soils using Burenkova method is given below in Figure-6.

**Figure-6.** Burenkova's chart for suffusion susceptibility of soils S1 to S5.

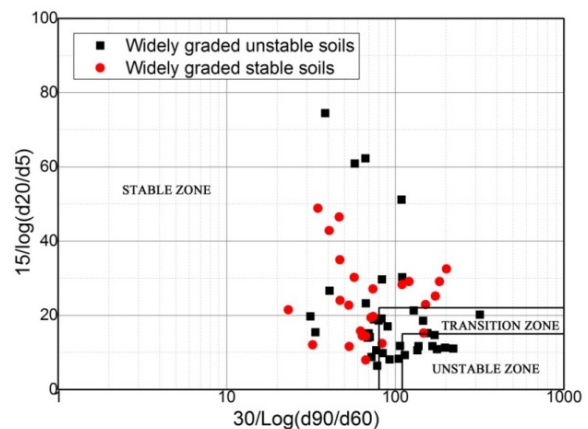
Burenkova method is found to give unsafe results. In view of its extensive application and inconsistent conclusions, this methodology gives scope for improvement to ensure better success rate.

MODIFICATION OF BURENKOVA METHOD

In order to refine this methodology, a broader analysis has been done using experimental data from other authors along with the data from current study. A total of 106 data points are used. 62 gradations used by Li (2008) to modify Kezdi and Kenney criteria are taken along with 6 gradations from Andrianatrehina (2012), 20 from Wan and Fell (2008), 3 from Chapuis (1996), 8 from Burenkova (1993), 2 from Lafleur (1989) and 5 from current study. The 106 gradations are categorized into two groups namely gap-graded and widely-graded soils. 39 gradations fall into the category of gap-graded soils while the rest 67 gradations are widely-graded soils.

Wan and Fell (2008) proposed a modified method for widely-graded soils. Applying their method to the chosen 106 gradations clearly shows that their method gives unsafe results (see Figure-7).

Burenkova originally suggested common limits for both gap-graded and widely-graded soils (see Figure-8). Though d_{90} , d_{60} and d_{15} values of a gap-graded soil and a widely-graded soil might be same, packing of soil particles differs between them as one or more particle sizes are found missing in gap-graded soils. Suffusion is affected by soil structure which represents the way soil particles are packed. This fact proves that common model cannot be proposed for both gap-graded and widely-graded soils. Further the data points also support the view of having separate models.

**Figure-7.** Wan and Fell method (2008).

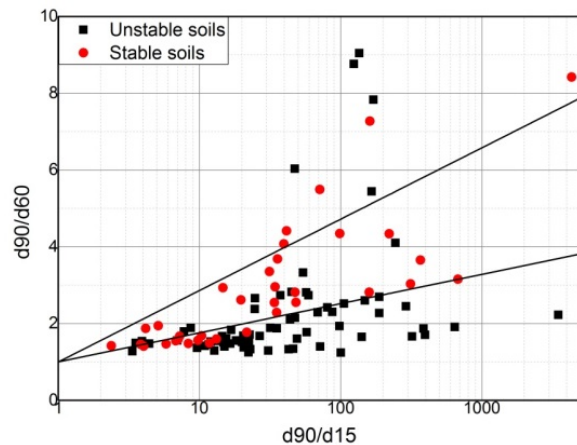


Figure-8. Soils plotted with Burenkova's original limits.

Logic behind the modification

The authors use the following two logics to propose modified models.

- Considering d_{90} as an index for coarser particles, d_{60} for medium-sized particles and d_{15} for finer particles, the value of d_{90}/d_{60} should increase when d_{90}/d_{15} increases for maintaining stability.
- If x is the value of d_{90}/d_{15} , then d_{90}/d_{60} should have value between 0 and x . If x value is small then number of data points between 0 and x will be less and the number of stable data points will be lesser. On the other hand if x value is high, then the number of data points between 0 and x will be more and the number of stable data points will also be more. Thus at lower values of d_{90}/d_{15} the stable zone will be small and as d_{90}/d_{15} increases, stable zone also increases.

The model which satisfies the above logics and also accommodates maximum data points correctly should be the most perfect model.

Proposed models for widely-graded soils

Based on the data points of widely graded soils it can be concluded that there exists a transition zone near the stable zone on both sides, where there is a mix of stable and unstable soils. If the data points of the unstable soils below a possible transition boundary are connected, a slope is obtained (see Figure-9).

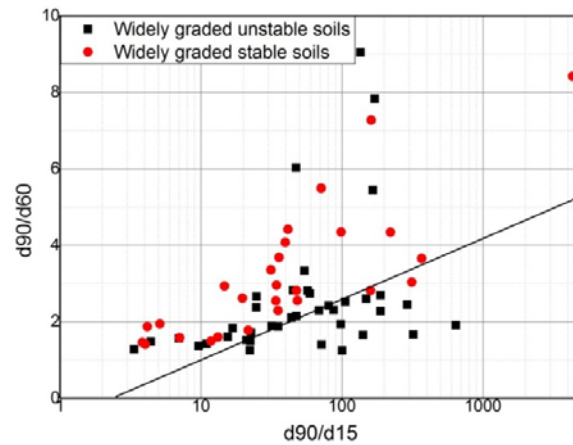


Figure-9. Slope for proposing models.

The authors feel a model having lower boundary of the lower transition zone with similar slope will be the most compatible model. Based on this idea, the authors have proposed five models (see Figure-10, 11, 12, 13 and 14).

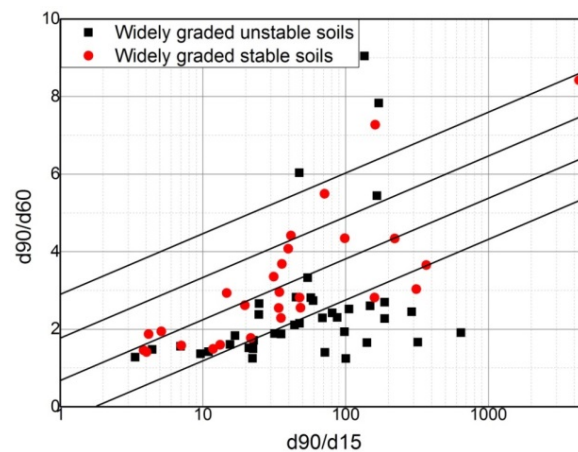


Figure-10. Widely-graded soils: model 1.

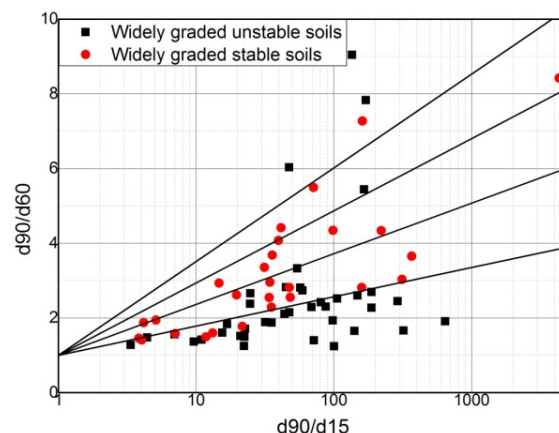


Figure-11. Widely-graded soils: model 2.

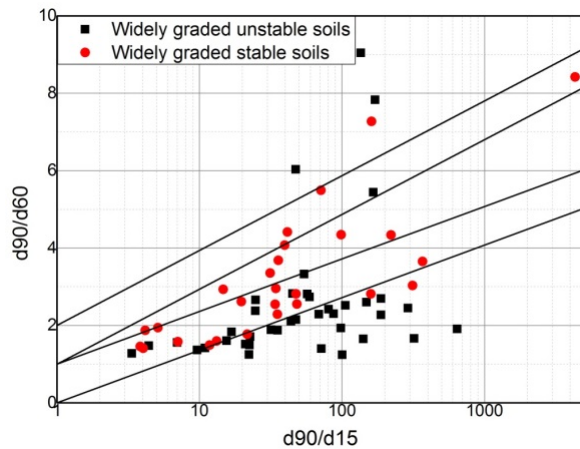


Figure-12. Widely-graded soils: model 3.

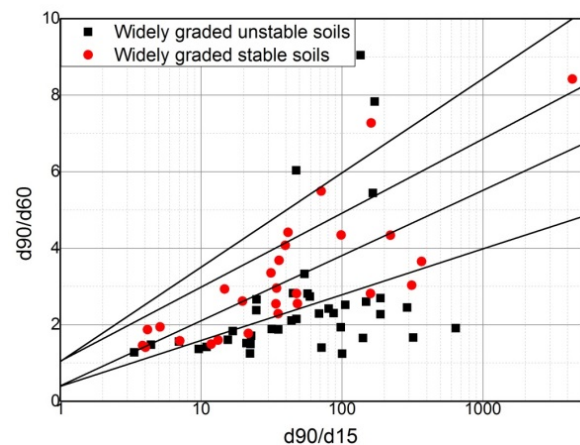


Figure-13. Widely-graded soils: model 4.

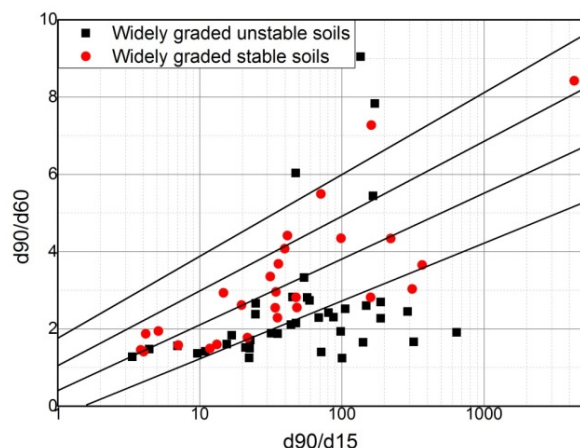


Figure-14. Widely-graded soils: model 5.

A point to note here is that the upper limit of the stable zone is restricted by the unstable soil S2 from current study with co-ordinates (165, 5). Soil S2 is proved to be unstable based on theoretical methods and not experimentally. The current theoretical methods are not

perfect and the experimental result may deem the soil S2 to be stable. As of now the authors consider the soil as unstable and have proposed the limits.

In the event of the soil being unstable but considered stable, then the current proposed limits may give unsafe results. Hence, the authors conservatively consider the soil as unstable. The authors suggest verifying Soil S2 experimentally in future and if it is found to be stable, the limits may be refined. In case of refinement the current stable zone area will get increased.

The model 1 gives illogical values for stable zone at lower values of d_{90}/d_{15} i.e., when d_{90}/d_{15} is 1, the model allows a stable soil to have d_{90}/d_{60} value greater than 1. Further the model does not obey the second logic of the modification. Though the models 3 and 5 obey the second logic of the modification, they give illogical values for the soils in transition zone.

The model in which least number of unstable data points in the transition zone and maximum number of unstable data points outside this transition zone are present is the best model. Model One has 14 unstable data points in the transition zone and 25 unstable data points outside the transition zone, model Two 7 unstable data points in the transition zone and 32 unstable data points outside the transition zone, model Three 14 unstable data points in the transition zone and 25 unstable data points outside the transition zone, model Four 10 unstable data points in the transition zone and 29 unstable data points outside transition zone and model Five 14 unstable data points in the transition zone and 25 unstable data points outside transition zone.

Considering the above discussions, model 2 can be concluded as the most appropriate model. One interesting aspect to note here is the upper boundary of the new stable zone and the lower boundary of the lower transition zone. The upper boundary of the new stable zone has the same equation as that of Burenkova's original stable zone upper limit and the lower boundary equation of the lower transition zone is same as that of Burenkova's original stable zone lower limit. Thus the refinement leads to reduction in the Burenkova's original stable zone and addition of new transition boundaries on either side of stable zone. The lower part of Burenkova's original stable zone has been refined as lower transition zone. The final refined boundaries are

Transition Zone Lower Limit: $h' = 0.76 \log(h'') + 1$

Stable Zone Lower Limit: $h' = 1.36 \log(h'') + 1$

Stable Zone Upper Limit: $h' = 1.86 \log(h'') + 1$

Transition Zone Upper Limit: $h' = 2.51 \log(h'') + 1$

Proposed models for gap-graded soils

Though a transition zone as in widely-graded soils may be proposed for gap-graded soils, inadequacy of data points compels authors to restrict the models with stable zone alone. However in future with sufficient data points the models may be refined to include transition zones. 4 models have been proposed here (see Figure-15, 16, 17 and 18).

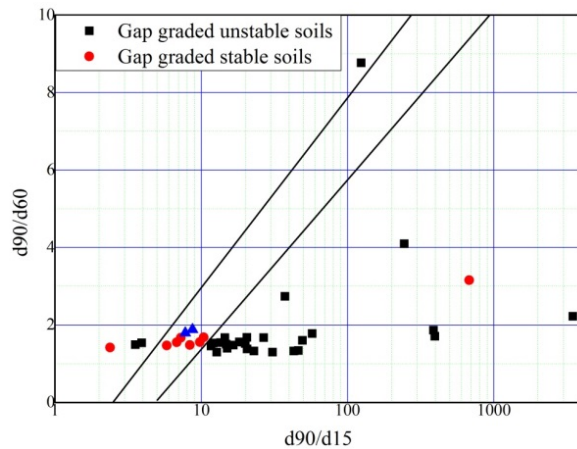


Figure-15. Gap-graded soils: model 1.

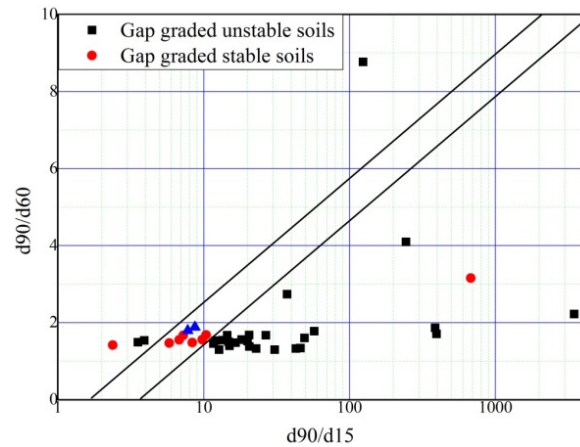


Figure-18. Gap-graded soils: model 4.

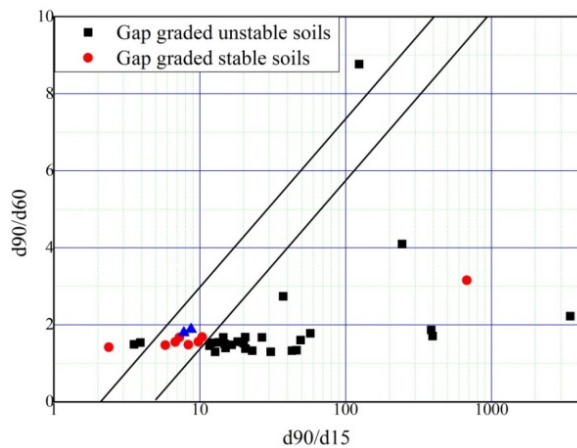


Figure-16. Gap-graded soils: model 2.

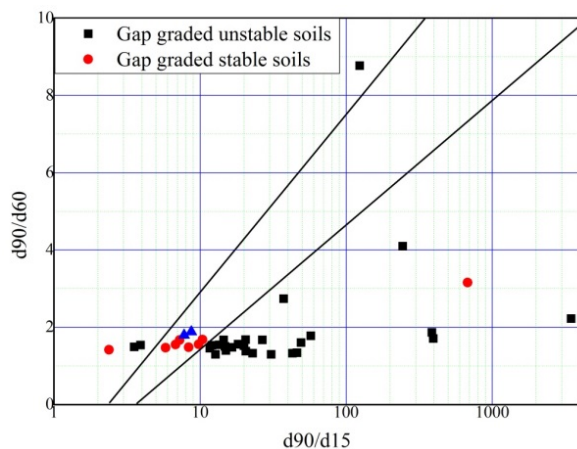


Figure-17. Gap-graded soils: model 3.

The two blue data points in the above Figures represent UNSW samples 13 and 9. UNSW deemed these soils as stable and Wan and Fell (2008) also stated the same. But Salehi (2012) assessed these two soils to be unstable. Hence there is difference of opinion. However, based on the trend of data points, the authors feel the soils 13 and 9 are stable.

If the boundaries of the stable zone for gap-graded and widely-graded soils are compared, the slope of the boundaries proposed for gap-graded soils is higher than that of widely-graded soil. The gap generally occurs below d_{50} . Consider two stable soils one in each type (gap-graded and widely-graded) having same d_{90} and d_{60} value. The voids between the coarser particles are filled by successive finer particles. In case of gap-graded soils, because of the absence of one or more particle sizes below d_{50} , the d_{15} value must increase to maintain stability. Thus for the same value of d_{90} and d_{60} , a gap-graded stable soil should have higher d_{15} value than a widely-graded stable soil which means that for the same value of d_{90}/d_{60} , the d_{90}/d_{15} value of a gap-graded stable soil should be smaller than that of widely-graded stable soil. Thus the slope of the boundaries proposed for gap-graded soils is higher than that of widely-graded soil.

Models 2 and 4 do not obey the second logic. The authors feel that it is safer to consider a gradual increase of stable zone in smaller steps along the x-axis and thus the authors consider model 1 to be more appropriate than model 3. The final refined boundaries are

Stable Zone Lower Limit: $h' = 4.37 \log(h'') - 3.00$

Stable Zone Upper Limit: $h' = 4.90 \log(h'') - 1.94$

PROPOSAL OF A NEW METHOD

Though many theoretical methods exist for assessing susceptibility to suffusion, no method is found to be perfect. There is some irregularity or discrepancy in each method. Many refine the methods but still irregularities exist though reduced. This compels the authors to find a new method which would be much better than the current available methods, though the aim is to



find a perfect flawless model at the end of the day. The same 106 gradations used for Burenkova method's refinement has been used for this. The newly proposed criteria is as follows:

Stable condition: $D/d_5 < 4$

Transition Condition: $D/d_5 = 4-6$

Unstable Condition: $D/d_5 > 6$

Where, d_5 - diameter (in mm) corresponding to 5% mass passing; if d_5 value is not available d_{10} can be used.

D - higher diameter (in mm) at the gap location for gap-graded soils (the black points in the Figure-19 represent D) or higher diameter (in mm) corresponding to highest value of division between two successive diameters with a difference of 10% (first division alone be for a difference of 5%, between d_{10} and d_5 ; if d_5 value is not available then this value might be skipped). See the Example-1 given below the Figure-19 to know how to calculate the D value for widely-graded soils.

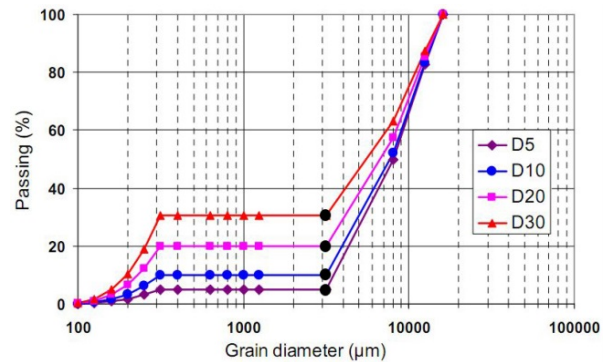


Figure-19. Location of D in gap-graded soils (Andrianatrehina, 2012).

Example-1. Consider a soil with the following grain-size diameters (Table-2).

Table-2. Grain size diameters of soil 'a' (Li, 2008).

d_5	d_{10}	d_{20}	d_{30}	d_{40}	d_{50}	d_{60}	d_{70}	d_{80}	d_{90}	d_{100}
0.14	0.18	0.28	1.02	2.18	2.98	3.74	4.51	5.52	7.05	9.45

The division between two successive diameters with a difference of 10% (first division alone being for a difference of 5%) is as follows:

Table-3. Division between two successive diameters with a difference of 10% (first division alone being for a difference of 5%) for soil 'a' (Li, 2008).

d_{10}/d_5	d_{20}/d_{10}	d_{30}/d_{20}	d_{40}/d_{30}	d_{50}/d_{40}	d_{60}/d_{50}	d_{70}/d_{60}	d_{80}/d_{70}	d_{90}/d_{80}	d_{100}/d_{90}
1.26	1.56	3.65	2.13	1.36	1.26	1.21	1.23	1.28	1.34

If the values in the Table-3 are observed, d_{30}/d_{20} has the highest value. Therefore, highest value of division between two successive diameters with a difference of 10% is 3.65 which corresponds to d_{30}/d_{20} (in case of 2 or 3 divisions having same value, the first instance should be taken). d_{30} is the higher diameter corresponding to highest value of division between two successive diameters with a difference of 10% and therefore value of $D = d_{30} = 1.02$ mm.

The authors feel that there might be influence of parameters other than particle size for soils in transition zone which prevents classifying them perfectly as stable or unstable. The authors suggest to future researchers to find out and study the parameters that influence the soils in transition zone. Possible parameters that might play a direct role in suffusion are soil mineralogy, fabric structure, atterberg limits and soil stress state.

97 gradations obey the newly proposed formula while 9 do not. All the 9 gradations that do not obey the above formula are UNSW (Wan and Fell, 2008) stable samples. However it should be noted that even in case of those 9 samples, the new method, though does not give correct results, gives safer results i.e. stable soils as

unstable. There might be parameters like soil mineralogy which make them stable though their particle size corresponds to unstable type. Some minerals create high interparticle attraction which prevents movement of particles. However, the authors suggest to re-check the UNSW stable soils experimentally again to confirm their stability. If again found stable, the influencing parameters might be studied.

CONCLUSIONS

- Based on most of the criteria given by earlier researchers, the authors conclude that all the five soils of the current study are susceptible to suffusion.
- It is suggested that sufficient precaution shall be taken when dealing with these soils.
- It has been noted that the initiation of suffusion for suffusive soils depends upon the hydraulic gradient. Hence the influence of hydraulic gradient in respect of soil suffusion shall be studied.
- It is observed that there is some variation in defining soil susceptibility to suffusion based on different criteria. From the various criteria for analysing



susceptibility to suffusion, literature suggests that the modified Kezdi and Kenney-Lau criteria are more conservative whereas Burenkova criterion gives unsafe results (Li, 2008).

- Refined Burenkova method by Wan and Fell (2008) for widely-graded soils gives unsafe results.
- In view of extensive usage of Burenkova method in the field of engineering and the inconsistent results given by the method, a study has been done to refine the method and refined boundaries for stable zone and new transition zones have been proposed. However, it is suggested to check the new boundaries for its conservativeness with more soils.
- The authors suggest a new method which is much better than currently available methods.

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