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DISTINCTIVE FEATURES OF THE RELATIONS BETWEEN GRINDING EQUIPMENT AND DEVICES INSIDE BALL MILL BODY

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ABSTRACT

The mathematical description of the parameters of spatial movement processes of the grinding bodies and their interaction with the inclined intermediate diaphragm in the rotating body of the ball mill is carried out to determine the influence of inside mill devices on the grinding load. It allows defining the kinematic and dynamic characteristics for each of the grinding bodies and their groups; the calculating method of the power consumed by their movement. An analytical expression is obtained to calculate the power consumed by the grinding media motion in the mill body with inclined intermediate diaphragm. The numerical computer and physical experiments with mill without inside mill devices are conducted to determine the power consumed by the grinding media motion. A quantitative estimation of the changes of kinematic parameters during the grinding media rotation, characterized by more intensive movement and changing their mode installation of the inclined intermediate diaphragm which contributes to the intensification of the process of grinding material is given.

Keywords: ball mill, grinding media, intermediate diaphragm, kinematic parameters, dynamic parameters, power consumption.

INTRODUCTION

Currently, the industry produces a large number of building materials and products: cement, lime, gypsum, glass, ceramic and others. During their manufacture raw components or products of technological processing are milled. Ball mills are widely used for fine materials grinding at the enterprises. Their work, along with a large enough capacity, the relative ease of maintenance and operation, is characterized by high specific power consumption caused by the irrational organization of the movement process of grinding media (g.m.). Mills used in various designs of energy-exchange devices provide intensive cross-longitudinal movement which increases the efficiency of the grinding material and allows reducing the specific energy consumption [1].

The model variation of the grinding media motion is mainly used to investigate the influence of energy exchange devices [2]. In our opinion, the finite element method is worthwhile to use for such kind of structures [3-5], and especially its variety Event driven algorithm [6-10].

MATERIALS AND METHODS

During the mathematical description of the parameters of the motion process let's consider g.m. in the form of spheres in mill with inclined intermediate diaphragm (IID) equal to zero at some instant, let's also admit the known radius vectors $\vec{r}_{i0}$, centers of mass (c.m.) velocity $\vec{V}_{i0}$ and angular velocities of the balls $\vec{\omega}_{i0}$. Without the collisions with other balls, lining and inside mill devices, the movement of the c.m. each grinding media is described by the equation:

$$\vec{r}_i = \vec{r}_{i0} + \vec{V}_{i0} \cdot t + \vec{g}t^2 / 2 , \quad (1)$$

where $\vec{g}$ - gravitational acceleration vector
t - time of the motion.

Let's define the motion time of the parabolic path for each ball (if there is one in the mill) using equation (1) before the collision with the drum t_{bi}, with another ball t_{sij} or with IID - t_{pik} (see Figure-1). Resulting all the times t_{bi}, t_{sij} and t_{pik} we have a minimum amount of time t_u, through which the ball will clash with either lining or with another ball or with inside mill device determined from the condition:

$$t_u = \min_{1\le i,j\le N, i<j, 1\le k\le K} \{t_{bi}, t_{sij}, t_{pik}\}, \quad (2)$$

where N – number of grinding media,
K – number of inclined surfaces.

The coordinates and velocities of the c.m. balls when $t=t_u$ are determined by the equations:

$$\vec{r}_i = \vec{r}_{i0} + \vec{V}_{i0} \cdot t_u + \vec{g}t_u^2 / 2 ;$$
$$\vec{V}_i = \vec{V}_{i0} + \vec{g}t_u; \quad (3)$$
$$\vec{\omega}_i = \vec{\omega}_{i0}.$$

If t_u is equal to one of the values t_{bi}, then we calculate the ball and drum collision, when t_u equal to one of the values t_{sij}, then we calculate the collision of two balls when t_u is equal to one of the t_{pik}, then calculate the ball collision by IID, i.e. determine the post-collision velocity: V_{i2}, V_{j2}, ω_{i2}, ω_{j2}.

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Then the intervals motion are calculated only for the balls involved in the collision (including post-impact velocities) to the following collisions with lining, the inclined diaphragm and other balls but the motion intervals are reduced to t_u before the collision of the remaining balls. Thus, if there is minimum time of the ball movement before the collision with IID, the times of this ball movement are redefined before the collision with the other balls, lining and IID. If the *i*- ball collides with *j*-ball, the times of their movement are redefined before the next collision with lining, IID and other balls. The procedure is then repeated.

Let's consider in more detail the time definition of the ball before its impact with the IID.

The equation of the median plane IID rotating with the drum has the form:

$$Ap \cdot z + Bp \cdot x + Cp \cdot y + Dp = 0, \quad (4)$$

where $Ap = \sin \alpha_k$;, (5)

$$Bp = -\sin \chi_k \cdot \cos \alpha_k ; \quad (6)$$

$$Cp = -\cos \chi_k \cdot \cos \alpha_k ; \quad (7)$$

$$Dp = -\sin \alpha_k \cdot a, \quad (8)$$

where χ_k - the angle between the minor axis of the ellipse *k*-th inclined diaphragm and Y axis;

α_k - the angle between the major axis of the ellipse *k*-th inclined diaphragm and X axis;

a - distance from the origin to the point of intersection of the inclined diaphragm all and the X axis

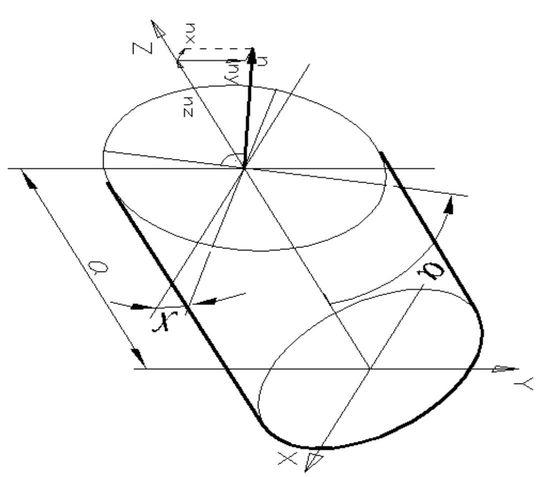

Figure-1. Scheme of the time calculation of the ball.

Let's denote the projections $\vec{r}_i$, $\vec{r}_{i0}$ and $\vec{V}_{i0}$ on the coordinate axes accordingly through x_i, y_i, z_i, x_{i0}, y_{i0}, z_{i0}, V_{ix0}, V_{iy0}, V_{iz0} , then the expression (3) takes the form:

$$x_i = x_{i0} + V_{ix0}t;$$
$$y_i = y_{i0} + V_{iy0}t; \quad (9)$$
$$z_i = z_{i0} + V_{iz0}t - gt^2/2 .$$

Equations (9) describe the movement of the ball in the mill body. At the moment t_{pik} when it touches IID the distance from c.m. of the ball to its middle surface must be equal to the sum r_{si} and to the half of the thickness of IID $\delta_k/2$:

$$d = \frac{|Ap \cdot x_i + Bp \cdot y_i + Cp \cdot z_i + Dp|}{\sqrt{Ap^2 + Bp^2 + Cp^2}} = r_{si} + \delta_k/2. \quad (10)$$

Otherwise: the distance from the c.m. of the ball to the plane which is parallel to the given one and is situated at the distance of the sphere radius and of the half the thickness of the IID should be equal to zero (see Figure-2).

$$d' = \frac{|Ap \cdot x_i + Bp \cdot y_i + Cp \cdot z_i + Dp'|}{\sqrt{Ap^2 + Bp^2 + Cp^2}} = 0,, \quad (11)$$

where:

$$Dp' = -\sin \alpha_k \cdot \left(a - \frac{(\delta/2 + r_{si})}{\sin \alpha_k} \right). \quad (12)$$

Angle χ_k is changed according to the law:

$$\chi_k = \chi_{k0} + \Omega_z t, \quad (13)$$

where χ_{k0} - the minor axis angle of the ellipse of the inclined plane at the ball flight initiation;

Ω_z - the angular velocity of the body rotation around the axis OZ.

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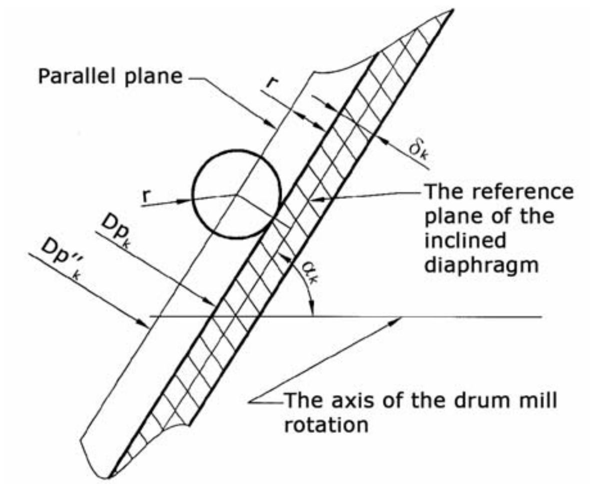

Figure-2. Scheme of the calculation of the ball position referred to the inclined diaphragm.

Substituting (5) and (6), (7), (9), (12) and (13) into (11) we obtain the equation to determine the time of the ball flight before the collision with the IID.

$$\sin\alpha_{k0}\left[\left(z_{i0}+V_{iz0}t\right)-a\right]- \\ -\cos\alpha_k\left[\left(\sin\left(\chi_{k0}+\Omega_z t\right)\right)\left(x_{i0}+V_{ix0}t\right)+\cos\left(\chi_{k0}+\Omega_z t\right)\left(y_{i0}+V_{iy0}t-{gt^2}/{2}\right)\right]+ \\ +\left(\delta_k/2+r_{si}\right)=0. \tag{14}$$

We define the ball movement before its impacts with the drum liner. This position in the space with the cylindrical shape of the drum and its axis which coincides with the X axis of the coordinate system is practically the same as on the plane perpendicular to the axis of the cylinder [11]. Thus, to determine the start ball movement t_{bi} or the collision with another ball or with IID, the equation is solved:

$$a_1 t_{bi}^4 + a_2 t_{bi}^3 + a_3 t_{bi}^2 + a_4 t_{bi} + a_5 = 0, \tag{15}$$

where

$$a_1 = g^2/4;$$
$$a_2 = -gV_{iy0};$$
$$a_3 = V_{ix0}^2 + V_{iy0}^2 - gy_{i0};$$
$$a_4 = 2\left(V_{ix0}\cdot x_{i0} + V_{iy0}\cdot y_{i0}\right);$$
$$a_5 = x_{i0}^2 + y_{i0}^2 - \left(R - r_{si}\right)^2.$$

where R - clearance radius of the mill drum.

Let's take only two balls i and j among their total quantity of them to determine the balls motion before the collision. At the moment of contact t_{sij} the condition must be satisfied:

$$\left|\vec{r}_i - \vec{r}_j\right| = r_{si} + r_{sj}.. \tag{16}$$

In coordinate form, it has the form:

$$\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2 + \left(z_i - z_j\right)^2 = \left(r_{si} - r_{sj}\right)^2. \tag{17}$$

Substituting (9) into (17) and after transformations we shall receive the equation to determine the movement times t_{sij} before the collision:

$$At_{sij}^2 + Bt_{sij} + C = 0; \tag{18}$$

$$A = \left(V_{ix0} - V_{jx0}\right)^2 + \left(V_{iy0} - V_{jy0}\right)^2 + \left(V_{iz0} - V_{jz0}\right)^2;$$
$$B = \left(x_{i0} - x_{j0}\right)\left(V_{ix0} - V_{jx0}\right) + \left(y_{i0} - y_{j0}\right)\left(V_{iy0} - V_{jy0}\right) + \left(z_{i0} - z_{j0}\right)\left(V_{iz0} - V_{jz0}\right);$$
$$C = \left(x_{i0} - x_{j0}\right)^2 + \left(y_{i0} - y_{j0}\right)^2 + \left(z_{i0} - z_{j0}\right)^2.$$

Normal positive root of the equation (18), determining the movement of balls before the collision will be:

$$t_{sij} = \frac{-\left(B + \sqrt{B^2 - 4AC}\right)}{2A}. \tag{19}$$

Let's consider the collision of the ball with the bot, on the lining or IID, regardless of the point of collision. Let's determine the post-collision characteristics of the ball and the body according to the theorem of the change of the number and angular momentum. It's necessary to consider the nature of the friction force between the ball and the IID in the collision. Depending on the values of the linear and angular velocities of the bodies frictional force may be:

a) absent during the whole collision,
b) equal to zero during the collision,
c) acting during the whole collision.

Obviously, the cases 1 and 3 are limiting for the second one; let's discuss it in details. We divide the time of interaction of the ball with IID into two intervals: from the start of the treatment until the impact of the friction force which is equal to zero and continue until the end of the blow. Let's denote the speed at the moment of the treatment of the friction force which is equal to zero by index "1" and form the equation separately for the first and second time intervals:

$$\begin{cases} m_i\left(\vec{V}_{i1}-\vec{V}_{i0}\right)=\vec{S}_{n1}+\vec{S}_\tau+\vec{S}_b; \\ I_i\left(\vec{\omega}_{i1}-\vec{\omega}_{i0}\right)=r_{si}\left(-\vec{n}\right)\times\vec{S}_\tau+r_{si}\left(-\vec{n}\right)\times\vec{S}_b; \\ M\vec{U}_1=-\vec{S}_{n1}-\vec{S}_\tau-\vec{S}_b; \\ I\left(\vec{\Omega}_1-\vec{\Omega}_0\right)=Av\left(-\vec{r}_{i0}\right)\times\vec{S}_{n1}+Bv\left(-\vec{n}\right)\times\vec{S}_\tau+Cv\left(-\vec{n}\right)\times\vec{S}_b; \\ m_i\left(\vec{V}_{i2}-\vec{V}_{i1}\right)=\vec{S}_{n2}; \\ I_i\left(\vec{\omega}_{i2}-\vec{\omega}_{i1}\right)=0; \\ M\left(\vec{U}_2-\vec{U}_1\right)=-\vec{S}_{n2}; \\ I\left(\vec{\Omega}_2-\vec{\Omega}_1\right)=Av\left(-\vec{r}_{i0}\right)\times\vec{S}_{n2}, \end{cases} \quad (20)$$

where $\vec{S}_{n1}$ and $\vec{S}_{n2}$ - force pulses of the normal pressure before and after the transformation of the friction force to zero;

$\vec{n}$ - the unit normal vector;

$I_i = 0.4\cdot m_i \cdot r_{si}^2$ - moments of the *i*-th ball inertia;

m_i - the i-th ball mass;

M - body weight with inside mill devices;

I - moments of body inertia with inside mill devices;

$\vec{U}$ and $\vec{\Omega}$ - accordingly body linear and angular velocity;

Av, Bv, Cv - coefficients depending on the radius vector of the ball impact.

Let'add the equation in accordance with the hypothesis of Newton:

$$\left(\vec{V}_{i2}-\vec{U}_2\right)_n = -k\left(\vec{V}_{i0}\right)_n.. \quad (21)$$

Let's write the equation for the force pulse using Coulomb's law which binds the force of normal pressure and friction:

$$S_\tau = fS_{n1}. \quad (22)$$

We write the condition of equality of the tangential velocities of the points of the ball and lining touched at the time of application of the zero friction force. Since the velocity of any point on the surface of the ball may be represented as a geometric sum of the rates the cm and speed of the ball in the rotational motion around the axis passing through the cm the condition for zero friction force can be written as:

$$\left(\vec{V}_{i1}-r_{si}\cdot\vec{\omega}_{i1}\times\vec{n}\right)=\left(\vec{U}_1-Av1\cdot\vec{\Omega}_1\times\vec{n}\right), \quad (23)$$

where *Av1* - coefficient depending on the radius vector of the ball at collision.

Thus, we obtain a closed system of equations (20) (23), describing ball and drum collision. Solving this system we find the values of normal and tangential impulse:

$$S_n = \frac{-(1+k)\left(V_{i0n}-\Omega_0\left(x_s\cdot n_y - y_s\cdot n_x\right)\right)}{\dfrac{1}{m_i}-\dfrac{\left(x_s\cdot n_y-y_s\cdot n_x\right)}{I_z}-\eta\dfrac{\left(x_s\cdot\tau_y-y_s\cdot\tau_x\right)}{I_z}\left(x_s\cdot n_y-y_s\cdot n_x\right)};$$

$$\eta = \min\left\{f, \frac{V_\tau\left[\dfrac{1}{m_i}-\dfrac{\left(x_s\cdot n_y-y_s\cdot n_x\right)^2}{I_z}\right]+V_n\left[\dfrac{\left(x_s\cdot\tau_y-y_s\cdot\tau_x\right)}{I_z}\left(x_s\cdot n_y-y_s\cdot n_x\right)\right]}{V_n\left[\dfrac{7}{2}\cdot\dfrac{1}{m_i}-\dfrac{\left(x_s\cdot\tau_y-y_s\cdot\tau_x\right)^2}{I_z}\right]+V_\tau\left[\dfrac{\left(x_s\cdot\tau_y-y_s\cdot\tau_x\right)}{I_z}\left(x_s\cdot n_y-y_s\cdot n_x\right)\right]}\right\};$$

$$S_\tau = \eta S_n$$

Using these expressions for shock pulse and the position that the power consumption of the mill is the change in kinetic energy of the drum [12], we obtain the formula for calculating the capacity of mills with devices inside body:

$$P = \frac{\sum_{j=1}^{n}\Delta E_{i\tau}^b}{t},$$

where t - time of calculated power, n - number of collision, $\Delta E_{i\tau}^b$ - kinetic energy change at the j-th collision of i-th ball with the mill body or inside body mill device is equal to:

$$\Delta E_{i\tau}^b = \frac{\left[S_n\left(x_s\cdot n_y-y_s\cdot n_x\right)+S_\tau\left(x_s\cdot\tau_y-y_s\cdot\tau_x\right)\right]^2}{2I_z}+\Omega_{0z}\left[S_n\left(x_s\cdot n_y-y_s\cdot n_x\right)+S_\tau\left(x_s\cdot\tau_y-y_s\cdot\tau_x\right)\right].$$

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RESULTS AND DISCUSSIONS

The calculations were made on the bases of the drum DxL = 0.45 x0. 8m; equipped with NMP [alfa] =67.5° and without it at a rate of [fi] =0.3 download g.m.; relative speed body [psi] =0.76 [psi]$_{cr}$; coefficient of restitution at collision k=0.9; coefficient of friction f=0.3; bulk density of the material g.m. in the form of balls: pl = 7800kg/m^3. Range g.m. had a ratio: 50 mm - 15.2%; 45 mm - 27.6%; 40 mm - 27.6%; 35% -16.0 mm; 30 mm - 13.6%. According to the results of numerical experiments carried out on the basis of the above mathematical description the position of g.m. inside the drum mill was determined [13-15]. Knowing the movement of each ball in the transverse and longitudinal directions the efficiency of devices in terms of increasing the mobility of the ball load was determined. For this purpose an indicator such as the average path g.m. equal to the sum of the cm displacement all g.m. per second divided by the number of g.m. was used.

The calculations showed that in the case with NRM the value of the averaged longitudinal path g.m. per second is 0.136 m and is 2 times higher than this indicator for the body without equal devices 0.066 m (Figure-3). The average value of the cross-way of g.m. in the drum equipped with IID equal to 0.309 m, is also in excess of 1.17 times. The average full path of g.m. increases in the case of the installed IID 1.25 times and is achieved by the increasing of the longitudinal path of g.m. This characterizes the motion of g.m. in body with IID as more intense.

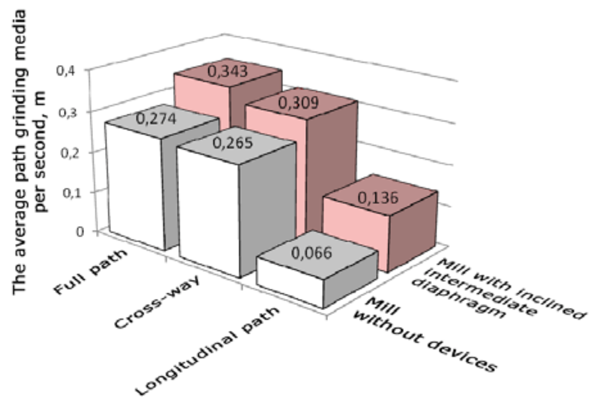

Figure-3. Average path of grinding media.

The assessment of the g.m. distribution in the direction of the longitudinal axis of the body was carried by their number in the selected volume (SV), which represent the areas bounded by the body and two planes perpendicular to its axis of rotation. According to the given the size of the body and g.m., the distance between the planes is assumed to be 0.08 m, the secreted volume in the body is 0.01272m^3.

In the cylindrical body without the inside mill devices the change of its rotation angle and the number of g.m. in the selected direction of the longitudinal axis of the volume varies slightly - from 70 to 80 units (Figure-4). The quantity of g.m. is lower directly at the end bottoms - 60 to 70 pieces. This is explained by a less dense packing of g.m. in contact with the end bottoms.

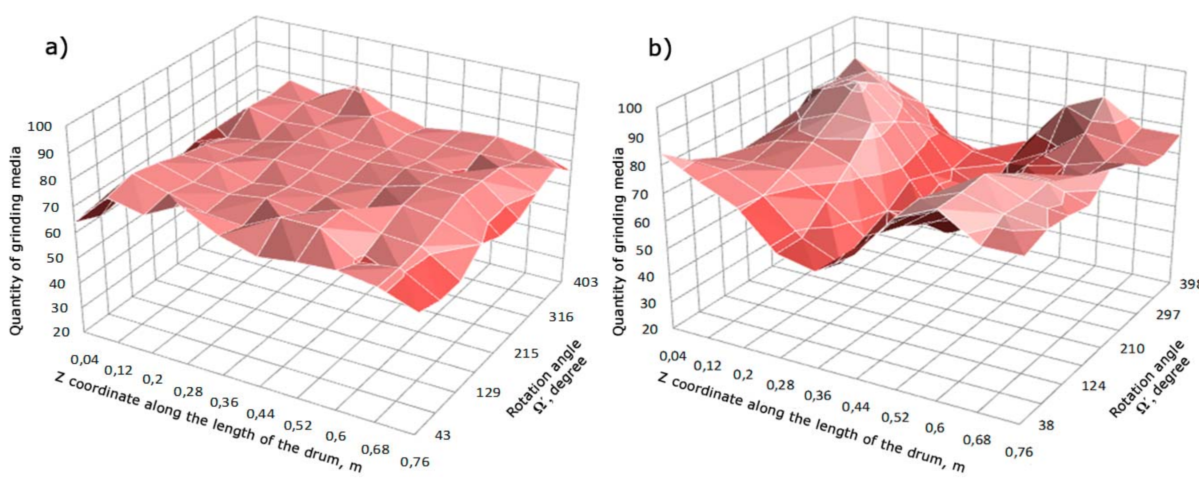

Figure-4. Longitudinal distribution g.m. on Z-axis direction in the mill body DxL=0.45x0.8м, [fi] =0.3; [psi] =0.76 a) without mill devices б) mill with IID.

When the IID is installed in the body during its turnover one maximum and one minimum quantitative change of g.m. is observed in isolated-represented volumes (Figures 4, b). One of the feature of the redistribution of g.m. at IID installation is a decrease of the number of g.m. in the central body to values 50 - 65 of g.m. highlighted by the volume and the increase of their number and magnitude of changes in the bottoms. In the preferred volume at the bottom (Z = 0.2 m and Z = 0.6 m) the number of g.m. changes in housing turnover from 68 to 92 unit. The distribution features of g.m. by rotating the enclosure with IIDe is thevidence of changing their mode of motion in the selected volumes. The place of intense exposure to IID grinding load extends over the entire length of the body.

The validation of the mathematical description of the process of movement g.m. performed on a test mill DxL = 0.45x0.5m with adjustable electric. Its body was machined together with cable ties to reduce the vibrations and the power fluctuations. The steel cylindrical device installed in the central holes in the end of the bottoms of the aligned body simulates the weight of g.m. The set of goods allowed to change a lot of devices, equivalent to a mass loaded into the mill g.m. at their load factor of 0.2 to 0.4 at intervals of 0.01. Power expended in motion g.m. housing is defined as the difference between the power consumed by the motor when loaded into the case g.m. and installation simulating weight of g.m. device.

The adequacy of the mathematical description of the parameters of the motion processes of g.m. and relations in the body were estimated according the the degree of relatedness values of the power expended on the g.m. motion and was obtained numerically and on the physical model. For this purpose the correlation ratio [eta] put forward by Pearson was uesd. [16] After the experiments the sample was made for variables obtained on the pilot plant by means of numerical experiments on the computer. Value [eta] was 0.96 indicating a close relationship between the experimental and numerical values and the adequacy of the mathematical description.

CONCLUSIONS

Ball mill is an efficient grinding units widely used for grinding materials. One of the areas of the improvement is the use of energy-exchange inside mill devices with different designs ensuring the reduction of specific energy consumption. Existing theories of g.m. motion do not provide sufficient quantitative and qualitative characteristics of their kinematic and dynamic parameters in the mill body equipped with energy exchange devices.

The mathematical description of the parameters of the process of g.m. movement in mill body with IID is developed allowing to expect coordinates, velocities centers of mass, angular velocities and collision energy of g.m. at any instant.

The analytical expression allows calculating the power consumed by the g.m. motion in the body with IID.

The software based on the mathematical description of the parameters of the spatial movement processes of g.m. and their interaction in the rotating mill body with IID was used. The computer numerical calculation of the kinematic and dynamic characteristics of g.m. in a rotating mill body DxL=0.45x0.8 m with or without IID was carried out. The regularities of kinematic parameters of g.m. which characterize the mill with IID as providing more intensive their movement were shown.

The computer numerical and physical experiments with mill DxL = 0.45x0.5m without inside mill devices was conducted to determine the power consumed by the g.m. motion under varying [fi] and [psi]. According to the correlations put forward by Pearson the close relationship between experimental and calculated values was established.

Findings

The carried out mathematical description adequately describes the process parameters of the spatial movement of the grinding media in a rotating mill body equipped with IID and without it; interaction inside mill device; allows to determine the kinematic and dynamic characteristics as each of grinding media and their groups.

The estimation of the values of the averaged path of grinding media in mill body with IID per second is 1.25 times in excess of the same indicator for the mill body without devices which characterizes the movement of g.m. in mill body with IID as more intensive.

The distribution amount of g.m. changing by the rotating mill body with IID characterizes the modes of their movement as changing. These changes have less extent in the mill without energy exchange devices.

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