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THE ANALYSIS OF STRATA FORM IN HOLOGRAPHIC EMULSION AND A VIEW OF AN IMAGE RECONSTRUCTED WITH A FOURIER-HOLOGRAM

A. G. Prygunov[1], S. A. Sinjutin[2], A. A. Prygunov[3] and E. S. Sinjutin[2]
[1]Department of Radio Electronics, Rostov Technological Institute of Service and Tourism, Southern State University of Economics and Service, Russia
[2]Department of Microprocessing Systems, Taganrog Institute of Technology, Southern Federal University, Russia
[3]Office of Operational Business Improvement, ARMZ Uranium Holding Co. (JSC 'AtomEnergoProm'), Russia
E-Mail: suncat_75@mail.ru

ABSTRACT

This paper examines the process of image reconstruction with volume Fourier-hologram and Fourier semi-transparent reflective hologram used in optical interferometer on the basis of spatial-spectral method of the holographic interferometry. The paper describes the configuration of strata in emulsion of holograms of this type. We have obtained the mathematical expression allowing calculating numbers of strata in a Fourier-hologram emulsion. The relation for a cumulative distribution function of space frequencies of reflection is also illustrated herein. It has been demonstrated that, with the placement of reflective mirror in immediate proximity behind the semi-transparent reflective-type Fourier-hologram and under a small angle to it, the interferogram in the form of elliptic-shaped strata is formed on a screen placed in front of this Fourier-hologram. It has been established in mathematical terms that for a mirror placed behind hologram, increase of mirror inclination angle in relation to the hologram results in reduction of linear dimensions of a central light spot and in increment of elongation of image ellipses. Interference holography can be used in the optical system of fiber-optic gyroscope (FOG). Processing interference pattern with FOG hologram increases the resolution of FOG to order and more, which in turn allows you to either improve the accuracy of the device while preserving the length of the optical fiber in a coil, or reduce the length of the optical fiber at a lower resolution of FOG. Both approaches allow to reduce the cost of FOG and accordingly to make it acceptable to use civilian vehicles.

Keywords: holographic interferometry, spatial spectral method, the Fourier-hologram, hologram emulsion, paraboloids of strata rotation, mirror incline angle, cumulative distribution function.

1. INTRODUCTION

The research results outlined in this paper were obtained with financial support from Ministry of Education and Science of the Russian Federation, as part of the execution of the project entitled "Establishment of high-tech production for the manufacture of complex reconfigurable systems of high-precision positioning of objects on the basis of satellite systems of navigation, local networks of laser and microwave beacons and technology MEMS", pursuant to decree of the government of the Russian Federation № 218 issued on April 09, 2010.

Research and development works in the field of holographic interferometry, conducted in last two decades, open new perspectives for wide practical use of holographic interferometers which have demonstrated the clear advantages in actual application in comparison with usual optical interferometers [1, 2, 7, - 10]. Being less complicated in technical implementation of optical channels, and differential to the errors caused by optical schematic components, holographic interferometers allow to ensure a higher responsivity and higher measurement accuracy. Spatial spectral method [7 - 10] is regarded to be the most promising and advanced for practical use among holographic interferometry methods. This method provides high responsivity to normal, tangential and angular movements of a controlled object, and it allows to form informational interferograms on the plane of photodetectors placed some distance away from the optical axis of signal light flux incident upon the reference Fourier-hologram, which considerably simplifies the analysis and processing of interferogram, and provides possibility to receive output information in digital form in real time. This opens wide possibilities of holographic interferometry spatial spectral method use for solving a number of application tasks which are connected with high accurate measurement of various physical values. For example, the given method can be used in developing and creating inertial navigation devices with the improved technical characteristics.

It can be noticed that holographic interferometry spatial spectral method is based on reference Fourier-hologram properties use.

Thus, the task of analysis of strata form in reference Fourier- hologram emulsion and a process of Image reconstruction in holographic interferometer on the basis of spatial-spectral method of the holographic interferometry have practical importance.

To analyze the strata form in the reference Fourier-hologram holographic emulsion and reconstructed image, we will examine Figure-1 with the following legend used: г - the reference Fourier-hologram; з - a reflective mirror; α - an angle between the hologram and a mirror; d - layer thickness of photographic emulsion; x', y', z' and x, y, z - the axes of the Cartesian rectangular coordinates; Si and Si* - real and virtual point light sources respectively; 0_α - the point where reflective mirror

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rotation axis passes through and around which it can rotate; r_a - distance from the beginning of coordinates to mirror rotation axis.

2. PROBLEM STATEMENT AND PRELIMINARIES

The upper plane of hologram and the upper plane of mirror intersect in point 0_α, which represents a projection of mirror rotation axis to the plane of this drawing. The mirror can be turned at an angle α relative to hologram plane and the hologram remains immobile. The axes of Cartesian coordinate systems on Figure-1 have the beginning in the point 0 and they form the right-handed frame. The first Cartesian system of coordinates has axes 0x', 0y', 0z' and it is immobile in relation to the mirror. The axis 0x' is located on the plane of mirror and it passes through point 0_α. The axis 0z' is perpendicular to the upper plane of mirror. The axis 0y' is perpendicular to the plane of this drawing and it is directed depthward. Second Cartesian coordinate system has axes 0x, 0y, 0z and is mobile in relation to the hologram, since its beginning is located in point 0, which can move together with the mirror. In addition to it, axis 0z is always perpendicular to the hologram plane, and axis 0x is parallel to the hologram plane. The hologram itself remains immobile in space, therefore orientation of the axes 0x, 0y, 0z does not change when mirror turns.

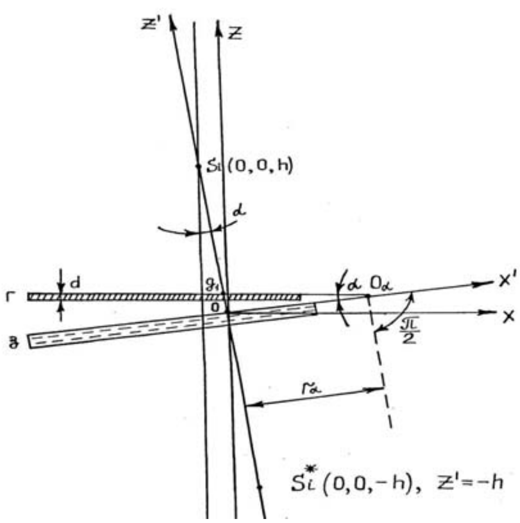

Figure-1. Analysis of strata form in reference Fourier-hologram emulsion and a process of Image reconstruction.

Surfaces of blackening in the form of two-sheet hyperboloids of rotation [6, 8] appear in the emulsion material of holographic photoplate as a result of light interference, when photoplate is exposed.

For the optical scheme shown on Figure-1, the shape of surfaces of blackening (strata) is described by the following expression:

$$\frac{(z')^2}{h_n^2} \quad \frac{(x')^2 + (y')^2}{(h^2 - h_n^2)} = 1 \quad , \qquad (1)$$

where

$$0 \leq h_n \leq h \; ; \; h_n = \frac{\lambda}{2} n \; ; \qquad (2)$$

n - the number of a surface of blackening, and where the value of n=0 refers to a degenerated case (when two-sheet hyperboloid turns into a plane with z'=0). As it can be seen from *Fig.1*, the plane z'=0 passes through a mirror rotation axis, and the focal points of surfaces of blackening (1) are located in points z = h and z = -h. Position of point with coordinates x ' =0, y ' =0, z ' =h determines the real point source Si when hologram is exposed. The position of point x ' =0, y ' =0, z ' = -h determines virtual light source Si* when hologram is exposed.

Virtual source Si* appears as a result of reflection of light from a mirror surface. All vertexes of two-sheet hyperboloids are located on z' axis, which geometrically serves for them as a rotation axis. The distance between vertexes of two neighboring hyperboloids in the coordinate system 0x'y'z ' is equal to $\frac{\lambda}{2}$, as it follows from expressions (1) and (2). The mirror rotation axis (with its projection in point 0_α) is set a distance of r_α from point 0 (Figure-1). Coordinates of Cartesian system 0xyz are connected to coordinates of Cartesian system 0x'y'z' through the following relations:

$$x = x' \cos\alpha - z' \sin\alpha;$$
$$y = y';$$
$$z = x' \sin\alpha + z' \cos\alpha \, . \qquad (3)$$

The crosspoint between axis **z'** and the upper plane of hologram is marked as g_1, and segment length $0g_1$ we denote as L and it can be found from this expression:

$$L = r_\alpha \operatorname{tg}\alpha. \qquad (4)$$

The surface number n_1, the vertex of which can be situated on the upper plane of the hologram, is found by division of L value by $\frac{\lambda}{2}$

$$n_1 = \frac{2r_\alpha}{\lambda} \operatorname{tg}\alpha \, . \qquad (5)$$

For small angles α the expression (5) can be written as [6]:

$$n_1 = \frac{2r_\alpha \alpha}{\lambda}. \qquad (6)$$

With r_α = 0, 05 m, λ = 1 µkm and with mirror rotation angle of 5°, L value is equal to 4375 µkm, and the number n_1=8750.

Let's consider and describe the process of reconstruction of an image exposed in volume Fourier-

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hologram material. According to [5] we consider that the hologram thickness parameter Q takes the value greater than or equal to 10. If we use thickness parameter Q, given in [5], and the relations presented in [5, 8], then the minimal thickness of volume hologram emulsion (with λ = 1 μkm) will come up to 0, 5 μkm approximately. It indicates that the material of thick hologram exposed in accordance to Figure-1, will have surfaces of blackening (strata) numbered from 8740 to 8750, i.e. from one stratum to ten strata in total.

With a focal distance h > 1m the strata in the material of hologram emulsion can be presented in the form of rotation paraboloids [6, 8]:

$$z_n' = h_n\left[1+\frac{1}{2h^2}(x^2+y^2)\right] \tag{7}$$

Relative error of such expression is less than

$$\left(\frac{L}{h}\right)^2 \times 100\% , \tag{8}$$

I.e. the order of 0, 0025 %.

Figure-2 illustrates one of the strata in the form of paraboloid, exposed in the emulsion of reference Fourier-hologram. The paraboloid rotation axis z' forms an angle α with the normal to the upper plane of hologram. The rotation radius of surface of blackening varies from zero to value ρ' not exceeding $h tg\beta \approx h\beta$. The angle β shows the direction of light emmision of point light source Si. The light rays that characterize light emmission direction of the source Si are directed to the hologram and marked with arrows. The dotted line denotes the paraboloids which are located outside layer d of the hologram emulsion. The dotted line also shows an extension of paraboloid surface which does not have silver blackening. The rays reflected from the surface of blackening (stratum) in Fourier-hologram emulsion will be directed upward. And these rays are parallel to the axis z' according to the properties of geometric optics [4], and under a condition that the light source is situated in the focus of parabola Si.

If we put the greatest value of rotation radius of surface of blackening in a Fourier hologram emulsion in formula (7), we will get the expression:

$$z_n'(\rho') = \left[1+\frac{1}{2}tg^2(\beta)\right] \tag{9}$$

The augend in square brackets of expression (9) defines the relative measure of inclination of reflective strata surface from the plane.

With β = 8° the value of tg β = 0, 1405, and stratum form inclination from the plane at surface edge is less than 1%.

The analysis of expressions (7) and (9) allows us to make a conclusion that the higher the sharpness of emission directionality of point source during Fourier-hologram exhibiting - the closer surface of blackening (stratum) is circumscribed by a circle, the plane of which is inclined at angle α in the hologram emulsion.

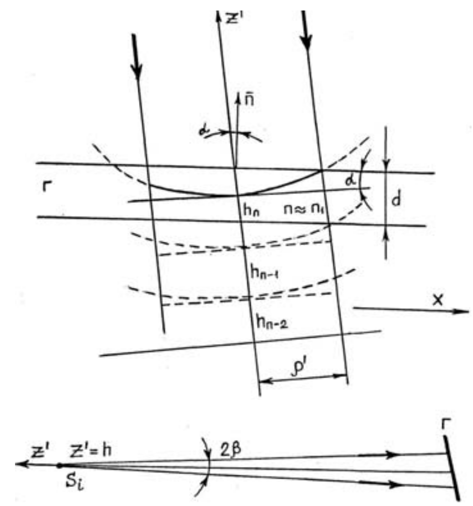

Figure-2. Principle of strata forming in emulsion of the reference Fourier-hologram.

According to geometric optics laws with reference to Figure-2 [4] each stratum forms a light spot of elliptic shape on plane z = 0. The major semiaxis of the spot is equal p', and the minor semiaxis is equal to p'cosα. The elliptic spot is prolated along axis y. Calculation of an optical field on elliptic spot plane is provided in detail in [8].

Within the framework of Fraunhofer difraction theory the amplitude of the electric field strength, which is formed as a result of light reflecting from surface of blackening (strata) of a given number, is described in screen plane by expression [8]:

$$E_n(x,y,z) = B(z')F_n(\omega_x,\omega_y)e^{-j\omega t} \tag{10}$$

where B(z') - is an exponential factor, and where

$$B(z') = B_0\frac{1}{\lambda z}\exp[jk(z\cos2\alpha - x\sin2\alpha)] ; \tag{11}$$

n = the number of surface of blackening;
n_1 = the number of the upper surface of a blackening;
z = distance to the screen;
B_0 = initial amplitude of incident emission;
j = imaginary unit;
$B(z')e^{-j\omega t}$ = plane wave, propagating along axis z';
$F_n(\omega_x,\omega_y)$ = spatial spectral reflectance function.

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Front of wave reflection forms an angle 2α with positive direction of axis x.

Function $F_n(\omega_x, \omega_y)$ describes the diffractional effects that occur at reflection of light and it is function of two non-dimensional variables which has meaning of space frequencies:

$$\omega_x = \frac{2\pi x}{\lambda z};\ \omega_y = \frac{2\pi y}{\lambda z}. \quad (12)$$

Cumulative distribution function values of space frequencies of reflection

$F_n(\omega_x, \omega_y)$ are calculated with the aid of the relation [8]:

$$F_n(\omega_x, \omega_y) = \frac{2\pi ab}{æ r}\left[U_R(1 - U_R)^{2(n_1 - n)}\right] J_1(æ r), \quad (13)$$

where

U_R = is the non-dimensional coefficient of single reflection which may take on the value $0 < U_R \ll 1$;
$J_1(æ r)$ = Bessel function of the first kind;
n_1 = a number of the upper reflective surface (stratum) in holographic emulsion , defined by the formula (5);
n = number of reflective surface (stratum) in holographic emulsion, and where $n \leq n_1$, and herewith increase of n decreases the value of reflected wave amplitude; parameter $æ$ is a value reciprocal of non-dimensional wave number of diffraction D [1,3, 8] and it is found by using the following formula:

$$æ = \frac{2\pi ab}{\lambda z}. \quad (14)$$

It should be noted here that the parameter $æ$ takes on small values, since Fraunhofer diffraction is considered here. The non-dimensional value r in the relation (13) is obtained from the following expression:

$$r = \sqrt{\left(\frac{x}{b}\right)^2 + \left(\frac{y}{a}\right)^2}. \quad (15)$$

This value r has mathematical meaning of radius in the generalized polar coordinates on a plane xoy [6, 8].

Let's put in a polar coordinate system to use in a plane:

$$x = c\, b \cos\Theta,\ y = c\, a \sin\Theta, \quad (16)$$

where $c \geq 0$, and Θ is an angle between radius-vector coming out of point 0, and axis x.

After substitution (16) for (15) we will get r = c. Relations (16) describe an ellipse in a parametric form.

The resultant field on the screen plane is formed by the aggregate of fields from separate reflectors. In accordance with Bragg, requirements for summation of separate fields diffracting from elliptic-shaped strata in the reference hologram emulsion, are examined in [8]. For a Fourier-hologram of reflective semi-transparent type, as it can be seen on Figure-1, a spherical wave from virtual point source Si* makes an additional contribution to the resulting field on the screen plane. According to the laws of geometric optics [4], the interference (on the screen parallel to the hologram plane) of a spherical wave from virtual source Si* and of wave with ellipsoidal front, formed as a result of light diffraction on the reflective semi-transparent Fourier-hologram, determines elliptic spatial spectral distribution of intensity of an optical field in interferogram on the image plane.

The eccentricity of elliptic fringes of the interferogram formed in such way is determined, as it is shown in [8], by an eccentricity of elliptic reflective fringes (strata) in the emulsion of reference Fourier-hologram.

From expressions (13), (15) and (16) it can be seen that ellipses on the plane xoy (*Fig. 1*) describe the lines of fixed level of cumulative distribution function of space frequencies of reflection $F_n(\omega_x, \omega_y)$. Semiaxes of ellipses (a, b) in the expressions (13), (14), (15) and (16) are found from the following formulas:

$$a = h\, tg\beta\, \cos\alpha;\ b = h\, tg\beta. \quad (17)$$

The analysis of formulas (17), with Figure-2 taken into consideration, show that with decrease of angle β the directionality of the source increases, and linear dimensions of surfaces of blackening (strata) in hologram emulsion reduce.

It is important to note that in practice, there are certain limits on data recording density in the hologram [2, 3, 5], however, increase of directionality of light source in addition to increase of record density essentially simplifies the mathematical model describing an image reconstruction by means of a simpler theory of plane mirrors.

The position of a central maximum of diffraction [8] is determined by using the first non-zero root of Bessel function ($J_1(3,832) = 0$). Assuming x = 0 in the expression (13), we determine the boundary of central maximum on the axis y. And assuming that y = 0 in the expression (13), we determine the boundary of central maximum on the axis x. As a result of that we get the following:

$$y_1 = 3{,}832 \times \frac{\lambda z}{2\pi h\, tg\beta};$$
$$x_1 = 3{,}832 \times \frac{\lambda z}{2\pi h\, \cos\alpha\, tg\beta}. \quad (18)$$

The analysis of expressions (18) shows that increase of mirror inclination angle α in relation to the hologram reduces linear size of the central spot along axis x and increases an oblongness of image ellipses along axis y.

3. CONCLUSIONS

The number of reflective (strata) in emulsion material of a volume Fourier-hologram does not exceed few tens, and elliptic-shaped strata direct light fluxes of elliptic shape into the plane of main image. Thus, in a holographic interferometer, on the basis of a spatial spectral method and as a result of interference of light stream diffracted from the reference Fourier-hologram, and interference of light flux generated by a virtual light source, the interferogram with elliptic spatial spectral distribution of optical field intensity is formed on the screen plane, in front of Fourier-hologram. With increase of directionality of light stream which is used for exposing hologram, linear dimensions of surfaces of blackening (strata) decrease, whereas density of holographic recording increases, and the mathematical model describing the process of image reconstruction becomes simpler. In the process of image reconstruction, increase of inclination angle of the mirror placed behind Fourier-hologram, in relation to the hologram reduces linear dimensions of a central light spot but increases an oblongness of image ellipses in the screen plane placed in front of the Fourier-hologram.

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