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CERTAIN ALGEBRAIC TESTS FOR ANALYZING APERIODIC AND RELATIVE STABILITY IN LINEAR DISCRETE SYSTEMS

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ABSTRACT

Simple algebraic test procedures are presented to analyze aperiodic as well as relative stability in a given stable linear time-invariant discrete system represented in the form of its characteristics equation $f(Z) = 0$. The proposed schemes are applied to various illustrations.

Keywords: linear discrete systems, stability analysis, aperiodic stability, relative stability, necessary condition.

INTRODUCTION

For a given absolutely stable linear time-invariant discrete system represented by its characteristics equation $f\,(Z) = 0$, with all the roots having z < 1, the aperiodic and relative stability can be obtained using either controller or compensator in the given system. In general, in instrumentation systems aperiodic stability is important [4] and relative stability is used in the design of two-dimensional digital filters [5, 6]. To infer the above situations, simple test procedures are suggested below.

LITERATURE SURVEY

Stability criterion for a continuous-time polynomial was established long time ago. These results were generalized for Hurwitz stability robustness property recently where a quick qualitative measure of stability robustness of more general types of polynomials (polyopic and multi-linear) than those that can be handled by Khqritonov theorems (interval polynomials) can be easily obtained. Similar results for discrete-time polynomials are even more desirable, for in this case even for the simplest case of interval polynomials; one has to do the computationally extensive edge polynomial stability checks. Nour-Eldin introduced Markov-like parameters for discrete-time case and established a stability criterion based on these parameters. This criterion was simplified first and then further in attributed.

A frequency-domain graphical (hodograph) approach was presented for left-sector (relative) stability robustness analysis of a given real or complex nominal polynomial was discussed by Katbab and Jury [8] which have to be tested for having interlacing real zero property. The Schur testing of the required edges of the cube was performed using three different methods, namely, the critical edge polynomial, edge stability as an eigen value problem, and edge stability using co-linearity conditions and comparison of these three methods was discussed by Kraus Mansour and Jury [9]. Katbab and Jury [6] in their Generalization to multidimensional digital filters with real and complex coefficients proved that the Applications of the results may be found in robust stability analysis of digital signal processing and control theory. Katbab and *et al.* [7] investigated on Markov-like parameters and defined for a discrete-time polynomial and proved that these results are generalized for Schur invariance property, and the maximum allowable variation in the associated parameters are obtained via evaluating some comer points with the results gives a quick qualitative measure of stability robustness of discrete-time polynomials. Karivaratha Rajan and Reddy [5] had corroborated procedures to test discrete scattering Hurwitz polynomials and simplified the complexity of the test procedure. The number of polynomials required to check robust stability is one, two, and three, respectively, instead of four. Furthermore, it is shown that for n 2 6, the number of polynomials for robust stability checking is necessarily four, thus further simplification was a proceeding given by Anderson and *et al*. [1]. Jury [4] proposed a note of aperiodicity of linear discrete system and corrected in corollary by Szaraniec and deals with both distinct and multiple roots in the interval.

Byrne [2] had revealed that Fitts' third-order counter example to Aizerman's conjecture is false for some values were proved mathematically. A simplified version of the conditions for the analytical absolute stability test was introduced by Jury [3] and revealed that the absolute stability test polynomial have no positive real zero. Fuller [10] proposed a methodology to check whether a control system is having the dead-beat condition (Aperiodic stability analysis) and revealed that the characteristic equations with real coefficients can be transformed into characteristic equations with complex coefficients.

APERIODIC STABILITY ANALYSIS

If the characteristics roots of $f(Z) = 0$ lie in the sector region $0 \leq Z < 1$ and all are simple (distinct), then the system becomes aperiodically stable since all the roots are positive then the system characteristic equation can be written as

$$F\,(z) = (z - x_1)\,(z - x_2) \ldots .\ (z - x_n) = 0 \qquad (1)$$

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Where, x_i are distinct roots of $f\ (Z)\ =\ 0$.

In general, the equation (1) can be arranged as

$$F\ (z) = Z^n - (x_1 + x_2 + x_3 + \ldots .\ x_n)Z^{n-1} + (x_1x_2 + x_2\,x_3 + \ldots .\ )Z^{n-2} - (x_1x_2x_3 + x_3\,x_4x_5 + \ldots .)Z^{n-3} + \ldots . + (x_1x_2 \ldots .x_n) = 0 \qquad (2)$$

From the equation (2), it is observed that the necessary condition for aperiodic stability, the sign of $Z^{|n-k+1|}$ must alternate; in other words the coefficients of $f\ (Z)\ =\ 0$ should alternate in sign.

The equation F (Z) is rewritten for simplicity as

$$F(z) = z^n - a_{n-1}z^{n-1} + a_{n-2}z^{n-2} - \ldots + a_0 = 0 \qquad (3)$$

with $Z\ =\ -Z$, the coefficients of equation (3) will become positive in the sector region, $-1\ <\ Z \leq 0$.

To test the aperiodic stability of a given linear time-invariant continuous system represented in the form of its characteristics equation,

$$F(s) = s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + \ a_0 = 0 \qquad (4)$$

Fuller [10] formulated a transformed equation of f(s) = 0 as

$$F(s) = f(s)|_{s=s^2} + s\left.\frac{df(s)}{ds}\right|_{s=s^2} = 0 \qquad (5)$$

The degree of F(s) is twice of f(s) and the Routh's test [1, 2] is applied for the equation (5). If the first column of Routh's table does not possess any sign change, then the system represented by the equation (4) is aperiodically stable. Thus, extending Fuller's idea, the following transformed equation is written for the equation (3) as

$$F(Z) = f(Z)|_{Z=Z^2} + Z\left.\frac{df(Z)}{dZ}\right|_{Z=Z^2} = 0 \qquad (6)$$

Thus, this transformed equation (6) can be handled by Routh's test verify the sufficiency condition for aperiodic stability in the Z-domain.

Thus, above proposed procedure is applied for the following illustrative examples.

Illustrations

Example-1

Consider a stable system having

$$F\ (z)\ =\ z^2 -\ 1.1z\ +\ 0.3\ =\ 0$$

Since the coefficients of f(z) alternate in sign, the necessary condition is satisfied for this $f\ (z)$ with $Z = -Z$ the transformed equation is formed as

$F\ (z) = (z^4 +\ 1.1z^2 +\ 0.3) +\ (2z^3 +\ 1.1z) =\ 0$

To test for sufficiency condition the Routh's table is formed for $f\ (z)\ =\ 0$.

1	-1.1	0.3
2	1.1	0
0.55	0.3	
0.01	0	
0.3		

Since all the elements in the first column of Routh's table are all positive it is assumed that the system is aperiodically stable, with f (z) =0.

Example-2

Let $f(z)\ =\ z^2 – 0.5z\ +\ 0.8\ =\ 0$

with $z\ =\ -z$, $f(-z)\ =\ z^2 +\ 0.5z\ +\ 0.8\ =\ 0$

The Routh table is formed

1	0.5	0.8
2	0.5	0
0.25	0.8	
$\frac{1.475}{0.25}$	0	
0.8		

Since there is a sign change in the first column of Routh's table, the discrete system represented by f(z)=0 is not aperiodically stable.

Example-3

For a given stable system with

$$f\ (z)\ =\ z^2 – 1.1z\ +\ k\ =\ 0$$

The range for K is evaluated for aperiodic stability with $z\ =\ -z$.

$$f\ (-z)\ =\ z^2 +\ 1.1z\ +\ k\ =\ 0$$

The Routh table is formed

1	1.1	k
2	1.1	
0.55	k	
$\frac{0.605 - 2k}{0.55}$		
k		

From the first column of the Routh Table, for aperiodic stability it is formed that

ARPN Journal of Engineering and Applied Sciences

(i) $k > 0$
(ii) $(0.605 - 2k) > 0$

(Or) $k < 0.3025$

Thus the range for K is obtain as $0 < k < 0.3025$

Note

In the formulation of Routh's table the coefficients of $f(Z) = 0$ are entered in the first row while the coefficients of $f'(-Z) = 0$ are entered in the second row. The remaining table is completed as per Routh's table.

RELATIVE STABILITY ANALYSIS

This analysis is important in the design of controller as well as compensator for stabilizing the output response in a quick manner; this factor mainly depends on the location of roots within unit circle. For absolute stability, $|Z_i| < 1$ [3]. Suppose these are designed situations for a given linear time-invariant discrete system [4] [7] having the characteristics $|Z_i| < \alpha < 1$ and $|Z_i| < \beta < 1$ respectively, and if $\alpha < \beta$, then the first designed situation is relatively stable than the second one. This implies that the characteristics roots of the first designed situation are nearer to origin then that of the second designed situation. The above is formation can be easily gathered with help of the coefficients of the characteristics equation $f(Z) = 0$ as discussed below.

Let

$$f_1(z) = z^n - s_1 z^{n-1} + p_1 z^{n-2} - p_2 z^{n-3} \ldots + p_n = 0 \quad (7)$$

and

$$f_2(z) = z_n - s_2 z^{n-1} + q_1 z^{n-2} - q_2 z^{n-3} \ldots + q_n = 0 \quad (8)$$

Where

s_1 = sum of all the roots
p_1= sum of product of the roots
p_2= sum of product of three roots
. .
. .
. .
p_n= product of all the roots

Similarly $s_2, q_1, q_2 \ldots. q_n$ represents the same in formations for the equation (8).

In general if

$$s_1 < s_2, p_1 < q_1, p_2 < q_2 \; and \; p_n < q_n \quad (9)$$

Then the first designed situation as given by the equation (7) is relatively stable [8] compared to the second situation dictated by the equation (8). In a lighter sense, this indicates that all the roots of $f_1(Z) = 0$ lie in the region nearer to the origin of unit circle compared to the origin enclosed by the roots of $f_2(Z)=0$. Thus the proposed procedure is simple to apply for relatively stability study in the z-domain as illustrated in the following examples.

Illustrations

Example-1
For a given linear time-invariant discrete system, let

$$F_1(z) = z^3 - 0.6z^2 + 0.28z - 0.04 = 0$$

$$F_2(z) = z^3 - 0.9z^2 + 0.38z - 0.06 = 0$$

$$0.6 < 0.9, \quad 0.28 < 0.38, \quad 0.04 < 0.06$$

Applications of the conditions given in the equation (9), it is inferred that the designed situation given by $f_1(Z) = 0$is relatively stable compared to the designed situation shown by $f_2(Z) = 0$

Example-2

Let a discrete system be designed and its three situations are specified as

$$F_1(z) = z^3 - 0.1z^2 + 0.04z + 0.02 = 0$$

$$F_2(z) = z^3 - 0.1z^2 + 0.14z + 0.04 = 0$$

$$F_3(z) = z^3 - 0.2z^2 + 0.02z + 0.02 = 0$$

Applying the results indicated by the equation (9), It is inferred that the first designed situation is relatively stable compares to the other two situations as indicated by $f_2(Z)$ and $f_3(z)$.

Example-3
Let two all pole systems be

$$G_1(z) = \frac{1}{z^2 - 0.6z + 0.3}$$

and

$$G_2(z) = \frac{1}{z^2 - 0.6z + 0.4}$$

Applications of the result given by the equation (9) to the denominator polynomial of$G_1(z)$ and $G_2(z)$, it is inferred that $G_1(z)$ is relatively stable system compared to $G_2(z)$.

Example-4 [11]
For a given unstable two-dimensional digital filter

$$H(z_1, z_2) = \frac{1}{1 - 0.95z_1 - 0.95z_2 - 0.5z_1 z_2}$$

The three designed situations for stabilization are given below:

a) $H_1(z_1, z_2) = \frac{1}{1 - 0.95z_1 - 0.55z_2 + 0.5z_1 z_2}$

b) $H_2(z_1,z_2) = \frac{1}{1-0.55z_1-0.95z_2+0.5z_1z_2}$

c) $H_3(z_1,z_2) = \frac{1}{1-0.55z_1-0.55z_2+0.5z_1z_2}$

As shown in future chapter, the respective one-dimensional equivalent systems are given below.

$$E_1(z) = \frac{1}{z^2 - 1.45z + 0.5} = E_2(z)$$

$$E_3(z) = \frac{1}{1 - 1.1z + 0.5}$$

Application of the proposed results given in the equation (9), it is observed that $E_3(z)$is relatively stable compared to $E_1(z)\ and\ E_2(z)$.

RELATIVE STABLITY WITH NECESSARY CONDITION

(a): In this section an another algebraic scheme is proposed for relative stability with the help of necessary conditions as shown in below.

For the equations (7) and (8) with$z = 1$,

$$F_1(1) = [1 - s_1 + p_1 - p_2 + p_3 + \ldots. + p_n] = T_1$$

$$F_2(1) = [1 - s_2 + q_1 - q_2 + q_3 + \ldots. + q_n] = T_2$$

$$\left.\begin{array}{l} \text{If, } s_1 < s_2,\ p_1 < q_1,\ p_2 < q_2 \ldots. p_n < q_n \\ \text{then the value , } T_1 > T_2 \\ \text{along with } |f_1(-1)| < |f_2(-1)| \\ \text{with } p_n < q_n \end{array}\right\} \quad (10)$$

The above results can be applied for the inference of relative stability in a given linear discrete system.

(b): Suppose if

$$f_1(z) = z^n + s_1 z^{n-1} + p_1 z^{n-2} + \ldots. + p_0 = 0$$

and $f_2(z) = z^n + s_2 z^{n-1} + q_1 z^{n-2} + \ldots. + q_0 = 0$

$$f_1(1) = T_1$$

$$f_2(1) = T_2$$

then the conditions for relative stability are

$$\left.\begin{array}{l} \text{(i) } T_1 < T_2 \\ \text{(ii) } |f_2(-1)| > |f_1(-1)| \\ \text{(iii) } |p_n| < |q_n| \end{array}\right\} \quad (11)$$

The results in the equations (10) and (11) are applied for the following illustrations.

Illustrations

Example-1

Let $f_1(z) = z^2 - 0.4z + 0.2 = 0$

and $f_2(z) = z^2 - 0.6z + 0.3 = 0$

for $z = 1$ and $z = -1$

$f_1(1) = T_1 = 0.8$ and $f_2(1) = T_2 = 0.7$

$f_1(-1) = 1.6$ and $f_2(-1) = 1.9$ and $0.2 < 0.3$

The application of the equation (10) shows that $f_1(z)$ is relatively stable compared to the designed situation of the discrete system represented by$f_2(z)$.

Example-2

Let $f_1(z) = z^2 + 0.4z + 0.06 = 0$
and $f_2(z) = z^2 - 0.7z + 0.12 = 0$

for $z = 1$ and $z = -1$

$f_1(1) = 1.56 = T_1$ and $f_2(1) = 1.82 = T_2$

$f_1(-1) = 0.56$ and $f_2(-1) = 1.82$ and $0.06 < 0.12$

The application of the equation (11) shows that $f_1(Z)$ is relatively stable compared to the designed situation of discrete system represented by $f_2(z) = 0$.

CONCLUSIONS

In this paper, the analysis of aperiodic stability of a given stable linear discrete system is presented with the help of Fuller's equation and Routh's table while the relative stability of a system under different designed situations is studied using proposed algebraic schemes which involve only inspection tests. The presented test procedures are simple in applications and are illustrated with the help of examples.

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