EQUATIONS OF STATE OF PLASTICALLY DEFORMED POLYCRYSTALLINE MEDIUM CONSIDERING PLASTIC DEFORMATION THERMAL EFFECT

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ABSTRACT
The article suggests a modeling method and shows relations of state of a polycrystalline body under severe plastic deformation in temperature range outside the sphere of phase transformations, where the body constitutes heterogeneous medium created by two interacting components - continual medium and grain boundaries structure imbedded into this medium. Evolution of grain boundaries, temperature and internal energy connected with changes of stress-strain state inside of and at grain boundaries are regarded from the point of view of regulations of thermodynamics of irreversible processes.

Keywords: polycrystalline body, grain boundaries, severe plastic deformation, equations of motion, Gibbs energy, stress-strain state.

INTRODUCTION
Mechanical properties of steel and alloys to a large extent depend on construction of their grain structure formed at a stage of workpieces plastic shaping in hammering and presswork and subsequent thermal treatment. In this regard, to optimize deformation technological parameters and to obtain required properties of workpieces, it is important to model their microstructure. Polycrystalline material microstructure may be modelled directly, via dynamics of disloccation motion and crystals plasticity, and indirectly, via relations based on dislocation density. Models using dislocation density are developed in works of Bergström [1, 2, 3] and so on. Direct relations require extensive computing resources; therefore nowadays indirect models are the ones that are developed to the fullest extent. In the sphere of polycrystalline bodies evolution phenomenological modelling the most popular approach is the approach based on Avraami’s works [4]. We know a dynamical recrystallization model based on a mesoscale mechanism (MesoScaleUnits model), which gives more precise results, in particular for processes with repeated straining impact [5, 6]. The advantage of MSU-models and the same-name designator of the operator

\[ \frac{\partial a}{\partial x_j} = \delta_{ij}, \]

where \( \Delta_0 \) is change of local volume due to grain boundaries migration;
\( F_{\Omega_0} \) is force of interaction between two components of the deformed medium;
\( \rho \) is deformed medium density (taken as an invariant one). From now on in subindex at a variable \( a \) we accept designator of the operator \( a_{ij} = \frac{\partial a}{\partial x_i} \), so for components

\[ \begin{align*}
\sigma_{ij,j} + \sigma_{ij,j} - \rho[(1 - f_p)w_j - (f_p + \rho \Omega_0)\Delta w_j] = 0, \\
\sigma_{ij,j} - \rho(f_p + \rho \Omega_0^{(m,p)})\Delta w_j - F_{\Omega_0} = 0,
\end{align*} \] (1)
of tensor $a_{ij}$ in space $D_3$ taking into account Einstein notation for repeated indices is denoted as:

$$a_{ij} = \sum_j \left( \frac{\partial a_{ij}}{\partial x_j} \right).$$

Unknown variable of interaction $F_{\Omega}$ in (1) will be defined using methods of thermodynamics of irreversible processes. Total entropy $S$ in medium’s volume $\Omega$ limited by size of a grain (or several grains), with surface $A$, segregates into entropy $S_i$ changed by the environment via this surface, and irretrievably increasing entropy $S_i$. According to Clausius–Duhem inequality:

$$0 = \dot{S} - \dot{S}_i \geq 0.$$  

(2)

Inequality (2) transformed into a local shape at grain boundaries leads to the following inequality:

$$\dot{S}^{loc} = \dot{S}^{loc} + \left[ q_i - \rho c \Delta w_i \Delta T \right] \geq 0,$$

(3)

Omitting derivation of the energy balance equation, let’s put the first law of thermodynamics for local volume, solving it regarding internal energy $U$:

$$\dot{U} = \sigma^v \dot{e}^v + \sigma^w \dot{e}^w + F_{\Omega} [f_{\rho} \Delta w_i - (1 - f_{\rho}) v_i] - (q_i + \rho c T_v \Delta w_i)$$

$$\delta U = \rho c \dot{\Omega}_{0,i} [\Delta w_i \dot{\delta}_j + c \Delta T]$$

where $\{\delta_j | \delta_j = 0 \text{ when } i \neq j, \delta_j = 1 \text{ when } i = j\}$ is a Kronecker symbol.

From (4) it follows that change of internal energy is caused by the work of external forces (summand in first brackets), internal friction caused by grain boundaries moving (second summand), and also conducted and delivered heat (last summand). Second equation in (4) is connected with change of internal energy $\delta U$, caused by heat transport during grain boundaries migrations and change of mass $\delta \Omega_{0,i}$, located at grain boundaries.

$$F = (1 - f_{\rho}) \left[ \frac{\partial F^v}{\partial (e^v_j - \delta_j e^v)} (e^v_j - \delta_j e^v) + \frac{\partial F^w}{\partial (e^w_j - \delta_j e^w)} (e^w_j - \delta_j e^w) + \frac{\partial F^w}{\partial (e^w_j - \delta_j e^w)} (e^w_j - \delta_j e^w) \right] + S^{loc}.$$  

(5)

Let’s define $\frac{\dot{U}^{loc}}{T}$ in (4) via Helmholtz free energy $F = U^{loc} - S^{loc} T$, which will be represented in the form of a functional $F = (e^v_j, e^w_j, T, f_{\rho})$, such one that there is a linear combination $F = (1 - f_{\rho}) F^v (e^v_j, T) + f_{\rho} F^w (e^w_j, T)$. In this case change of free energy $F$ will be equal to:

(6)

where $\dot{e}^v_j = \dot{e}^v_j - \delta_j \dot{e}$ are components of rate-of-strain deviator, $\dot{e} = \frac{1}{3} \dot{e}^v_j \delta_j$.

Taking into account (4), (5), the inequality (3) modified to the local shape will be as follows:

As far as, according to (6), the following defining relations are true
then correspondence (10) is performed identically, if

\[ P_D = F_{\Omega} \left[ f_\rho \Delta w_i - (1 - f_\rho) \nu_i \right] \geq 0, \]

(8)

where \( P_D \) is a power dissipation function connected with expenditure of energy for structure formation caused by relative (over the field of velocities \( \Delta w_i \)) movement of grain boundaries.

Taking into account fulfillment of inequality (8), we can write expanding of \( F_{\Omega} \) in series:

\[ F_{\Omega} = \sum_{n=1}^{N} b_n \left[ f_\rho \Delta w_i - (1 - f_\rho) \nu_i \right]^{2n-1}, \]

(9)

which, considering the first member of the line when \( n = 1 \), it follows that:

\[ F_{\Omega} = b \left[ f_\rho \Delta w_i - (1 - f_\rho) \nu_i \right], \]

(10)

Free energy \( F \) is a scalar value of the invariant of state of rate of deformation and temperature. After finding values of partial derivatives of \( F \) at thermodynamic equilibrium point \((0,0,0,0,T)\) as characteristics of the material properties, which is equivalent to Taylor’s serial expansion coefficients similar to (6), we will find defining relations for the deformed polycrystalline medium:

\[ \sigma'_y = \frac{\partial F'}{\partial \varepsilon'_y - \delta_y \varepsilon'} + \frac{\partial F'}{\partial \varepsilon'} \delta_y, \quad \sigma''_y = \frac{\partial F''}{\partial \varepsilon'_y - \delta_y \varepsilon''} + \frac{\partial F''}{\partial \varepsilon''} \delta_y, \]

and inequality \(- \left[ q_i - \rho c \Delta w_i \Delta T \right] \Delta T_i \geq 0\) leads to implementation of heat-transfer law

\[ q_i = - \lambda \Delta T_i + \rho c \Delta w_i \Delta T, \]

(7)

then correspondence (10) is performed identically, if

\[ P_D = F_{\Omega} \left[ f_\rho \Delta w_i - (1 - f_\rho) \nu_i \right] \geq 0, \]

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(10)

Power dissipation, connected with thermophysical restructuring, is determined by internal forces capacity and change of heat flow inside of volume and passage of heat via its surface, which follows from the correspondence (4) for substantial derivative of the internal energy \( \dot{U} \):

\[ \dot{S} = \left[ f_\rho \Delta w_i \left( \frac{\partial f_\rho}{\partial u} + f_\rho \frac{\partial f_\rho}{\partial b} \right) \right] + \frac{\partial F}{\partial s} \Delta s + \frac{\partial F}{\partial \rho} \Delta \rho + \frac{\partial F}{\partial T} \Delta T \]

(11)

The last correspondence together with (11) and heat-transfer law (7) lead to a generalized equation for heat transfer in a polycrystalline medium:

\[ \dot{S}^* = \left[ f_\rho \Delta w_i \left( \frac{\partial f_\rho}{\partial u} + f_\rho \frac{\partial f_\rho}{\partial b} \right) \right] + \frac{\partial F}{\partial s} \Delta s + \frac{\partial F}{\partial \rho} \Delta \rho + \frac{\partial F}{\partial T} \Delta T \]

(12)

We can calculate the coefficient \( b \) from the polycrystalline medium motion law in the form (13) after analyzing (12) in stationary conditions of isothermal restraint at grain boundaries:

\[ b = \left(-1\right)^n \frac{\left( c_n^w - c_n^w \right) \Delta T_i}{(f_\rho \cdot \Delta w_i)^2}, \]

(13)

where \( n \) defines average direction of grain size change under dynamical recrystallization \((n = 0 - \text{grain is increasing}, \ n = 1 - \text{grain is decreasing})\).

Taking into account (13) and the fact that main volume of the deformed continuous medium falls on the continual constituent, and the structure formation coefficient \( f_\rho \) is of quantum nature, heat-transfer equation equals to the system:

\[ \begin{align*}
\mu_{\Omega}^v &= \frac{\partial F'}{(\hat{\varepsilon'} \hat{\eta} \hat{t})^2}, \\
\mu_{2p}^v &= \frac{\partial F'}{(\hat{\varepsilon'} \hat{\eta} \hat{t})^2}, \\
\end{align*} \]
\[ \lambda \nabla^2 T = c_{\alpha_0} \nabla^2 T + c_{\alpha_1} \dot{\varepsilon}^w - \text{inside of grain} \]

\[ (-1)^n \frac{(f_w^w \cdot \Delta w_i - (1 - f_w^w) v_j)^2}{(f_w^w \cdot \Delta w_i)^2} \delta_{ij} \nabla T = \frac{\dot{\varepsilon}^w}{\alpha_m} \quad \text{in local domain at grain} \]

We will find the coefficient of heat migration of grain boundaries \( \alpha_m \) using the phenomenological approach and supposing that grain size is proportional to deformation \( \varepsilon \) and some reference temperature found experimentally.

Let's present the second equation in (14) in the differential form

\[ d \varepsilon_i^w = (-1)^n \alpha_m \left(1 - \frac{f_w^w}{f_p^w} \right)^2 dT, \quad f_{\alpha_i} = \frac{\Delta w_i}{v_i} \]

and replace differentials with finite increments, and after transformations we will obtain the system of equations describing the process of dynamical recrystallization during grain size change in the direction of axis \( i \) from \( d_0 \) to \( d_i \).

\[ \frac{d\Delta w_i}{dt} = -[a_0 + a_1 \dot{\varepsilon}^w] \left(1 - \frac{f_w^w}{f_p^w} \right)^2 \frac{\Delta w_i}{v_i} \]

\[ \text{where unknown coefficients } a_0, a_1, \text{ and } a_2 = \frac{1 - f_w^w}{f_p^w} \left( \dot{\varepsilon}^w \right)^2 \text{ are found experimentally with the help of dynamical recrystallization diagrams; } T, T_c \text{ are current temperature and some reference temperature found experimentally.} \]

To define coefficients of connection between linkage parameters of movement of polycrystalline medium and its microstructure let's state functional relations between field parameters describing grain boundaries migration and microstructure geometrical parameters which can be found by means of optical metallography. Let’s put 12 coefficients of connection with the form \( \Delta w_i = \Theta(v_i) : f_0 = \frac{\Delta w_i}{v_i} \) - 3 coefficients,

\[ f_{\alpha_i} = f_{\alpha_{ij}} \frac{v_i}{v_j} + f_{\alpha_{ji}} \]

where

\[ f_{\alpha_{ij}} = f_{\alpha_{ij}}(\Delta w_{ij} - \frac{v_i}{v_j}), \quad f_{\alpha_{ji}} = f_{\alpha_{ji}}(\Delta w_{ji} - \frac{v_i}{v_j}) \]

Let’s introduce some functions \( \psi_i, \psi_{ij} \), such that the following will be performed:

\[ \frac{\Delta w_{ij}}{v_j} = \frac{\psi_i}{v_j} + \psi_{ij}, \quad \frac{\Delta w_{ji}}{v_i} = \frac{\psi_i}{v_i} + \psi_{ij}, \quad \text{then if } f_{\alpha_i} = f_{\alpha_{ij}}, \text{ the following relations are true:} \]

\[ \dot{\varepsilon}_{\alpha_i} = f_{\alpha_{ij}}(\dot{\varepsilon}_{\alpha_j} + \psi_i), \quad \dot{\varepsilon}_{\alpha_{ij}} = f_{\alpha_{ij}} \left(1 + \frac{\psi_i}{v_j} \right) \dot{\varepsilon}_{\alpha_{ij}}, \quad \psi_{ij} = v_i \left[ f_{\alpha_{ij}}(1 + \frac{v_i}{v_j}) - 1 \right] \]

If, for example, we put \( \psi_{ij} = 0 \), which is equal to \( \psi_{ij} = 0 \), then we will obtain:

\[ \dot{\varepsilon}_{\alpha_i} = f_{\alpha_{ij}} \dot{\varepsilon}_{\alpha_j}, \quad \dot{\varepsilon}_{\alpha_{ij}} = f_{\alpha_{ij}} \dot{\varepsilon}_{\alpha_{ij}} \]

Physical meaning of the value \( \psi_i \) is illustrated by Figure-1, where \( f_{\alpha_i} = \text{deformation } \dot{\varepsilon}_{\alpha_i} \) rate step due to grain division after reaching critical value \( \left( \dot{\varepsilon}_{\alpha_i} \right)_{cr} \).

Approximating (fitting) curve is defined by the relation:

\[ \dot{\varepsilon}_{\alpha_i} = f_{\alpha_i} k_{\alpha_i} \dot{\varepsilon}_{\alpha_{ij}}, \]

\[ \text{where } k_{\alpha_i} \text{ is a coefficient of grain boundaries structure sensitivity towards deformation intensity. Let’s define coefficients } k_{\alpha_i} \text{ in such a way that they could be found via grain size change in three projections.} \]
Let it be

\[ k_{ii} = (1 + \text{sgn}(\epsilon_{ii}))(k_{ii}^f - \dot{\epsilon}_{ji}^f), \]

\[ k_{ji} = (1 + \text{sgn}(\epsilon_{ji}))(k_{ji}^f - \dot{\epsilon}_{ji}^f), \]

then for 3-dimensional deformation we will have:

\[ k_{ii}^{\text{new}} = \begin{pmatrix} 0 & k_{ji} & k_{ij} \\ k_{ji} & 0 & k_{jj} \\ k_{ij} & k_{jj} & 0 \end{pmatrix} \]

- coefficient matrix,

\[ k_{ii}^{\text{new}} = \frac{k_{ii} - \dot{\epsilon}_{ii}^f}{\dot{\epsilon}_{ii}^{\text{true}}} = \frac{k_{ii} - \dot{\epsilon}_{ij}^f}{\dot{\epsilon}_{ii}^{\text{true}}} \]

- coefficient connection equations.

For plane deformation state:

\[ \dot{\epsilon}_{ii}^w = f_0 k_{ii} \dot{\epsilon}_{ii}^w = f_0 \left(1 + \text{sgn}(\dot{\epsilon}_{ii}^w) \right) k_{ii} \dot{\epsilon}_{ii}^w \]

(22)

\[ \mu_\Sigma \nabla^2 v_i + \sigma_j^w - \mu_\Omega \alpha_m \nabla T_{ji} + F_{\Omega f} - \rho(1 - f_\rho) v_i = 0, \]

(24)

where \( f_{ij} = f_{ij} \frac{\psi_j(x_i)}{v_j} \) and according to (18), (20)

\[ \psi_j = \left( \frac{f_{ij} k_{ii} - 1}{f_{ij}} \right) \frac{\dot{\epsilon}_{ii}^w}{v_i}. \]

With some approximation we can accept that average pressure is equal for both constituents of the medium \( \sigma_j^w \approx \sigma_j^w = \sigma_j \), then (24) will have the following form:

\[ \mu_\Sigma \nabla^2 v_i + \sigma_j^w - \mu_\Omega \alpha_m \nabla T_{ji} + F_{\Omega f} - \rho(1 - f_\rho) v_i = 0, \]

(25)

Thus, 6 equations (25) connect 3 unknown constituents of the medium flow rate \( v_i \), 3 unknown values of hydrostatic pressure gradient \( \sigma_j \), 3 constituents of structure formation internal force \( F_{\Omega f} = b_j [f_{\rho f} f_{ij} - (1 - f_\rho)] v_i, b_j = (-1) \frac{(c_\rho - c_\Omega) \Delta T_s}{(f_\rho \cdot f_{ij})}, \]

and, if ignoring heat passage from local volume for high
rates of deformation, law of transformation of plastic
deformation energy into heat energy \( T_s = k_T^v \dot{\varepsilon}_s^v \).

Calculations and experimental part

The most difficult task in finding invariables
appearing in the motion equations (25) is to find a
coefficient of shear modulus for grain boundaries \( \mu_{w}^v \). As
an example let’s take hot deformation of titanium alloy
BT9. Value \( \mu_{w}^v \) for BT9 is estimated on the basis of
coefficients found according to the dynamical
recrystallization model in the form (16) [9]. The
dynamical recrystallization model according to (16) is
visually presented in Figure-2. Models were obtained
under a priori accepted values of coefficients \( n=1, a_0=0.001. \) Calculated coefficients and statistics corresponding
to them are shown in Table-1.

![Figure-2. Dynamical recrystallization surface (mathematic model visualization) and experimentally obtained points for hot slump of titanium alloy BT9.](image)

Table-1. Design coefficients of the dynamical recrystallization mathematic model.

| Titanium alloy BT9. \( a_3 = 1 \), \( a_{re} = 0.9 \cdot 10^{-5}, \frac{Q}{R}=38.7 \cdot 10^3 \) | K, initial grain size - \( d_0 = 20 \mu m \). The model’s confidence level - 90 % |
|---|---|---|
| \( a_1 \) | \( 0.141 \) | \( 0.00097 \) | \( 145.541 \) |
| \( a_2 \) | \( 0.565 \) | \( 0.09272 \) | \( 6.095 \) |
| \( T_1 \) | \( 1191.465 \) | \( 16.4696 \) | \( 55.767 \) |

Value of shear modulus \( \mu_{w}^v \) for the continual
constituent can be found in reference data or in
experimental data. In our case deformation proceeds under
high temperatures \( T_0=1223K, \) when yield limit is
determined by temperature factor, in this case it is
\( \mu_{w}^v = \frac{\sigma_y}{3} = 18.3MPa \) [10].

Using value of \( \mu_{w}^v \), and also Hall-Petch law [11-
13], let’s estimate the shear modulus \( \mu_{w}^v \) for the grain
boundaries constituent.

Plastic deformation local power is estimated via
expression \( (1 - f_p^w)\sigma_i^v \varepsilon_i^v + f_p^w \sigma_i^w \varepsilon_i^w \), and usually during
the process of deformation, under conditions of all-around
non-uniform compression, grain is atomized. Even during
large deformation, grain is atomized to a certain limit
under permanent increase of plastic deformation power;
therefore, for local volume at grain boundaries the
following inequality is true:

\[
(1 - f_p^w)\sigma_i^v \varepsilon_i^v \geq f_p^w \sigma_i^w \varepsilon_i^w \\
(1 - f_p^w) \mu_{w}^v \cdot (\dot{\varepsilon}_2^w)^2 \geq f_p^w \mu_{w}^v \cdot (\dot{\varepsilon}_2^w)^2,
\]

from which we obtain an upper estimate:

\[
\mu_{w}^v \leq \mu_{w}^v \frac{1 - f_p^w}{f_p^w} \left( \frac{\dot{\varepsilon}_i^v(v_i)}{\dot{\varepsilon}_2^w(v_i, f_p)} \right)^2
\]

(24)

Let’s estimate the ratio \( \frac{\dot{\varepsilon}_2^w}{\dot{\varepsilon}_2^v} \) in (25). For this purpose, to
simplify computations, let’s take a plastically deformed
state for which \( v_3 \equiv 0, \dot{\varepsilon}_{11} = -\dot{\varepsilon}_{22}, \) and besides, according
to (22) the following is performed:

\[
\dot{\varepsilon}_{ij}^v = f_0 k_{e1} \dot{\varepsilon}_{ij}^v,
\]

(25)

where \( k_{e1} = 1 + k, \) \( k_{e2} = 1 - k \)

Accordingly, for intensities of deformation rates
of continual and grain boundaries constituents we have the
following relations:
\[
\varepsilon_2^v = \frac{2}{\sqrt{3}} \left( (\varepsilon_{11}^v)^2 + 2(\varepsilon_{12}^v)^2 \right)^{1/2}, \quad (26)
\]

Then the upper estimate (24) of shear modulus \( \mu^w \) in this case will be as follows:

\[
\mu^w < \frac{\mu^v_{\infty}}{2} \left( 1 - \frac{f^w}{f_0^w} \right)^2 \quad (28)
\]

Sizes of an \( \alpha \)-phase of a grain, obtained under high-speed extrusion of workpieces of blades made from titanium alloy BT9 [14] (Table-2):

Table-2. Parameters of microstructure of titanium alloy BT9.

<table>
<thead>
<tr>
<th>Coefficient ( f^w_0 )</th>
<th>Coefficient ( f_0 )</th>
<th>Size of ( \alpha )-phase in initial grain, ( \mu m )</th>
<th>Size of ( \alpha )-phase in grain after deformation, ( \mu m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.92761</td>
<td>0.1377</td>
<td>4.2</td>
<td>2.85</td>
</tr>
</tbody>
</table>

Using (28), we obtain the stiffness factor upper estimate

\[
\mu^w_{\infty} < 2,058 \mu^v_{\infty} \quad (29)
\]

It is known that the classical Hall-Petch law describes relation between yield limit \( \sigma^w_\infty \) and average size of grain \( d \) of polycrystalline material:

\[
\sigma^w_\infty = \sigma_0 + K d^{-1/2}, \quad (30)
\]

where \( \sigma_0 \) is a certain friction stress necessary for glide of dislocations in a monocrystal, and \( K \) is a material constant often called “a Hall-Petch coefficient”. This law is well-fulfilled for polycrystals, grains of which are of size more than 1 \( \mu m \), and this good correspondence usually preserves up to a very small grain of size nearly 100 nanometers [13].

Using known Tabor’s law [16] \( \sigma = CH \), which states proportional relations between microhardness \( H \) and yield limit \( \sigma_\infty \) with proportionality coefficient \( C \) (\( C=0.4...0.6 \)), let’s estimate, in accordance with experimental data obtained for titanium in the work [17], value of Hall-Petch coefficient:

\[
K = (5...7.8) \sigma_\infty, \text{ MPa} \cdot \mu \text{m}^{1/2}, \quad \square \sigma_\infty \text{ is a microhardness increment according to the data of [17].}
\]

Therefore, if we know the estimation of \( \sigma^w_\infty |_{d=d_0} = \sigma_0 \) under grain size \( d_0 \), then we have:

\[
\sigma^w_\infty = \sigma_0 + K (d^{-1/2} - d_0^{-1/2})
\]

Because \( \sigma^w_\infty \), given due to Hall-Petch correlation, determines lower value leading to yield, then the estimation of \( \mu^w_{\infty} \) below is valid:

\[
\mu^w_{\infty} > \frac{1}{3} (\sigma_0 + K (d^{-1/2} - d_0^{-1/2})) \quad (31)
\]

On the basis of (29) and (31), we obtain boundaries for estimation of \( \mu^w_{\infty} \)

\[
(1,522...1,814) \mu^w_{\infty} < \mu^w_{\infty} < 2,058 \mu^v_{\infty} \quad (32)
\]

Values of coefficients \( \nu^w_\Omega \) and \( \nu^w_\Omega \) can be calculated using known data (Table-3).

Table-3. Thermophysical data for titanium alloy BT9 [18].

<table>
<thead>
<tr>
<th>( C_p, \text{ J/kg K} )</th>
<th>( \alpha_m, \text{ K}^{-1} )</th>
<th>( Q/R, \text{ K} )</th>
<th>( a_T, \text{ [Aut.]} )</th>
<th>( T_b, \text{ K} )</th>
<th>( \rho, \text{ kg/m}^3 )</th>
<th>( m_y, \text{ g/mole} )</th>
<th>( \nu^w_\Omega ), \text{ MPa/K} (11)</th>
<th>( \nu^w_\Omega ), \text{ MPa/K} (11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T=273K )</td>
<td>( T=1223K )</td>
<td>( T=273K )</td>
<td>( T=1223K )</td>
<td>( 530.8 )</td>
<td>( 633.9 )</td>
<td>( 8E-6 )</td>
<td>( 1E-5 )</td>
<td>( 3.87E+4 )</td>
</tr>
</tbody>
</table>
According to the Table data, increment of internal stress in grain border area due to deformation thermal effect will be correspondingly 13.9 and 2.49 Mpa for each 1 K of temperature increment.

CONCLUSIONS
a) We suggested a modelling method and obtained relations of state of a polycrystalline body under severe plastic deformation in the temperature range outside the phase changes area, where a body is a heterogeneous medium consisting of two interacting constituents - a continual medium and a structure of grains boundaries imbedded into this medium.

b) Equations of medium state, expressed via medium’s flow rate and grain boundaries migration rate, include per-unit force of interaction of continual medium and grain boundaries, which is proportional to the rate of relative displacement of grain boundaries, and this force is determined by invariables which can be found in the dynamical recrystallization diagram.

c) On the basis of microstructure change during dynamical recrystallization (change of size of a grain or a subgrain) we can estimate parameters of the stress-strain state if we know a 2-level model including:

- defining relations between stress fields and deformations (deformation rates) and, correspondingly, the deformation hardening law;
- dependence of parameters of microstructure (size of a grain or a subgrain) on the specified field functions.

REFERENCES


Symbols

\( V_l \) - continual medium motion speed, \( l = i, j, k \);

\( \square W_l \) - speed of relative motion (migration) of polycrystalline medium grain boundaries;

\( t \) - current time;

\( A \) - surface area;

\( \Omega \) - volume;

\( f_p \) - volume coefficient of grain boundaries structure amount;

\( f_p^w \) - value of \( f_p \) in local domain at grain boundaries;

\( [k_{ij}] \) - matrix of coefficients of sensitivity of deformed medium grain boundaries constituent’s deformation to the stress-strain state form;

\( \dot{f}_{ij} = \frac{\square W_{i,j}}{V_{i,j}} \) - relative variation of speed of grain boundaries migration in \( j \) axis direction;

\( \alpha_m^w \) - line expansion coefficient;

\( \alpha_m^w \) - coefficient of thermal migration of grain boundaries;

\( \varepsilon_{ij}^v, \varepsilon_{ij}^w \) - components of tensor of strain of continual and grain boundaries constituents of deformed medium;

\( \dot{\varepsilon}_{ij}^v, \dot{\varepsilon}_{ij}^w \) - components of tensor of rate of strain of continual and grain boundaries constituents of deformed medium;

\( \sigma_{ij}^v, \sigma_{ij}^w \) - components of tensor of stress of continual and grain boundaries constituents of deformed medium;

\( F_\Omega \) - per-unit-volume power used for changing size and shape of grain;

\( U \) - internal energy;

\( F \) - Helmholtz free energy;

\( S \) - entropy;

\( c \) - thermal capacity;

\( c_\Omega \) - isochoric thermal capacity;

\( c_\sigma \) - thermal capacity at constant pressure;

\( \lambda \) - thermal conductivity;

\( q_i \) [\( J/m^2c \)] - coordinates of heat flow vector;

\( k_f^T \) [K] - coefficient of plastic deformation energy conversion into heat energy;

\( T \) [K] - temperature;

\( T_0 \) [K] - forging temperature.